EUCLIDEAN ECONOMICS

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"There is no royal road to geometry"

The General Method for Construction and Application of Economic Models

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Summary

The General Method for Construction and Application of Economic Models is the revised and updated version of ***Euclidean Economics*** published in Greek by the University of Cyprus in November 2006. The monograph establishes that the science of economics can be studied and utilised on the basis of a minimum of fundamental hypotheses and the general laws of mathematics. In order to reveal the strength of the Euclidean approach, references and bibliography are intentionally excluded. In this conjunction, in-depth knowledge of economics and extensive experience with mathematics are essential. Economists should be conversant with linear algebra and differential equations while mathematicians should be conversant with advanced micro and macro economics.

Starting from twelve major propositions, it is illustrated that closed economies are replicated by systems of 2n first-order non-homogeneous linear differential equations, which simulate household, business, financier and government receipts and payments. Such exchange networks, called Euclidean, are defined in the (R^{2n}, t) vector space, where n is the number of institutional sectors and corresponding markets or sovereign countries and corresponding currencies, while t is the elapsed time. As a result, monetary flows and financial stocks do not suffer from aggregation problems; they are perfectly distributed among sectors and markets as well as countries and currencies.

Euclidean economic models reproduce takings and outgoings as functions of behaviour coefficients and policy parameters per relevant period. Elements of irrationality or idiosyncrasy that often characterise agents are reflected by the coefficients of behaviour and the stickiness of quotations. Division of annual turnovers of services, products and securities, respectively, by average salaries, prices and values, yields the demanded years of labour, pieces of output and units of securities. Quantities demanded only fortuitously coincide with quantities available. The potential inflation or deflation rate is proportional to the weighted deficit or surplus of market goods.

The equilibrium vector's uniqueness, stability and controllability are functions of i) sector, country or zone behaviour coefficients and ii) monetary, fiscal and other policy parameters. Modern software enables examination of the economic system's steady state, dynamic stability and controllability properties under various scenarios. Among other advantages, the methodology makes possible i) composition of mathematical models which

mirror economic reality and ii) solution of related problems on the principles of simplicity, uniformity and generality.

Such exchange networks yield sector and market as well as country and currency monetary flows, balance sheets, quantity gaps, etc, as functions of the exogenous variables. The endogenous variables may be digitally animated according to realistic or imaginary scenarios about the autonomous parameters. Computer simulation opens new avenues for dialogue among parties and the teaching of economics at all levels. The monograph is intended for academic and professional economists as well as for undergraduate and postgraduate physicists, engineers and other scientists engaged in simulation theory and practice. It is relevant to financial and government organisations interested in improving their forecasting capabilities through all-inclusive economic models.

Biographical Note

Dr Sophocles Michaelides was educated in the US under a Fulbright Scholarship. He earned his BA from Wake Forest University, North Carolina, and his MA and PhD from Cornell University, New York. Dr Michaelides served as assistant professor of economics at the University of the Philippines and at Eastern Michigan University. Upon returning to his native Cyprus, he joined the Central Bank as economic research officer. During his career at the Central Bank, Dr Michaelides headed a number of departments and represented the Bank on a number of intergovernmental and European Union committees.

Between 1979 and 1982, he was responsible for the investment of Cyprus' international reserves and the management of the island's foreign debt. From 1983 to 1999, he was in charge of the Central Bank's Offshore Business Department, coordinating Cyprus' development as an international business and financial centre. As Director of Exchange Control and Foreign Investments, he contributed significantly to liberalization of capital flows in line with EU requirements.

In 2002, Dr Michaelides became Director of the Statistics Department where he supervised implementation of European Central Bank regulations and standards regarding electronic collection and dissemination of monetary financial statistics. Between 2004 and 2006, he headed the Economic Research Department where, among other things, he oversaw monetary analysis and policy. At this time, his book ''EUCLIDEAN ECONOMICS: *The General Method for Construction and Application of Economic Models*" was published by the University of Cyprus. He participated in numerous international conferences and lectured on economic affairs and business matters in Cyprus and overseas. In 2006, he was promoted to Senior Director.

Following his retirement in 2010, he served as the managing director of a financial services company and has been active as non-executive director as well as consultant and author in the fields of economics and finance. During the financial crisis in 2013 he was elected Chairman of the Bank of Cyprus Interim Board of Directors to carry through recapitalisation with own resources.

THE GENERAL METHOD FOR CONSTRUCTION AND APPLICATION OF ECONOMIC MODELS

by Dr Sophocles Michaelides

Part II Elementary Examples and Applications

SYNOPSIS: A mathematical framework for solution of economic problems

By Dr. Sophocles Michaelides

Euclidean Economics, published in Greek by the University of Cyprus in 2006, and its English translation subtitled ***The General Method for Construction and Application of Economic Models*** demonstrate that market systems of any dimension and complexity can be replicated and utilized on the basis of a minimum of fundamental hypotheses and the general laws of mathematics. Starting from twelve major propositions, it is shown that a closed system of *2n* first-order non-homogeneous linear differential equations simulates demand for and supply of *n* services, products, securities, etc., traded by *n* institutional sectors (grouped under sovereign countries and currency zones).

The approach is very versatile and can be summarised as follows:

1) Every sector is characterized by the sources of its receipts (assets) and the destinations of its payments (liabilities). Each sector determines the distribution of its outlay subject to its budget constraint. The magnitude and the division of each sector's revenue are determined by all other sectors. Attitudes and capabilities of natural and legal persons – reflected by the proportions of incomings and outgoings – are among the determinants of sector - market and thus country - zone interactions per time unit.

2) Households, enterprises, financiers, governments, etc., receive bank deposits from other sectors and pay bank deposits to them for services, products, securities, taxes, subsidies, etc., according to each one's resources and requirements. Monetary financial institutions record economic activity as debits and credits. Due to the linear form of balance sheets and the twelve fundamental hypotheses, a closed system's monetary flows and financial stocks are perfectly allocated among sectors and markets as well as countries and currencies.

3) Monetary financial institutions comprise the only sector entitled to expand its balance sheet at the stroke of a pen or keyboard within limits set by public policy. All other sectors are constrained to operating with primary surpluses, i.e. current payments less than current receipts, so as to avoid bankruptcy, a terminal condition in the economic game.

4) The global banking balance sheet is the sum of the balance sheets of national and supranational monetary institutions, expressed in international currency. Countries are

connected by the global banking balance sheet that is indirectly controlled by the financial powers and their representative organizations. Banks – through simultaneous generation of loans and deposits – redistribute takings and outgoings of sectors and countries worldwide.

5) For convenience and comprehension purposes, the calendar year has been adopted as the time unit. Within this period, salaries, prices, values, etc., change autonomously because of relentless interference by trade unions, company boards, government bodies and international organisations. As a result, demanded quantities only fortuitously coincide with available quantities. The annual turnover of any service, product or security divided by its average quotation yields the man years, output pieces and security units traded.

6) Gaps between demanded and accessible labour, output, liquidity, etc., are functions of average quotations as well as of magnitudes and allotments of sector demand and bank credit. Potential inflation (deflation) is analogous to the weighted scarcity (plethora) of market goods. Dynamic stability is a function of behaviour coefficients and policy parameters worldwide. When sales and purchases converge, monetary flows, financial stocks and the corresponding quantities move toward their steady states. Under certain conditions, a market system may not possess a unique, stable and controllable steady state.

7) National monetary tools comprise the annual expansion, sector allocation and relative cost (remuneration) of loans (deposits) denominated in the home currency. National fiscal tools include the withholding tax and subsidisation rates applied to traded goods by the home government. Due to convertibility of deposits and loans – by means of exchange rates, trade arrangements and payment systems – all economies influence each other. The external impact of internal policies is a function of the relative size of the initiating country or currency zone.

Every economic model constructed according to the Euclidean approach incorporates both sides of each transaction and thus does not suffer from aggregation problems. The *2n* first order non-homogeneous linear differential equations, which simulate demand for and supply of *n* services, products, securities, etc., traded by *n* institutional sectors, comprise a vector space; therefore, they can be solved as a mathematical system in order to evaluate, for example, *n* policy parameters that would bridge *n* sector-market or country-zone deficits and surpluses. The methodology allows classification and enumeration of all relevant factors and thus examination of policy and related issues with simplicity, uniformity and generality. Within this framework, (endogenous) sector, market, country and currency flows as well as balance sheets, quantity gaps, inflation rates, etc., are functions of the following six sets of (exogenous) variables:

- The institutional sectors (markets) and sovereign countries (zones) which comprise the complete system;
- The available real and financial resources, including years of services, amounts of products, units of securities, etc.;
- The prevailing quotations regarding man years, output pieces, security units etc., (expressed in international currency).
- The percentages of each sector's (or country's) annual revenue from services, products, securities, etc., worldwide;
- The percentages of each sector's (or country's) annual outlay on labour, output, securities, etc., worldwide;
- The parameters of monetary, fiscal, exchange and other policies implemented (or contemplated) by the interacting countries (or zones).

No sector, market, country or currency is an island. When the desired level of analysis is high, international statistics covering the six groups of autonomous factors are needed for each year of assessment. The endogenous variables may be digitally animated on the basis of historical, projected or hypothetical data about the exogenous variables. Advanced mathematical software enables exploration of the network's dynamic stability, steady state and controllability properties under a variety of scenarios. Computer simulation enhances dialogue among citizens, politicians, academics, journalists etc.

Some of the most interesting inferences from this novel approach are presented below:

1) If yearly salaries, product quotations, bond values, administered prices, etc., were continuously shaped by supply and demand, the corresponding markets would clear straight away. In such case, there would be no economic but other kinds of issues to worry about. Inflexible quotations and their consequences lead to calls for government intervention. As a matter of priority, national authorities should enact legislation that encourages free competition and market play.

2) Observed surpluses (or shortages) and potential deflation (or inflation) are two sides of the same phenomenon. Until structural measures take effect, there might be social and

political discontent. The usual response is to adjust i) the expansion and allocation of bank credit, ii) the sources and destinations of public funds as well as iii) the pricing and timing of capital transactions with the hope of ameliorating quantity gaps and quotation trends. Whenever an exogenous parameter is changed, all incomings and outgoings vary simultaneously (but unevenly) in accordance with their interlinked relationships.

3) The financial statements of banking institutions around the world – combined by means of exchange and interest rates – form the universal monetary balance sheet. Natural growth of resources and downward inflexibility of quotations necessitate yearly expansion of global credit. Demand for and supply of labour, output, securities, etc., depend on the latter's volume, apportionment and pricing.

4) Incomes and expenditures are linear functions of the universal liquidity increment's magnitude and division. It is shown that without outside money, sector-market (and thus country-zone) interaction would be a homogeneous linear equation system, which would possess only a trivial solution (i.e. the null vector). Unceasing magnification and alteration of the worldwide banking balance sheet is a leading cause of domestic and international economic fluctuations.

5) Any unilateral attempt to control the time paths, sector portions, purchasing values, etc., of a nation's loans and deposits is faced with serious obstacles because of unavoidable reaction by other sovereign countries and currency areas. Balanced financial portfolios – and thus harmonious economic relations – depend upon coordination of liquidity growth, annual discount and exchange rate policies. Hence, national monetary and fiscal institutions should heed the advice of supranational organizations, such as the IMF, the ECB, etc.

6) Domestic and worldwide authorities should not modify monetary, fiscal and exchange parameters arbitrarily because gross turnovers and quantity mismatches would change haphazardly. Central banks and sovereign governments may guide the global steady state to their preferred position when: i) they coordinate their decisions via a mathematical economic model, ii) the network's endogenous flows converge to a unique equilibrium vector and iii) the policy tools match the policy targets.

7) Given sufficient instruments, the authorities may improve the steady state as long as the economic system has free resources and remains dynamically dirigible. The latter implies that during monetary, fiscal and exchange fine-tuning, behaviour coefficients and

policy parameters stay within the ranges where market transactions tend to converge. In this case, the mathematical model can be solved (with the help of advanced software) for the credit allocation shares, withholding tax percentages, foreign exchange rates, etc., which abolish unbearable quantity gaps and intolerable quotation trends during the ensuing period. It goes without saying that it is easier to estimate than to implement the policy parameters that clear all markets.

Euclidean methodology is appropriate for examining all kinds of economic issues, including i) management of risks threatening monetary institutions, ii) impact of discount and exchange rates on balance sheets, iii) distribution of gains and losses after currency unification, etc. Provided they avail of the international system's statistical data and mathematical model, central banks, sovereign governments and supranational organizations may weigh the effects of alternative policies beforehand and take pre-emptive action as befits each case. Financial coordination among important players is absolutely essential for smooth functioning of domestic and international networks. Without such unifying influence, competitive wage, price and value spirals would be everyday phenomena. In particular, when considering the monetary financial increment's yearly magnitude and global distribution, banking authorities should pay attention to national capacities for employment and production. Inflation and deflation rates are simply mirror images of demand and supply gaps. Planning and executing economic policy without a suitable mathematical model and the relevant worldwide data resembles:

- sailing to remote places without a map, a compass and a sextant, or
- launching a manned satellite without knowing its orbit in advance.

The original Greek version of Euclidean Economics (in electronic form) is available at:

http://ucyweb.ucy.ac.cy/pek/index.php?option=com_content&view=article&id=82:euklidia-oikonm&catid=19:mediterranean-publications-gr

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Part I. Micro Foundations and Clarifications

Chapter 1. Introduction to the Euclidean Approach

Euclid was born in 325 B.C. and lived in Alexandria during the reign of Ptolemy I, who was the founder of both the eponymous dynasty and the greatest library of the Hellenistic world. Interestingly, knowledge regarding geometric phenomena had accumulated long before Euclid. The pyramids, for instance, with their astonishing relations to both terrestrial and heavenly objects, had been built more than two thousand years earlier. However, prior to Euclid, there was no logical theory linking the various types of lines, shapes or structures. Euclid was the first to demonstrate with simplicity, uniformity and generality that all geometric phenomena can be related and treated mathematically according to five axioms based on the abstract ideas of 'point', 'line' and 'circle'. In this monograph we follow Euclid's methodology. We endeavour to discover whether the spectrum of measurable economic phenomena can also be explained through a minimum of fundamental hypotheses.

We commence our approach from the ordinary concepts of 'economic agent', 'budget constraint' and 'market behaviour', which are analogous to the Euclidean ideas of 'point', 'line' and 'circle'. From the outset, we take for granted that economic agents can be classified under a finite number of institutional sectors. Institutional sectors are grouped under sovereign countries and currency zones. Each resident or non-resident sector (i.e. consumer households, productive businesses, financial institutions, government organisations, etc.) demands monetary income by selling a characteristic good and supplies monetary expenditure by buying other market goods (e.g. services, products and securities). While engaging in transactions, institutional sectors:

- combine rational optimisation, instinctive self-preservation and pursuit of power;
- distribute their receipts among possible sources and allocate their payments among pertinent destinations in accordance with their characteristic behaviour;
- face budget restrictions regarding their annual as well as lifetime income and expenditure.

In Chapter **2** we explain and justify twelve major propositions, which are assumed to govern the conduct of sectors and markets. The behaviour of institutional sectors, the rules of market exchange, the mathematical specification of functions and, most importantly, the acquisition of assets and the issue of liabilities by monetary financial institutions (banks) are duly elaborated. The twelve propositions are permeated by the Euclidean ideas of point, line and circle; consequently, analysis and synthesis of transactions as well as construction and application of models based on them are subject to the laws of geometry and, therefore, mathematics. The premise that an agent's or sector's income and expenditure are simultaneously determined as a vector (i.e. an ordered configuration of numbers), for the first time enables academics and professionals to address classic economic problems and current economic issues using a simple, uniform and general modelling methodology that incorporates the fundamental rule of accountancy, in addition to the aforementioned laws. This particular assumption also establishes correspondence between models of physical reality (where every action is accompanied by an equal and opposite reaction) and models of economic reality (where every credit is accompanied by an equal and opposite debit), and thus justifies similar mathematical treatment.

1.1. Details and effects of the propositions

Every discipline studies, from its own perspective, the interaction and direction of the variables constituting its subject matter. Economics investigates the course of variables which obey the rules of sector behaviour, voluntary exchange and collective coercion. Human instincts – such as the profit motive (i.e. optimisation of assets and liabilities), the retentive complex (i.e. preservation of exogenous wealth) and the pursuit of power (i.e. promotion of self-serving policies) – are determinants of such rules. Over the years, numerous theories have been developed in order to explain how institutional sectors and market platforms operate and influence each other. Euclidean approach to economic modelling is extensively influenced by neoclassical theory. All neoclassical hypotheses are adopted except 'non-measurable consumer utility' and 'automatic price adjustment'. The aforementioned are replaced by the concepts of 'measurable monetary flow' and 'monetary flow adjustment', which are fundamental as far as our simulation methodology is concerned. While developing the elementary model, which is the building block for larger structures, we also assume that:

- banking institutions accept and issue guaranteed consols (deposits),
- economic agents, institutional sectors, etc., are mostly myopic, and
- monetary flows are perfectly convertible into capital and vice versa.

Future receipts and payments should be probability weighted before mathematical operations. Provided this is done properly, Euclidean economic models may include all kinds of financial instruments or contracts. Successive financial crises since 2007 have confirmed that most banking institutions, hedge funds, other investors as well as their auditors have tended to disregard the expected value principle when accounting for assets and liabilities. This imprudent practice has hindered responsible economic management by distorting global financial statistics. Given appropriate sets of external parameters, Euclidean models generate the steady state, dynamic stability and controllability conditions, which enable knowledgeable citizens and government authorities to improve their expectations regarding the future. As to the capacity to predict developments without scientific assistance, most people can barely say what they will have for supper or estimate their own net worth, let alone foretell the next blue moon. To the extent necessary, we introduce supplementary assumptions which leave the fundamental structure of the economy untouched but facilitate mathematical representation of a problem at hand. Secondary hypotheses may be modified or replaced by other well-justified postulates in order to study the resulting differences. As will be shown, the same mathematical form (i.e. a system of first-order non-homogeneous linear differential equations) duplicates all circulation networks which abide by the twelve propositions.

Chapter **3** demonstrates that a pair of mathematical functions generates demand for revenue and supply of outlay by each agent, sector, country, etc., as they engage in economic exchange. Consumer households, business enterprises, financial institutions, government organisations, etc., receive (bank) money from and pay (bank) money to other institutional sectors for services, products, securities, taxes, subsidies, etc., according to each one's resources and requirements. It can be shown that sector and market interaction without outside money would be a homogeneous linear equation system, which would possess only a trivial solution (i.e. the null vector). Exchange networks and their mathematical models acquire unique and non-singular solutions because monetary institutions issue financial liabilities, which the other sectors have to accumulate as financial assets. Differentiation of the banking balance sheet is one of the most important factors determining nominal magnitudes of an economy per annum. With regard to public policy, it is noted that egotistical money-accepting agents, such as consumers and producers, are:

- neither capable of acting collectively so as to exact delivery of optimum expansion and allocation of bank credit;
- nor capable of generating the aggregate demand that would optimise employment of available resources per year.

Households and businesses often become trapped in counter-productive combinations of revenue and outlay (in relation to the available quantities of goods) because of the blind pursuit of their own psychological motives and pecuniary interests. Market interaction among consumers, producers, bankers and politicians is driven by a combination of rational optimisation, preservation of wealth and desire for power and, thus, cannot be distinguished from a game of vital interests. In this framework, the electorate's representatives would not pursue full employment policies, unless they stood to gain greater financial, political and social strength.

1.2. Accounting aspects of the methodology

In an elementary Euclidean economy, exemplified in Chapters **4** and **5**, the main policy tools are i) the annual increase of the banking system balance sheet and ii) the allocation of new bank liabilities as non-bank sector liquid assets. Monetary financial institutions borrow through the issue and lend through the acceptance of guaranteed bonds. The banking system enjoys the right to expand its obligations, which are generally accepted as means of payment and store of value. All the other sectors are obliged, but also willing, to accumulate the said bonds or liquid securities, indefinitely. Due to the above mentioned arrangements, monetary financial institutions are the net debtors while consumer households, productive businesses, etc., are the net creditors of the exchange network. Banks – through issue of loans or purchase of perpetual securities as well as through acceptance of deposits or sale of perpetual securities – redistribute the monetary financial balance sheet. In this way, they influence the income and expenditure of every institutional sector. For instance, when banks grant more loans to:

- households compared to businesses, the percentage of liquid deposits belonging to consumers increases;
- producers compared to households, the percentage of liquid deposits belonging to businesses increases.

Market exchange is associated with giving and taking, buying and selling, borrowing and lending, etc. Although it is recognised that income (credit) cannot exist without corresponding expenditure (debit), most econometric and DSGE models do not include both sides of each transaction. Their statistical or mathematical equations ignore causality, i.e. the relationship between an event (the *cause*) and a second event (the *effect*), where the second event is understood as a consequence of the first. During any relevant period – which in our methodology is the calendar year – differences between the revenue and outlay of non-bank sectors are determined by the amounts of new credit provided (or consols purchased) by monetary financial institutions. Conversely, payments and receipts (i.e. debits and credits) concerning any good or market are always in perfect balance. The twelve major propositions, by incorporating the fundamental double-entry rule in the construction and application of mathematical economic models, have eliminated the aggregation problem that hitherto troubled econometricians and other modellers, from both a theoretical and a practical point of view. The endogenous variables of Euclidean economic models are the annual incomes and expenditures of households, businesses, financiers, etc. Such exchange networks consist of n pairs of first-order non-homogeneous linear differential equations. The n pairs of sector flows form a V_{2xn} vector space. Due to its mathematical form, a Euclidean economic model may be depicted equivalently as:

- a set of equations, tables or matrices, which record sector and market revenue and outlay per relevant period of time;
- a moving or static polygon, the vertices of which are formed by the revenue and outlay pairs of institutional sectors;
- a dynamic space where sector receipts and payments change in proportion to their distances from the respective steady-state flows;
- a network of monetary flows that circulate according to pull and push pressures exerted by apportionment of incremental liquidity among sectors, countries, etc.

From the standpoint of debit and credit and, therefore, action and reaction, Euclidean economic models of sectors and markets may be compared with systems of interconnected body masses or energy flows. Bearing in mind that the laws of thermodynamics hold in (closed) physical systems, we will examine whether they also hold in (closed) economic systems. Chapter **11** includes a number of examples which highlight similarities between monetary flows of closed exchange networks and mass or energy movements of closed physical systems. The analogy between economic and physical systems deserves further investigation but lies outside the scope of this monograph. Replication of any exchange network as a set of circuits leads us to conclude that ‘‘*Every economic policy proposal should first be studied as a mathematical problem dealing with control and regulation of interdependent flows*’’.

1.3. Mixing theory and practice

The coefficients of a Euclidean model's linear differential equations emanate from the habit-dominated but variable allocation of sector income and sector expenditure among market goods. The percentages of annual revenue derived from and annual outlay spent on services, products, securities, taxes, subsidies, etc., by consumers, producers, bankers, governments, etc., may be estimated by means of yearly statistical surveys focused on i) the credits and debits of the respective bank accounts or on ii) the sources and destinations of the respective monetary flows. Over any recent period deemed sufficient, it is possible to calculate the percentages of income and expenditure derived from and spent on labour, output, bonds, taxes, etc., by institutional sectors. Due to *de facto* changing *milieu*, it cannot be taken for granted that households, businesses, financiers, officials, etc., will continue to engage in transactions according to the average behaviour of previous years. Euclidean simulation approach does not compel us to extrapolate the past into the future. Instead, we may assume any hypothetical scenarios (e.g. combinations of normal and extreme behaviour coefficients) deemed justified for examining the steady state, dynamic stability and controllability conditions of the exchange network during subsequent years.

Exogenous factors, such as liquidity-providing and liquidity-absorbing operations, alter the incomes and expenditures of institutional sectors and, therefore, their receipts from and payments on traded goods. Modifications of the monetary financial balance sheet generate new cycles of sector and market interactions, which may converge or diverge. When they converge, the steady-state turnovers of services, products or securities may be divided by the average salary, price or value to yield the years of labour, pieces of output or units of liquidity traded per annum. If economic agents could enter and exit the exchange network bringing in or taking out real resources as necessary, the related gaps would be eliminated without need for salaries, prices and values to fall or rise automatically. During any calendar year:

- the numbers of market participants and the quantities of market goods are limited by scarcity;
- the salaries of workers, the prices of products and the values of securities tend to be inflexible;
- changes in quotations may require union approval, board agreement or official permission.

In view of the above, quantities demanded only fortuitously coincide with quantities available. Markets cannot clear without intervention regarding the expansion and allocation of bank credit, the sources and destinations of government funds as well as the quotations of services, products, securities, etc., in terms of local and international currencies. Monetary, fiscal and other authorities possess the requisite tools for moving the circulation network's equilibrium from one position to another, provided the steady state consists of unique, stable and positive flows. As shown in Chapters **6**, **7** and **8**, when the economic system and its mathematical model incorporate monetary, fiscal and exchange policy instruments, the number of intervention tools rises so that it becomes equal to the number of endogenous variables. According to the theory of dynamic system control, equilibrium vectors possessing the aforementioned four attributes are perfectly dirigible.

Hitherto, econometric and other types of models were *sui generis* or unique creations. Henceforth, thanks to Euclidean methodology, exchange networks of any magnitude and complexity can be built and used according to the same underlying principles. The twelve premises (backed by appropriate software) enable solution of any economic problem through the simplicity, uniformity and generality of mathematics. After constructing the Euclidean model of a closed economic system, interested parties may assign historical, projected or invented values to the external parameters in order to predict the endogenous flows of institutional sectors, the quantity gaps of market goods as well as the potential deflation or inflation rates, domestically and worldwide, during any relevant year.

1.4. Mathematical qualities and advantages

An economic model's initial assumptions are as significant as the final model's mathematical properties. Due to the twelve premises, Euclidean exchange networks are structured as systems of first-order non-homogeneous linear differential equations. The behaviour coefficients reveal how households, businesses, financiers, etc., distribute revenue among possible sources and allocate outlay among applicable destinations per year. The matrix consisting of sector or country behaviour coefficients fully identifies the exchange or circulation network, just as an iris, a finger print or DNA helix fully identifies a human being. Furthermore, powerful mathematical software and modern computer technology facilitate analysis and synthesis of linear dynamic systems. For example, the monetary flows of sectors determine the gross turnover of markets. Proceeds are endogenous while quotations are exogenous. Therefore, division of every market good's steady-state turnover by the corresponding salary, price and value, yields the amount traded per annum. Demanded quantities may be compared to available quantities in order to draw conclusions about inflationary or deflationary pressures on sectors, countries and zones.

Differences between demanded q_i^* and available $q_i^\#$ amounts tend to trigger interventions for adjustment of i) market quotations p_i and ii) official policies during subsequent periods. Such reaction scenarios may be easily embodied in economic simulations spanning a number of years. Thus, given *t* successive sets of exogenous parameters, a Euclidean model is capable of yielding *t* steady-state vectors $\mathbf{X}^*$(income flow, expenditure flow, quantity traded)$_t$ consisting of nx3 elements, where n is the number of distinct sectors-markets-quantities which comprise the exchange network. The economic system's n endogenous triplets $\mathbf{X}^*(y_i^*, e_i^*, q_i^*)_t$ constitute a V_{3xn} vector space and, thus, comprise a polyhedron that is free of aggregation problems. Neither econometric nor DSGE models are defined as vector spaces. Their equations do not comprise closed systems which obey the rules of logic and analogy. Such models will continue to run into aggregation problems as long as they are constructed without regard to the laws of geometry or mathematics.

1.5. Data and statistical issues

Euclidean models suffer from a major weakness, i.e. shortage of relevant worldwide data. This problem needs to be solved so that specialists may effectively begin to annually:

- compare the economic structures of countries and zones;
- prepare forecasts according to probable exogenous parameters;
- contribute to dialogue among those shaping national and international policies.

Implementation of Euclidean methodology is facilitated by the fact that the European System of Accounts (ESA) divides economies into the following major and minor sectors:

1) Non-financial Organisations. This sector includes legal persons whose main activity is the production of market goods and non-financial services.

2) Financial Organisations which include:

i. Monetary Financial Institutions (MFIs)

(a) the central bank

(b) all other MFIs

ii. intermediary financial organisations

iii. ancillary financial agencies

iv. insurance business and pension funds

3) **General Government** which includes:

i. central administration

ii. state administration

iii. local administration

iv. social security funds

4) **Households**. This sector includes persons or groups of persons in their capacity as consumers and, eventually, as businessmen. Households as consumers are defined as small groups of persons who live under the same roof, merge part or all of their income and wealth and consume certain types of goods collectively, especially housing and food. The main resources of these units derive from income from work, income from property, transfers from other sectors and income from the disposal of market goods.

5) **Not-for-Profit Organisations which Serve Households.** This sector consists of non-profit organisations, which are separate legal entities serving households. Their main resources, apart from those which arise from occasional sales, derive from voluntary contributions in money or in kind by households in their capacity as consumers, from payments by the public sector and from property income.

The units making up each sector may belong to one of two residential categories:

A) **Residents:** This category consists of institutional units which interact on the territory of the country.

B) **Non-residents:** This category consists of non-residents who carry out transactions with residents. This category's accounts provide a comprehensive overview of the relations which link the domestic to the international economy. All institutional units, which fulfil the criteria under this category, are also classified **by country** based on the focus of their economic interest. The residential classification allows analysis of a country's transactions with different regions of the world, and in particular with:

a. member states of the EU and the Euro area;

b. third countries and international organisations.

National central banks of European Union (EU) member states submit to European Central Bank the flow and stock financial accounts of their institutional sectors on a quarterly basis. As a result, most of the requisite data is readily available in the EU. When non-EU countries start compiling their economic statistics according to ESA, the simplest economic model of an EU member state would consist of 30 first-order non-homogeneous linear differential equations, as follows:

- 5 pairs for the monetary flows of the 5 major sectors of the EU member state,
- 5 pairs for the monetary flows of the 5 major sectors of the rest of the EU,
- 5 pairs for the monetary flows of the 5 major sectors of the non-EU countries.

As soon as the economic models of EU member states are structured as vector spaces according to Euclidean methodology, the economic model of the EU will become the sum of the models of EU member states. Econometric and DSGE models are not built as vector spaces and, thus, are not subject to mathematical operations. With a view to highlighting the aforementioned property, Chapter **8** provides:

- a complete example of uniting the mathematical models of two sovereign countries or currency zones;
- a comparison of two economies before and after operating under common trade and payment systems.

From the mid-1930s up to the present day, tens of thousands of econometric and other models have been built and tested. It is only reasonable to ask whether the investment of time, money and other resources has yielded the anticipated returns. Whatever the answer to this rhetorical question, Euclidean modelling methodology appears very promising and thus deserves similar investment.

1.6. Utilization of Euclidean models

In this monograph we apply Euclidean methodology to all types of theoretical and practical issues for the purpose of assessing and proving its merit and scope. Such topics include:

- relationships among monetary flows, average quotations and available quantities;
- convergence by various countries with a view to adopting a common currency;
- discount and exchange rate impacts on the balance sheets of institutional sectors;
- management of various risks threatening monetary financial institutions;
- replication of economic history and growth through automated calculations;
- advantages and disadvantages of monetary, fiscal, exchange and other policies;

- gains and losses of sectors, markets, countries and zones following globalisation.

Mathematical economic models built in accordance with the twelve major propositions confirm classic and modern deductions. Euclidean simulations bridge the gap between mathematical / microeconomic and statistical / macroeconomic reproductions of exchange networks. Systems of first-order non-homogeneous linear differential equations are easily solved and studied, especially with software such as *Mathematica*. The twelve propositions enable us to forecast an economy's static, dynamic and controllability features during any year in respect of which we have made assumptions or projections regarding the exogenous variables. Euclidean models operate in accordance with the same structural principles; as a result, differences among their equilibrium solutions, stability conditions and controllability limits stem from the portrayed system's exogenous variables. The autonomous parameters, which bestow a unique identity upon each exchange network, are classified under six categories:

1. sectors, markets, countries and currencies which make up the system;
2. behaviour coefficients, i.e. i) percentages of income derived from and ii)
3. percentages of expenditure destined for sectors or markets and countries or zones;
4. monetary, fiscal, exchange and other policy instruments;
5. average quotations of labour salaries, product prices and security values;
6. available quantities, i.e. years of labour, pieces of output, number of consols, etc.

The above categories are particularly useful for systematic comparisons of national economies and currency zones. Granted the relevant data, we should, first, confirm the uniqueness and stability of the equilibrium solution (vector) and, subsequently, the positive sign of its constituent elements (flows). With *Mathematica*'s assistance, we may express an economic system's steady-state solution, stability conditions and controllability limits, as functions of a number of policy parameters in order to simulate the resulting:

- receipts and payments of institutional sectors;
- annual sales of services, products and securities.

One of the purposes of such exercises would be the adjustment of policy parameters so as to achieve the chosen targets during the following year, if everything else remained the same. In this connection, it is noted that economic systems rarely move by leaps and bounds. Just like living organisms, exchange networks of sectors and markets (and, therefore, sovereign countries and currency zones) rarely internalise radical change immediately. In the near future, monetary, fiscal and other authorities – by utilising a Euclidean replica of the global system – will be able to ponder the consequences of moving from one set to another set of policy parameters that, in their view, would improve the annual flows of institutional sectors and the quantitative gaps of traded goods. Under all circumstances, planning and implementing economic policy without a suitable mathematical model resembles:

- sailing to remote places without a map, a compass and a sextant, or
- launching a manned satellite without knowing its orbit in advance.

1.7. General guidance to the contents

The monograph establishes that the science of economics can be studied and utilised on the basis of a minimum of fundamental hypotheses and the general laws of mathematics. In this conjunction, in-depth knowledge of economics and extensive experience with mathematics are essential. Economists should be conversant with linear algebra and differential equations while mathematicians should be conversant with advanced micro and macro economics. Perusing the monograph also presupposes comprehensive knowledge of statistics and accounting. Chapters **5**, **6**, **7** and **8** provide specific examples of economic systems consisting of three, four and five sectors or markets – or six, eight and ten first-order non-homogeneous linear differential equations – which may be used as building blocks for larger constructs. The paradigms are aimed at demonstrating the effects of various hypothetical scenarios on:

- the annual receipts and payments of institutional sectors and the quantitative gaps of market goods;
- sovereign countries and currency zones linked through the international trade and payment systems.

Chapters **9**, **10**, **11** and **12** explain Euclidean methodology's connections with classical philosophy and modern physics. Mathematical operations are carried out with the help of suitable software. Readers who are familiar with and have access to such software enjoy considerable advantages. The author would be pleased to respond to requests to provide mathematical proofs by e-mail. Computations are made available to interested parties in order to become acquainted with the relevant techniques.

Chapter 2. Fundamental Hypotheses or Axioms?

Adoption of a minimum number of basic assumptions is necessary for carrying out analysis and synthesis of an exchange network and, thus, construction and exploitation of its mathematical model. With this in mind, it is emphasised that monetary flows, financial stocks and real goods, associated with economic activity, are characterised by distinct essential qualities and should, therefore, be measured on separate dimensions of the vector space. Services, products, securities, etc., may be combined in single dimension mathematical operations – in order to draw conclusions about aggregates – if, and only if, they are evaluated in terms of the same measurement units. Economic agents weigh the relative significance of traded goods not in terms of prices or quantities *per se* but in terms of shares in their annual revenue or outlay. To facilitate recording, comparison and accumulation of various flows and stocks, market participants quote salaries, prices or values in terms of the generally understood *numeraire*. Every equation, system or model that combines dissimilar real variables (by ignoring the above facts) is methodologically questionable. In this chapter we present the transaction rules, sector motives, mathematical prescriptions and financial arrangements which characterise today's monetary economies. In our view, twelve major hypotheses explain modern exchange networks and govern their mathematical simulation models. The order of assumptions is immaterial because they should be understood and applied as a whole. The premises are followed by elaborations which leave the main arguments untouched but facilitate comprehension of their totality.

2.1. Rules of exchange

i) Market participants compare, evaluate and trade flows and stocks of different goods by means of bank accounting units or nominal money.

Barter of services for products and *vice versa* – without a generally accepted unit of account or finance – has long been abandoned because workers, entrepreneurs, etc., had to:

- price labour and output in terms of all the services and products traded in the open market;
- assess their incomes and expenditures in terms of every kind of labour and output bartered;
- incur transport and storage costs for the products they had to accumulate for services.

Modern trade is efficient because valuation, exchange and amassing of assets and liabilities is realised in terms of a generally understood *numeraire*. In developed economies, it would be unthinkable for anyone to buy or sell labour, output, capital, etc., without direct or indirect reference to the general measure of value and store of wealth. Bank accounting or monetary units have made possible comparison, recording or warehousing of receipts and payments with simplicity, uniformity and generality. Euclidean methodology has also adopted the calendar year in response to the need that all variables entering a mathematical model be measured in the same time unit.

ii) Salaries, prices and values are modified intermittently by those immediately concerned. Gaps between demanded and available quantities of labour, output and liquidity move opposite to the respective quotations.

If economic agents could enter and exit the exchange network bringing in or taking out real resources as necessary, the related gaps would be eliminated without salaries, prices and values having to fall or rise. It is highlighted that during any year:

- the numbers of market participants and the quantities of traded goods are limited by scarcity;
- the salaries of workers, the prices of products and the values of securities tend to be inflexible;
- changes in quotations may require union approval, board agreement or official permission.

Consequently, markets do not clear without intervention regarding the supply, the demand or the quotation of labour, output and liquidity. Within this framework, the monetary financial authority is empowered to modify national discount and exchange rates by buying and selling guaranteed securities in order to ameliorate internal and external imbalances.

iii) Exchange of services, products and securities is recorded by banking institutions in terms of the official accounting or monetary unit.

Labour, output and liquidity – which are respectively controlled by households, businesses and financiers – become part of the exchange network by being bought and sold in terms of bank accounting units. Supply of and demand for services, products and securities are inextricably linked with monetary flows and financial stocks because transactions are recorded by banks as pairs of debits and credits on behalf of buyers and sellers. The accounting records of banks – covering deposits and loans – are appropriate sources for sector incomes, market turnovers and other statistics.

2.2. Behaviour of sectors

iv) Economic agents are grouped into institutional sectors each one of which is distinguished by its pair of linearly independent income and expenditure functions.

Grouping agents under sectors (countries, zones, etc.) yields behaviour coefficients which are less variable and more meaningful. Aggregation reduces the number of sectors and markets to be dealt with, but necessitates calculation of average prices in respect of the corresponding goods. ESA divides economies into five main sectors: Households, Non-financial Organisations, Financial Organisations, General Government and Not-for-profit Organisations Serving Households. Each sector includes two residential categories: Residents and Non-residents. Over time, the five main sectors and their respective objects remain the same, although their subdivisions or components may change. The basic economic model – suited for use as module or paradigm for larger constructs – incorporates the three primary of the five main sectors. Accordingly, it is composed of:

- **Consumer households**, which primarily sell their labour and dispose of their income to buy the output of businesses, while lending any surplus or borrowing any deficit through monetary financial institutions.
- **Productive enterprises**, which primarily sell their output and dispose of their income to buy the labour of households, while lending any surplus or borrowing any deficit through monetary financial institutions.
- **Banking institutions**, which issue and accept guaranteed consols (i.e. binding agreements for the exchange of monetary flows with financial stock and *vice versa*) while buying services from households and products from businesses, as necessary.

For a number of reasons – including efficiency of financial services, optimisation of wealth creation and promotion of coherent policies – governments have historically:

- granted the right of being net liquidity issuer only to the banking sector;
- imposed the obligation of being net liquidity acceptor on all other sectors.

Banks form the money-issuing sector while all the rest form the money-accepting sector. Consumers, producers and financiers derive their takings from goods bought by other agents and sectors and direct their outgoings to goods sold by other agents and sectors. The spectrum of services, products and securities supplied and demanded around the world is determined by biological needs, climatic conditions, educational achievements, industrial resources, transport links and other characteristics of countries and zones.

v) Acting upon their 'human instincts' or 'animal spirits', agents and sectors distribute their income among possible sources and allocate their expenditure among pertinent destinations. The percentages of each one's revenue sources and outlay destinations, respectively, add up to 100%.

The terms 'animal spirits' and 'human instincts' imply that the best interests of agents and sectors may not always be served by their allocations of revenue and outlay among traded goods. In this context, it is noted that households, businesses, financiers, etc., become used to combinations of income sources and expenditure destinations which express their respective ways of living, operating and thinking. While consumers, producers, bankers, etc., consistently behave in accordance with this mathematical rule, exogenous factors (i.e. cultural influences, scientific inventions, fashion trends, etc.) may induce them to modify the apportionments of their incomes and expenditures from one year to the next.

vi) Market participants consistently maximise income by distributing it among feasible sources, minimise expenditure by allocating it among relevant destinations, while adjusting expenditure to income in line with monetary budgets and credit policies.

In their endeavour to maximise the present value of receipts (assets) and minimise the present value of payments (liabilities), households, businesses, governments, etc., uphold the prevailing substitution elasticities among income sources as well as among expenditure destinations, while staying within the respective budget constraints. Regarding divergences between revenue and outlay, it is impossible for all institutional sectors to be creditors (or debtors) at the same time. Constitutional and legal provisions entail that the issuing sector's annual deficit (increase of liquid liabilities) is equal to the accepting sectors' annual surplus (increase of liquid assets). The yearly variation and sector allocation of new credit (liquidity) are determined by the reasoning capacity of bankers and the monetary policy of the authorities. In a changeable environment, consumers, producers and other accepting sectors fear the future, suffer from survival anxiety and, thus, aim at self-preservation by all means. The legally enforced rule to operate as net creditors leads money acceptors to:

- consume and invest their income minus the instalment (k) on their indebtedness (k/r) to the banking system;
- stay solvent and go bankrupt only due to unpredictable factors (e.g. natural disasters and inappropriate policies).

The assumption that agents – aside from the monetary system – control their payments so as not to exceed their receipts is absolutely fundamental because otherwise:

- bankruptcies would not be due to exogenous and unpredictable factors (e.g. natural disasters and inappropriate policies) but rather due to an agent's free will;
- the portrayed exchange network could not be considered as 'economic' in the sense of being based on the thrifty management of each agent's scarce resources.

Along with human dignity and time-honoured behaviour, bank lending practices should restrain households, entrepreneurs, governments, etc., from recklessly wasting their wealth and thus receiving the severe penalties that often accompany bankruptcy.

2.3. Mathematical specifications

vii) Institutional sector income and expenditure are simultaneously determined as an ordered pair of real numbers (a vector) during operation of the economic system. As a result, the monetary flows and the corresponding stocks of institutional sectors comprise vector spaces.

The revenue of each economic agent is the result of outlay by other agents. Similarly, payments by any institutional sector end up as part of the receipts of other sectors. Interaction of sectors and markets entails simultaneous debits and credits. The twin monetary flows of consumers, producers, bankers, etc., although related, are not linearly dependent functions. Income and expenditure of institutional sectors – associated with selling and buying market goods – are generated endogenously as ordered pairs of real numbers, which form an integral whole (vector space) and thus obey the laws of mathematics. From the standpoint of any agent, sector, country or zone, revenue and outlay differ by the amount of net bank credit. From the entire system viewpoint, receipts and payments are by definition in perfect balance. On the above basis, it is noted that:

- the monetary flows of institutional sectors move close to or away from the system's equilibrium vector depending on their dynamic interaction;
- in a dynamically stable economic network, liquid financial wealth fluctuates around the value determined by the monetary authorities.

viii) The twin mathematical functions, which generate each sector's market behaviour, are linearly homogeneous of degree one. The domains of mathematical functions, which depict sector income and expenditure, as well as the ranges of independent variables, which determine sector income and expenditure, are measured in terms of the same accounting or monetary unit.

The measurement units of dependent and independent variables are integral parts of the equations, functions or system involved. Common sense, everyday practice and scientific logic substantiate measurement of:

- the domains of mathematical functions, which depict the income and expenditure of institutional sectors, as well as
- the ranges of the independent variables, which determine each sector's income and expenditure per time period,

in terms of the generally accepted *numeraire*. At the heart of the mathematical approach to building and applying economic models lies the uniform estimation and comparison of economic variables. Market participants are neither able to think nor able to act otherwise. Mathematical operations on independent variables should not alter the specified dimension and, consequently, the evaluation unit of dependent variables. This essential result is obtainable if, and only if, the mathematical functions – which generate the market conducts of households, businesses, governments, financiers, etc., – are linearly homogeneous. A general category of functions possessing the aforementioned property is:

$$w = x^{\eta} \cdot y^{(1-\eta)\cdot\theta} \cdot z^{(1-\eta)\cdot(1-\theta)}, \text{ where } \eta + \theta = 1. \qquad (2.1)$$

This particular form is very popular and well known in economics because, among other advantages, it yields the following desirable results:

- The rate of change of the dependent variable is equal to the linear combination of the rates of change of the independent variables.

- The rates of marginal substitution among the independent variables are diminishing and, therefore, the dependent variable is characterised by convexity to the origin.

ix) Most economic agents lack knowledge about the future while engaging in market transactions. Without scientific help, they tend to view the future as a projection (repetition) of the present and to discount anticipated receipts and payments using the consol yield as an objective factor connecting future flows with present values.

Most models which claim to predict the future are backward looking, in the sense of mainly taking into consideration data regarding the past. The 'rational expectations theory' has attempted to correct this weakness by assuming that decisions based on all available information almost determine what happens in the future. It is argued that on average, people can quite correctly predict future conditions and take actions accordingly, even if they do not fully understand the cause-and-effect (causal) relationships underlying the events and their own thinking. Thus, while they do not have perfect foresight, they construct their expectations in a rational manner so that, more often than not, they turn out to be correct. Any error that creeps in is usually due to random (non-systemic) and unforeseeable causes. On the contrary, in our approach, market participants resemble spectators at a theatrical performance who think they can anticipate the plot of a play although they are unable to figure out what is going on behind the curtain. Institutional sectors, while pursuing economic optimisation, wealth preservation and power politics, are not endowed with the qualities or means to foresee the future without scientific help. For any combination of external parameters likely to hold during the next and subsequent periods, Euclidean models provide the steady state, dynamic stability and controllability conditions of the depicted exchange network and, thus, facilitate formulation of rational expectations by those concerned. The proposed methodology professes:

- scientific certainty regarding the internal structure and operation of the global economic system;
- uncertainty about future behaviour coefficients, price quotations, real resources and public policies.

2.4. Monetary arrangements

x) Per calendar year, monetary financial institutions (i.e. banks) issue and accept guaranteed securities, which are equivalent to probability weighted perpetual bonds. The

nominal value of each consol is determined by the exogenous accounting unit (*numeraire*) and the exogenous (annual) discount rate.

In exchange for each issued (sold) consol, banks promise to pay to the acceptor (buyer) the *numeraire* unit annually forever. The present value of each perpetual bond is the inverse of the discount rate. Banking institutions act as intermediaries for conversion of monetary flows into financial stocks and *vice versa*, according to the demands of sectors, including the government. Perpetual bonds (bank deposits) are used by everyone as means for market transactions and wealth accumulation because their present value is fully known. By selling (issuing) perpetual bonds to consumers, producers, etc., banks:

- immediately decrease the lender's liquid assets and spending capacity by the amount of the deposit;
- annually increase the depositor's liquid assets and spending capacity by the amount of credited interest.

By buying (accepting) perpetual bonds from households, businesses, etc., banks:

- immediately increase the borrower's liquid assets and spending capacity by the amount of the loan;
- annually decrease the debtor's liquid assets and spending capacity by the amount of debited interest.

Transactions affecting the banking system balance sheet are called monetary financial operations because they either provide liquidity to or absorb liquidity from the other institutional sectors. In contrast to exchanges of services and products, agreements about purchases and sales of guaranteed securities – entered into by banks – are accompanied by permanent obligations and rights regarding receipt or payment of interest. Obligations to pay and, therefore, rights to receive interest in perpetuity are officially enforced. All contracts relating to borrowing and lending by monetary financial institutions require the unceasing sanction and attention of executive, judicial and parliamentary authorities in order to be implemented by all concerned, indefinitely. The aforesaid arrangements – including annual yield, quality valuation, accounting unit, etc., of loans and deposits – are intended to allow no doubt about the banking system balance sheet. Every year, uninterrupted flows of debited and credited interest affect the incomes and expenditures of institutional sectors and, therefore, the operation of the entire exchange network.

xi) Monetary financial institutions enjoy the right to repay their liabilities with the issue of additional perpetual bonds (bank deposits), which the other sectors must accept as means of payment and store of value.

Any sector's surplus (asset) is the mirror image of another sector's deficit (liability) and *vice versa*. Given choice, all sectors would prefer to operate with the largest possible annual surplus so as to accumulate the highest possible net worth. However, it is impossible for all market participants to be net creditors at the same time. In response to the underlying conflict regarding who will be the creditor sector(s), executive, judicial and parliamentary authorities in every country have assigned to their monetary financial institutions the perpetual right to cover deficits by increasing liabilities; this right has been granted on condition that banks will acquire services, products and securities from accepting sectors in ways that would be conducive to achievement of broader economic, political and social objectives. Under this constitutional framework, the supply of money may be expanded or contracted at the stroke of a pen or the touch of a keyboard. The guaranteed liabilities issued by banks are accepted at face value as liquid assets by the other sectors of the economy. The total supply of money equals the guaranteed liabilities of banking institutions and, thus, the corresponding liquid assets of households, businesses, etc. In order to strengthen economic confidence, encourage stable behaviour and promote financial deepening, negative present value of banking system flows must be covered by on and off balance sheet capital that includes parliamentary authorisations, government guarantees, deposit insurance, gold reserves, real estate, pledged collateral, etc.

xii) The monetary authority may influence banking institutions to increase their liquid liabilities (deposits) and allocate the corresponding assets (loans) among other sectors in accordance with explicit economic, political or social criteria.

Constitutional provisions entitle banking institutions to operate as issuers of money and oblige households, businesses, etc., to operate as acceptors of money. Banks acquire assets – including services, products and securities – by issuing their own liabilities, which appear as assets of the accepting sectors. In accordance with their respective reasoning, agents, sectors, countries, etc., choose the revenue percentage to be earned from financial capital and the outlay percentage to be spent on financial debt. However, it is entirely up to banking institutions, including the monetary authority, to decide the amount and allocation of new credits and, thus, the difference between the twin flows of each accepting sector.

In Sections 3.2 to 3.4 we will discuss how constitutional and legal arrangements governing monetary financial operations set the constants which permit the economic system and its mathematical model to become determinate or non-homogeneous. Through the right to intervene regarding increase, allocation and quotation of bank assets and liabilities, monetary authorities may influence the years of labour, pieces of output, units of liquidity, etc., bought and sold per annum. Bank credit and debit operations expand or contract monetary flows of institutional sectors and, consequently, aggregate demand for market goods. Bankers consider a number of factors (including quantitative deficits or surpluses of services, products and securities as well as potential rise or fall of salaries, prices and values) when resolving how to increase and allocate their balance sheets. The exchange network's structure – formed by apportionment of consumer, producer, government, financier, etc., receipts and payments – determines whether the equilibrium vector is unique, sustainable and, therefore, dirigible (capable of being directed, controlled, or steered) per time unit. The macroeconomic system's configuration is inextricably linked to the micro behaviour of its component parts and *vice versa*.

During any period, the percentage of bank credit allocated to one sector may rise if, and only if, the percentage of bank credit allocated to another sector may fall. Any redistribution of the yearly money supply among accepting sectors (also classified under sovereign countries and currency zones) is filtered through the circulation network and affects domestic and foreign consumers, producers and bankers unevenly. Annual liquidity allotment is a source of conflict and confrontation, but also of competition and cooperation, among those who are favoured and those who are ignored. Effectiveness of national central banks and global financial institutions is often measured by the ease with which their monetary, credit and exchange policies are received by the sectors, countries or zones concerned. In the following chapters, we will show that the twelve propositions (along with other secondary assumptions) enable construction and utilisation of mathematical models (of any justifiable size) which:

- define exchange networks in terms of market and financial principles;
- combine micro and macro theories with statistical and accounting practices;
- replicate monetary economic systems with simplicity, uniformity and generality;
- enable formation of rational expectations given the exogenous parameters.

Table (2.1) summarises the major premises of Euclidean economic models.

(2.1) The pieces of the puzzle

Rules of exchange

1) Market participants compare, evaluate and trade flows and stocks of different goods by means of bank accounting units or nominal money.

2) Salaries, prices and values are modified intermittently by those immediately concerned. Gaps between demanded and available quantities of labour, output and liquidity adjust contrary to the respective quotations.

3) Exchange of services, products and securities is recorded by banking institutions in terms of the official accounting or monetary unit.

Behaviour of sectors

4) Economic agents are grouped into institutional sectors each one of which is distinguished by its pair of linearly independent income and expenditure functions.

5) Acting upon their human instincts or animal spirits, agents and sectors distribute their income among possible sources and allocate their expenditure among applicable destinations. The percentages of each one's revenue sources and outlay destinations, respectively, add up to 100%.

6) Every market participant, consistently, maximises income by distributing it among relevant sources, minimises expenditure by allocating it among pertinent destinations, while adjusting expenditure to income in line with monetary budgets and credit policies.

Mathematical specifications

7) Institutional sector income and expenditure are simultaneously determined as an ordered pair of real numbers (a vector) during the operation of the economic system. As a result, monetary flows and corresponding stocks of institutional sectors comprise vector spaces.

8) The twin mathematical functions, which generate each sector's market behaviour, are linearly homogeneous (of degree one). The domains of mathematical functions, which depict sector income and expenditure, as well as the ranges of independent variables, which determine sector income and expenditure, are measured in terms of the same accounting or monetary unit.

9) Most economic agents engage in market transactions with lack of foresight. Without scientific help, they tend to view the future as a projection (repetition) of the

present and to discount anticipated receipts and payments using the consol yield as an objective factor connecting future flows with present values.

Monetary arrangements

10) Per calendar year, monetary financial institutions (i.e. banks) issue and accept guaranteed securities, which are equivalent to probability weighted perpetual bonds. The nominal value of each consol is determined by the exogenous accounting unit (*numeraire*) and the exogenous (annual) discount rate.

11) Monetary financial institutions enjoy the right to repay their liabilities with the issue of additional perpetual bonds (bank deposits), which the other sectors must accept as means of payment and store of value.

12) The monetary authority may persuade banking institutions to increase their liquid liabilities (deposits) and allocate the corresponding assets (loans) among other sectors in accordance with explicit economic, political or social criteria.

Chapter 3. Mathematical Imitation of Economic Behaviour

While putting together the modular network, we will repeat major hypotheses and explain adopted symbols. Endogenous variables are the incomes y_i and expenditures e_i of a closed economy's institutional sectors. Households h, businesses b and financiers f :

- supply characteristic goods while earning money from their financial assets;
- spend money on their financial liabilities and on goods supplied by other sectors;
- adjust outlay to revenue in response to exogenous changes of their liquid wealth.

Consumers earn wages on their labour w and interest on their financial assets s; they spend on business output u and financial liabilities s. Producers earn revenue on their output u and interest on their financial assets s; they spend on labour services w and financial liabilities s. Households and businesses are legally obliged to be the acceptors of money and, therefore, adjust their outgoings so as to be equal to their incomings minus the exogenous increase of their liquid assets. Bankers earn interest on their financial assets s; they spend on household labour w, business output u and financial liabilities s. Bankers are legally empowered to be the issuers of money and consequently adjust their outgoings so as to match their incomings plus the exogenously decided increase of their liquid liabilities. Net debt of banks should be covered by official guarantees, gold bullion and other capital.

3.1 Meaning of prescripts and postscripts

Labour, output and securities are denoted by w, u, s, respectively. The calendar year has been adopted as the most practical answer to the prerequisite that a mathematical system's variables be measured in the same time unit. Therefore, behaviour coefficient:

- α_{ij} denotes the percentage of annual revenue derived by institutional sector $i = h, b, f$ from market good $j = w, u, s$.
- β_{ij} denotes the percentage of annual outlay allocated by institutional sector $i = h, b, f$ to market good $j = w, u, s$.
- κ_h and κ_b denote the annual proportions of bank credit k_f allocated to h and b.

Animal spirits, as Keynes would have called human instincts, steer derivation of income from possible sources and allocation of expenditure to applicable destinations. Behaviour coefficients:

- α_{hj} and β_{hj} reflect idiosyncrasies, social norms, biological needs, resource endowments and other attributes affecting the lifestyle of consumers;
- α_{bj} and β_{bj} reflect production technologies, management practices, distribution channels and other factors affecting the operation of enterprises;
- α_{fj} and β_{fj} reflect thrift, risk taking, need for power and other characteristics impelling the behaviour of bankers.

The percentages of sector *i* revenue sources ($\alpha_{iw} + \alpha_{iu} + \alpha_{if}$) and outlay destinations ($\beta_{iw} + \beta_{iu} + \beta_{is}$) respectively add up to 100%. The Double Entry Principle implies that the annual increase of bank loans k_f is allocated as additional deposits $(\kappa_h + \kappa_b)\cdot k_f$ among consumers and producers. According to our premises, there can be no monetary takings without corresponding outgoings and *vice versa*. Relative to this, it is worth mentioning that economic agents resist 'government taxes' and 'intermediation fees' because they cannot associate specific benefits to either category of expenditure. Taxes, like intermediation fees, can be easily collected only if disguised as percentages of payments willingly made by households and businesses for market goods. In Chapter **6**, where the public sector is integrated in the mathematical model, we most certainly allow for the aforesaid reasoning.

3.2 Network of receipts and payments

Table (3.1) summarizes interaction among consumers, producers and bankers regarding labour, output and liquidity; it confirms that a closed economic system, which has been constructed according to the twelve major premises, does not suffer from aggregation problems. Among other details, it has been assumed that:

- households *h* do not pay for the services *w* they acquire from themselves;
- businesses *b* do not pay for the products *u* they acquire from themselves.

This particular behaviour is true only in an elementary network. As soon as the system is made more complex – with presence of two (or more) kinds of labour and output supplied by two (or more) consumer and producer subsectors included in the corresponding broader sectors – workers and enterprises do purchase what is supplied by their own broadly defined sectors. Combination of more modules (Lego bricks) results in bigger more diversified but also more realistic models. Modern exchange networks encompass many

(3.1) Income sources and expenditure destinations

	COUNTRY	MARKET						
	SECTOR	**Labour**		**Output**		**Finance**		**Annual Stream**
	Consumer Household	Supply	Demand	Supply	Demand	Supply	Demand	
1	Endogenous Income	$\alpha_{hw}\cdot y_h$		-		$\alpha_{hs}\cdot y_h$		$(\alpha_{hw} + \alpha_{hs})\cdot y_h$
2	Endogenous Expenditure		-		$\beta_{hu}\cdot e_h$		$\beta_{hs}\cdot e_h$	$(\beta_{hu} + \beta_{hs})\cdot e_h$
3	Exogenous Surplus or Net Deposits							$(y_h - e_h) = k_h$
	Business Corporation							
4	Endogenous Income	-		$\alpha_{bu}\cdot y_b$		$\alpha_{bs}\cdot y_b$		$(\alpha_{bu} + \alpha_{bs})\cdot y_b$
5	Endogenous Expenditure		$\beta_{bw}\cdot e_b$		-		$\beta_{bs}\cdot e_b$	$(\beta_{bw} + \beta_{bs})\cdot e_b$
6	Exogenous Surplus or Net Deposits							$(y_b - e_b) = k_b$
	Monetary Institution							
7	Endogenous Income	-		-		$\alpha_{fs}\cdot y_f$		$(\alpha_{fs}\cdot y_f) = (\beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b)$
8	Endogenous Expenditure		$\beta_{fw}\cdot e_f$		$\beta_{fu}\cdot e_f$		$\beta_{fs}\cdot e_f$	$(\beta_{fs}\cdot e_f) = (\alpha_{hs}\cdot y_h + \alpha_{bs}\cdot y_b)$
9	Exogenous Deficit or Net Loans							$(e_f - y_f) = k_f$
10	***Aggregate Demand***	$(\beta_{bw}\cdot e_b + \beta_{fw}\cdot e_f)$		$(\beta_{hu}\cdot e_h + \beta_{fu}\cdot e_f)$		$(\beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b) + (\beta_{fs}\cdot e_f)$		
11	***Aggregate Supply***	$\alpha_{hw}\cdot y_h$		$\alpha_{bu}\cdot y_b$		$(\alpha_{hs}\cdot y_h + \alpha_{bs}\cdot y_b) + (\alpha_{fs}\cdot y_f)$		
12	***GROSS TURNOVER***	$(\beta_{bw}\cdot e_b + \beta_{fw}\cdot e_f) = \alpha_{hw}\cdot y_h$		$(\beta_{hu}\cdot e_h + \beta_{fu}\cdot e_f) = \alpha_{bu}\cdot y_b$		$(\beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b + \beta_{fs}\cdot e_f) =$ $= (\alpha_{hs}\cdot y_h + \alpha_{bs}\cdot y_b + \alpha_{fs}\cdot y_f)$		
13	***Earnings from Self***					$(\beta_{fs}\cdot e_f)$		
14	***VALUE ADDED***	$(\beta_{bw}\cdot e_b + \beta_{fw}\cdot e_f)$		$(\beta_{hu}\cdot e_h + \beta_{fu}\cdot e_f)$		$(\beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b)$		
		A		B		C		D

kinds of services, products and instruments. However the number of sectors and markets must be held in check, if the economic system's mathematical replica is to remain manageable. Grouping agents under a great number of sub-sectors and sub-markets complicates matters by necessitating calculation of an equal number of average quotations and quantity gaps. In this monograph, we have embraced ESA, which divides economies into five major sectors, each one of which is subdivided into two residential categories. In the examples of worldwide models to be discussed in Chapters **7** and **8**, we presume existence of two or more sovereign countries connected via the global trade and payments network; their households, businesses and financiers may buy and sell labour, output and liquidity internationally. These worldwide models open the way for construction and application of Euclidean exchange networks of any size. Let us now take a closer look at Table (3.1). Among the methodology's advantages is the perfect apportionment of nominal and real flows. Annual sales and purchases of services ρ_w, products ρ_u and securities ρ_s – reflected by cells A12, B12 and C12 – are copied below for ease of reference:

Labour market	$\rho_w = \beta_{bw}\cdot e_b + \beta_{fw}\cdot e_f = \alpha_{hw}\cdot y_h$	
Output market	$\rho_u = \beta_{hu}\cdot e_h + \beta_{fu}\cdot e_f = \alpha_{bu}\cdot y_b$	(3.2)
Liquidity market	$\rho_s = \beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b + \beta_{fs}\cdot e_f = \alpha_{hs}\cdot y_h + \alpha_{bs}\cdot y_b + \alpha_{fs}\cdot y_f$	

Relations (3.2) show that funds spent on and received from particular goods are functions of behaviour coefficients (α_{ij}, β_{ij}) and sector flows (y_h, e_h, y_b, e_b, y_f, e_f). By dividing the turnovers of services, products and securities (ρ_w, ρ_u, ρ_s) by the corresponding quotations (p_w, p_u, p_s), we obtain the quantities (q_w, q_u, q_s) of labour, output and liquidity traded each year:

Labour years	$q_w = (\beta_{bw}\cdot e_b + \beta_{fw}\cdot e_f) / p_w = \alpha_{hw}\cdot y_h / p_w$	
Output pieces	$q_u = (\beta_{hu}\cdot e_h + \beta_{fu}\cdot e_f) / p_u = \alpha_{bu}\cdot y_b / p_u$	(3.3)
Liquidity units	$q_s = (\beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b + \beta_{fs}\cdot e_f) / p_s = (\alpha_{hs}\cdot y_h + \alpha_{bs}\cdot y_b + \alpha_{fs}\cdot y_f) / p_s$	

It is reminded that monetary financial institutions issue and accept securities which guarantee to pay to the holder the *numeraire* unit annually forever. The present value of each perpetual bond is equal to the inverse of the discount rate. Furthermore, we should bear in mind that:

- bank income ($\alpha_{fs}\cdot y_f$) from accepted securities (loans) is necessarily equal to non-bank expenditure ($\beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b$) on issued securities (loans) and
- non-bank revenue ($\alpha_{hs}\cdot y_h + \alpha_{bs}\cdot y_b$) from security holdings (deposits) is necessarily equal to bank outlay ($\beta_{fs}\cdot e_f$) on securities outstanding (deposits).

From Table (3.1) and the above points, we derive the mathematical relationships governing endogenous takings of institutional sectors:

Household revenue	$\boldsymbol{y_h} = (\beta_{bw} \cdot e_b + \beta_{fw} \cdot e_f) + (\beta_{fs} \cdot e_f - \alpha_{bs} \cdot y_b)$	
Business revenue	$\boldsymbol{y_b} = (\beta_{hu} \cdot e_h + \beta_{fu} \cdot e_f) + (\beta_{fs} \cdot e_f - \alpha_{hs} \cdot y_h)$	(3.4)
Financier revenue	$\boldsymbol{y_f} = (\beta_{hs} \cdot e_h + \beta_{bs} \cdot e_b)$	

The tautological expressions regarding sector outgoings are as follows:

Household outlay	$\boldsymbol{e_h} = \beta_{hu} \cdot e_h + \beta_{hs} \cdot e_h$	where $\beta_{hu} + \beta_{hs} = 1$	
Business outlay	$\boldsymbol{e_b} = \beta_{bw} \cdot e_b + \beta_{bs} \cdot e_b$	where $\beta_{bw} + \beta_{bs} = 1$	(3.5)
Financier outlay	$\boldsymbol{e_f} = \beta_{fw} \cdot e_f + \beta_{fu} \cdot e_f + \beta_{fs} \cdot e_f$	where $\beta_{fw} + \beta_{fu} + \beta_{fs} = 1$	

Sets (3.4) and (3.5) are linearly dependent because the one arises from elementary operations on the rows of the other. Despite consisting of six equations and six unknowns (y_h, e_h, y_b, e_b, y_f, e_f), sets (3.4) and (3.5) do not form a single system with a unique solution. They feature only three independent equations and, therefore, any three of the six unknown variables are free for assignment of an infinite number of values which would satisfy the three equations. The aforesaid values include the pairs of parity between consumer ($y_h = e_h$), producer ($y_b = e_b$) and bank ($y_f = e_f$) income and expenditure, where no sector is either borrowing or lending. In the absence of at least one sector entitled to buy goods by issuing guaranteed securities or money indefinitely and one sector obliged to sell goods by accepting guaranteed securities or money indefinitely, the system of linear equations that represents the exchange network is homogeneous. Such economic models do not have a numerical solution apart from the null vector. Exchange networks which, for any reason, do not enforce the rights lenders and the obligations of debtors (concerning funding of deficits and investment of surpluses) in perpetuity are overwhelmed by indeterminacy and therefore anarchy.

3.3. The indispensable legal framework

According to our hypotheses, banking institutions are entitled to expand their liabilities (deficits) while consumers, enterprises, etc., are obliged to expand their assets (surpluses) *ad infinitum*. All branches of government enforce the obligation of borrowers to pay and the right of lenders to receive interest. Constitutional arrangements regarding annual issue and sector allocation of credit (money) provide the constant terms which endow the economic system and its mathematical model with non-homogeneity and thus determinacy.

In the case at hand, the algebraic relations between expansion ($dM/dt = k_f$) and distribution (k_h and k_b) of the banking sector balance sheet are as follows:

$$\begin{aligned} &(a)\quad dM/dt = k_f = e_f - y_f = (y_h - e_h) + (y_b - e_b) = k_h + k_b > 0 \\ &(b)\quad k_f / k_f = (k_h + k_b) / k_f \;\Rightarrow\; (\kappa_h + \kappa_b) = 1 \end{aligned} \qquad (3.6)$$

Increase of the liquid liabilities of banks ($dM/dt = k_f$) is necessarily reflected as increase of the liquid assets of non-banks ($k_h + k_b$). Monetary institutions keep track of their financial operations as well as of household, business, government, etc., credits (deposits) and debits (loans) per annum. Table (3.7) presents the feasible combinations of deficits and surpluses which could be realised by an elementary economy's institutional sectors.

(3.7) Sector deficit and surplus combinations

SECTORS	**i.**	**ii.**	**iii.**	**iv.**	**v.**	**vi.**
Households	*Deficit*	*Deficit*	Surplus	Surplus	*Deficit*	Surplus
Businesses	*Deficit*	Surplus	*Deficit*	Surplus	Surplus	*Deficit*
Financiers	Surplus	Surplus	Surplus	*Deficit*	*Deficit*	*Deficit*

Exchange networks and economic models without permanent financial deficits and surpluses (as well as annual interest payments and receipts) end up at the null vector. Banking system control over the amount and quotation of new loans makes possible marginal redistribution of deposits among consumers, producers, governments, etc. The slightest rearrangement of liquid wealth is accompanied by reshuffle of interest flows, and thus of incomes and expenditures. Elected and other officials tend to create allies and opponents among those affected by policies perceived to favour some and to discriminate against others. Any differentiation of the monetary financial balance sheet creates opportunities and threats for institutional sectors, sovereign countries or currency zones. The interconnected problems should be tackled by a banking system authority in ways that would maximise resource utilisation and minimise inflation pressure in all sectors, countries, etc., over subsequent periods. In a worldwide context, an international financial organisation (IFO) should develop the indispensable tools for bringing this mission to a successful conclusion. Like all leaders, central bankers should also be endowed with diplomatic and other skills for imposing their will with the minimum resistance. The ultimate test of a monetary financial authority's efficiency is the facility with which its policies are understood and accommodated by affected sectors, countries and zones.

3.4. Putting together the jigsaw puzzle

As reflected by set (3.4), every sector is aware that its endogenous revenue y_i depends on its share of the other sectors' outlay $e_1 + \ldots + e_n$. The following three linear differential equations replicate mathematically the dynamic adjustment of household, business and financier receipts emanating from services, products and securities:

$$\begin{aligned}
&\text{(a)} \quad dy_h/dt = (\beta_{\mathrm{bw}} \cdot e_b + \beta_{\mathrm{fw}} \cdot e_f) + (\beta_{\mathrm{fs}} \cdot e_f - \alpha_{\mathrm{bs}} \cdot y_b) - y_h \\
&\text{(b)} \quad dy_b/dt = (\beta_{\mathrm{hu}} \cdot e_h + \beta_{\mathrm{fu}} \cdot e_f) + (\beta_{\mathrm{fs}} \cdot e_f - \alpha_{\mathrm{hs}} \cdot y_h) - y_b \qquad (3.8) \\
&\text{(c)} \quad dy_f/dt = (\beta_{\mathrm{hs}} \cdot e_h + \beta_{\mathrm{bs}} \cdot e_b) - y_f
\end{aligned}$$

According to the sixth major proposition:

- accepting sectors modify their outgoings so as to equal their endogenous takings minus the increase of their liquid assets arising from sale of guaranteed securities to the banking sector;
- the issuing sector modifies its outgoings so as to equal its endogenous takings plus the increase of its liquid liabilities directed to purchases of guaranteed securities from accepting sectors.

The following three linear differential equations reproduce (in mathematical terms) the closed loops according to which consumers, producers and bankers adjust their annual payments to their annual receipts, as the difference between each pair of sector flows is exogenously altered by the decisions of monetary financial institutions:

$$\begin{aligned}
&\text{(a)} \quad de_h/dt = y_h - \mathrm{k_h} - e_h \\
&\text{(b)} \quad de_b/dt = y_b - \mathrm{k_b} - e_b \qquad (3.9) \\
&\text{(c)} \quad de_f/dt = y_f + \mathrm{k_f} - e_f
\end{aligned}$$

In Chapters **7** and **8**, we will show that banks (through their financial transactions with households, businesses, governments, etc.) also decide the differences between revenues and outlays of sovereign countries and currency zones per time period. The diverse factors, which impel interaction of sector, country and zone incomes and expenditures, complicate design and implementation of monetary, fiscal and other policies. The fundamental assumptions and everything they entail (e.g. permanent financing of gaps, uninterrupted flows of interest, redistribution of balance sheets, etc.) have enable us to:

- abolish the collinear relationship between revenue and outlay;
- place the necessary constants in the mathematical functions;
- eliminate free variables from the simultaneous equation system;
- define the exchange network in the (R^{2n}, t) vector space.

Prototype model (3.10), derived from (3.8) and (3.9), comprises six linear differential equations which describe the six endogenous variables. System (3.10) duplicates interaction of consumer (y_h, e_h), producer (y_b, e_b) and banker (y_f, e_f) incomes and expenditures per time unit or calendar year:

$$
\begin{aligned}
&\text{(a)}\quad dy_h/dt = \beta_{bw}\cdot e_b + \beta_{fw}\cdot e_f + \beta_{fs}\cdot e_f - \alpha_{bs}\cdot y_b - y_h \\
&\text{(b)}\quad de_h/dt = y_h - \mathrm{k_h} - e_h \\
&\text{(c)}\quad dy_b/dt = \beta_{hu}\cdot e_h + \beta_{fu}\cdot e_f + \beta_{fs}\cdot e_f - \alpha_{hs}\cdot y_h - y_b \\
&\text{(d)}\quad de_b/dt = y_b - \mathrm{k_b} - e_b \\
&\text{(e)}\quad dy_f/dt = \beta_{hs}\cdot e_h + \beta_{bs}\cdot e_b - y_f \\
&\text{(f)}\quad de_f/dt = y_f + \mathrm{k_f} - e_f
\end{aligned}
\qquad (3.10)
$$

The receipts and payments **[**(y_h, e_h), (y_b, e_b), (y_f, e_f)**]** of institutional sectors comprise the endogenous variables of the elementary model; aggregated separately, they form the economy's income and expenditure vector **[**($y_h + y_b + y_f$), ($e_h + e_b + e_f$)**]** = **(**Y, E**)**. Euclidean economic models assume the mathematical form **dX/dt = A·X – K**, where:

- **X** and **dX/dt** are columns which, respectively, represent sector monetary flows and sector flow differentiations per period;
- **A** is a square matrix that duplicates the structure of sector and market interaction;
- **K** is a column consisting of the annual increase and sector allocation of bank credit.

Dynamic exchange networks of the general type $\mathbf{dX}_t/\mathbf{dt} = \mathbf{A}_t \cdot \mathbf{X}_t - \mathbf{K}_t$ yield:

- Routh-Hurwitz stability conditions, which are functions of institutional sector behaviour coefficients (α_{ij} and β_{ij}) per time unit t;
- Cramer's rule equilibrium solutions which are functions of sector behaviour (α_{ij} and β_{ij}) and policy parameters (e.g. $\mathrm{k_f}$, κ_h, κ_b) per time unit t.

Whenever sector choices, policy shifts or external shocks alter the proportions of revenue sources, outlay destinations and bank loans, the entire economy's monetary circulation is redistributed accordingly. Figure (9.2) in Chapter **9** portrays the network of receipts and payments – in respect of real goods and financial paper – which comprise an elementary closed economy such as (3.10). Projections or forecasts regarding takings and outgoings should consider likely and unlikely developments regarding i) income and expenditure shares as well as ii) credit expansion and apportionment throughout the relevant years. Due to the scalar nature of salaries, prices and values – adopted as the second major proposition – it is possible to obtain the traded quantities of services, products and securities as well as the balance sheets of households, businesses and financiers per time unit. In subsequent chapters, we will explore the limits within which monetary, fiscal and other authorities may regulate demand for labour, output and liquidity so as to:

- maximise the utilisation of the economy's resources;
- minimise potential inflation or deflation in all markets.

Economic substance and mathematical implications are evenly treated throughout this monograph. In Chapters **4**, **5** and **6**, it will come to light that annual receipts $Y^* = (y_1 + y_2 + \ldots + y_n)\cdot k_f = c\cdot k_f$, annual payments $E^* = (e_1 + e_2 + \ldots + e_n)\cdot k_f = c\cdot k_f$ and annual sales $R^* = (\rho_1 + \rho_2 + \ldots + \rho_n)\cdot k_f = c\cdot k_f$ are always equal and perfectly allocated among the n institutional sectors and n traded goods comprising the exchange network. As a result, every Euclidean model's essential properties are captured by a sphere with radius $c\cdot k_f$ on axes Y, E and R. The sphere and its environs comprise a three dimensional dynamic space, where sector takings and outgoings as well as market proceeds change until they coincide with the respective steady states $\boldsymbol{Y}^*(y_1^*, y_2^*, \ldots, y_n^*)$, $\boldsymbol{E}^*(e_1^*, e_2^*, \ldots, e_n^*)$ and $\boldsymbol{R}^*(\rho_1^*, \rho_2^*, \ldots, \rho_n^*)$. Within this framework, it will be shown that n vertices $\boldsymbol{1}(y_1, e_1, \rho_1)$, $\boldsymbol{2}(y_2, e_2, \rho_2)$, … , $\boldsymbol{N}(y_n, e_n, \rho_n)$ represent the economic system's equilibrium position, structure and circulation. The polyhedron's form is decided by sector / country behaviour coefficients as well as by monetary, fiscal and other policy parameters while its magnitude is decided only by liquidity expansion per time unit. It is understood that negative revenue, outlay and turnover are unacceptable to economic agents in general and to policy makers in particular. Under certain conditions to be discussed, the authorities may i) adjust sector-market and country-zone equilibrium flows so as to confine them in the aforesaid sphere's positive quarter and ii) optimise demand for labour, output and finance in order to promote stability of salaries, prices and values world-wide. International policy planning and execution are discussed in Chapters **7** and **8**.

Chapter 4. The Prototype Exchange Network

Thanks to the twelve major propositions – which include the tendency of households and businesses to maximize their wealth as well as the privilege of bankers to allocate credit on the basis of economic, political and other criteria – we have formulated the mathematical model of an elementary closed economy shown below:

$$
\begin{aligned}
&\text{(a)} \quad dy_h/dt = (\beta_{\mathrm{bw}} \cdot e_b + \beta_{\mathrm{fw}} \cdot e_f) + (\beta_{\mathrm{fs}} \cdot e_f - \alpha_{\mathrm{bs}} \cdot y_b) - y_h \\
&\text{(b)} \quad de_h/dt = y_h - \mathrm{k_h} - e_h \\
&\text{(c)} \quad dy_b/dt = (\beta_{\mathrm{hu}} \cdot e_h + \beta_{\mathrm{fu}} \cdot e_f) + (\beta_{\mathrm{fs}} \cdot e_f - \alpha_{\mathrm{hs}} \cdot y_h) - y_b \\
&\text{(d)} \quad de_b/dt = y_b - \mathrm{k_b} - e_b \\
&\text{(e)} \quad dy_f/dt = (\beta_{\mathrm{hs}} \cdot e_h + \beta_{\mathrm{bs}} \cdot e_b) - y_f \\
&\text{(f)} \quad de_f/dt = y_f + \mathrm{k_f} - e_f
\end{aligned}
\qquad (4.1)
$$

System (4.1) reflects the fact that consumer and producer income and expenditure differ by the amount of additional loans / deposits generated by monetary financial institutions. If bankers were not entitled to increase their guaranteed liabilities (by operating with annual deficits) and if consumers and producers were not obliged to increase their liquid assets (by operating with annual surpluses) *ad infinitum*, dynamic interaction of sectors and markets would always end up at the trivial vector. Prototype model (4.1) is a mathematical system consisting of six first-order non-homogeneous linear differential equations. The exchange network assumes the general form $\mathbf{dX/dt} = \mathbf{A} \cdot \mathbf{X} - \mathbf{K}$ that replicates circulation of money and goods among an elementary economy's sectors and markets. Deviations between temporary $\mathbf{X} = (y_i, e_i)$ and equilibrium $\mathbf{X}^* = (y_i^*, e_i^*)$ flows cause convergent or divergent reactions which affect receipts and payments of households, businesses and financiers. Per time unit, differential equation system (4.1) may yield one or more vectors $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ consisting of three pairs of coordinates. Before any policy recommendation, it is necessary to investigate whether the economy depicted by model (4.1) possesses i) a unique and ii) stable equilibrium solution $\mathbf{X}^*$ that iii) assigns positive (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*) revenue and outlay to consumers, producers and bankers. In this chapter, we explain, among other things, why and how dynamic systems characterised by the aforesaid three properties are controllable within certain well-defined limits.

4.1. Uniqueness of the equilibrium vector

Steady state $\mathbf{X}^*$ of prototype dynamic model (4.1) is the solution of the associated time-invariant system $\mathbf{A} \cdot \mathbf{X}^* = \mathbf{K}$, expressed by (4.2) in matrix form:

$$\begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bw} & 0 & \beta_{fw} + \beta_{fs} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 & 0 & \beta_{fu} + \beta_{fs} \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & \beta_{hs} & 0 & \beta_{bs} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h^* \\ e_h^* \\ y_b^* \\ e_b^* \\ y_f^* \\ e_f^* \end{vmatrix} = \begin{vmatrix} 0 \\ k_h \\ 0 \\ k_b \\ 0 \\ -k_f \end{vmatrix} = \begin{vmatrix} 0 \\ \kappa_h \\ 0 \\ \kappa_b \\ 0 \\ -1 \end{vmatrix} \cdot k_f = \mathbf{A} \cdot \mathbf{X}^* = \mathbf{K} \quad (4.2)$$

Proof of existence and uniqueness of the above linear system's solution $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ involves a routine mathematical procedure. According to Cramer's rule, equilibrium vector $\mathbf{X}^*$ can be expressed in terms of $\mathbf{A}$ (i.e. the relative origin and destination of sector and market flows) and $\mathbf{K}$ (i.e. the increase of bank assets and the allocation of bank liabilities) provided determinant $|\mathbf{A}| \neq 0$. The behaviour coefficients of households (α_{hj}; β_{hj}), businesses (α_{bj}; β_{bj}) and financiers (α_{fj}; β_{fj}) decide whether $|\mathbf{A}|$ is negative, positive or zero. Transactions among the three sectors, regarding labour, output and liquidity, lead the exchange network to a unique solution when $|\mathbf{A}| \neq 0$, or to an indeterminate solution when $|\mathbf{A}| = 0$. The probability of determinant $|\mathbf{A}|$ being exactly equal to zero is infinitesimal. If the probability of such coincidence were large, many economies would frequently experience periods of indeterminacy and breakdown, something that is rarely the case.

Within any time unit, the percentages of sector receipts α_{ij} and payments β_{ij} are taken as fixed, although they may vary from one year to the next. Adoption of another hypothesis would have complicated the model's form without adding anything to its substance. In the medium and long run, exogenous causes – such as scientific inventions, cultural influences, religious beliefs, natural disasters or military operations – compel consumers, producers, bankers, etc., to alter their revenue sources and outlay destinations. Euclidean modelling methodology, assisted by modern mathematical software, enables us not only to simulate but also to animate the endogenous variables and their dependent functions over any number of years in respect of which we have embraced assumptions or predictions about the autonomous and policy parameters. Ultimately, real and financial exchange within and among sovereign countries, currency zones, etc., is governed by sector behaviour and credit policy. The latter is reflected by the annual change of the banking sector balance sheet.

4.2. Conditions for dynamic stability

The elementary monetary economy depicted by (4.1) is globally stable if, and only if, the Routh-Hurwitz conditions or determinants, derived from the corresponding homogeneous dynamic system (4.3), have positive signs.

$$\begin{vmatrix} dy_h/dt \\ de_h/dt \\ dy_b/dt \\ de_b/dt \\ dy_f/dt \\ de_f/dt \end{vmatrix} = \begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bw} & 0 & \beta_{fw}+\beta_{fs} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 & 0 & \beta_{fu}+\beta_{fs} \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & \beta_{hs} & 0 & \beta_{bs} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h \\ e_h \\ y_b \\ e_b \\ y_f \\ e_f \end{vmatrix} = \mathbf{A \cdot X} \qquad (4.3)$$

Homogeneous linear differential equation system (4.3) possesses the general form **dX/dt** = $\mathbf{A \cdot X} = \boldsymbol{\lambda} \cdot \mathbf{I} \cdot \mathbf{X}$ and, therefore, has a non-singular solution **X** when the characteristic equation $|\mathbf{A} - \boldsymbol{\lambda} \cdot \mathbf{I}|$ or (4.4) is equal to zero.

$$|\mathbf{A} - \boldsymbol{\lambda} \cdot \mathbf{I}| = \begin{vmatrix} -1-\lambda & 0 & -\alpha_{bs} & \beta_{bw} & 0 & \beta_{fw}+\beta_{fs} \\ 1 & -1-\lambda & 0 & 0 & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1-\lambda & 0 & 0 & \beta_{fu}+\beta_{fs} \\ 0 & 0 & 1 & -1-\lambda & 0 & 0 \\ 0 & \beta_{hs} & 0 & \beta_{bs} & -1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 1 & -1-\lambda \end{vmatrix} \qquad (4.4)$$

As shown above, determinant $|\mathbf{A} - \boldsymbol{\lambda} \cdot \mathbf{I}| = a_0 \cdot \lambda^6 + a_1 \cdot \lambda^5 + a_2 \cdot \lambda^4 + a_3 \cdot \lambda^3 + a_4 \cdot \lambda^2 + a_5 \cdot \lambda + a_6 = 0$, is a sixth degree polynomial in λ. Polynomial coefficients a_0 to a_6 are functions of household, business and financier behaviour coefficients (α_{hj}, β_{hj}; α_{bj}, β_{bj}; α_{fj}, β_{fj}) regarding traded goods. Polynomial (4.4) has six real or complex characteristic roots λ_i, $i = 1, \dots, 6$. Corresponding to each root there is a solution x_i that is globally stable if, and only if, the real parts of the six characteristic roots λ_i are negative. According to Routh-Hurwitz, systems (4.1) and (4.3) are globally stable provided the six principle minors of matrix (4.5) – composed of the coefficients of polynomial (4.4) – are greater than zero.

$$\begin{vmatrix} a_1 & a_0 & 0 & 0 & 0 & 0 \\ a_3 & a_2 & a_1 & a_0 & 0 & 0 \\ a_5 & a_4 & a_3 & a_2 & a_1 & a_0 \\ 0 & a_6 & a_5 & a_4 & a_3 & a_2 \\ 0 & 0 & 0 & a_6 & a_5 & a_4 \\ 0 & 0 & 0 & 0 & 0 & a_6 \end{vmatrix} \qquad (4.5)$$

The six Routh-Hurwitz conditions, $rh_1, \dots, rh_6$, of system (4.1) are presented below:

$$rh_1 = a_1 > 0 \;\; ; \;\; rh_2 = (a_1 \cdot a_2 - a_0 \cdot a_3) \lessgtr 0 \;\; ; \;\; rh_3 = a_3 \cdot (rh_2) - a_4 \cdot a_1^2 + a_5 \cdot a_0 \cdot a_1 \lessgtr 0 \;\; ;$$

$$rh_4 = a_4 \cdot (rh_3) - a_5 \cdot (-a_0 \cdot (a_1 \cdot a_4 - a_0 \cdot a_5) + a_2 \cdot (rh_2)) + a_6 \cdot (a_1 \cdot (rh_2)) \lessgtr 0 \;\; ; \qquad (4.6)$$

$$rh_5 = a_5 \cdot (rh_4) - a_6 \cdot (a_3 \cdot (rh_3) - a_1 \cdot a_5 \cdot (rh_2) - a_6 \cdot a_1^2) \lessgtr 0 \;\; ; \;\; rh_6 = a_6 \cdot (rh_5) \lessgtr 0 \;\; .$$

Provided Routh-Hurwitz determinants $rh_1, \dots, rh_6$ are positive, household, business and financier receipts and payments converge toward steady-state vector $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$, irrespective of the initial composition of flows $\mathbf{X}^0 = (y_h^o, e_h^o, y_b^o, e_b^o, y_f^o, e_f^o)$ that may have resulted from a shock. Column $\mathbf{K} = (0, k_h, 0, k_b, 0, -k_f) = k_f \cdot (0, \kappa_h, 0, \kappa_b, 0, -1)$ mirrors the new loans to accepting sectors and the new deposits of issuing banks. $\mathbf{K}$ is not part of characteristic equation $|\mathbf{A} - \lambda \cdot \mathbf{I}|$ and thus does not affect the stability of dynamic systems (4.1) and (4.3). It can be easily proven that there exist household (α_{hj}; β_{hj}), business (α_{bj}; β_{bj}) and financier (α_{fj}; β_{fj}) income distributions and expenditure allocations which convert the Routh-Hurwitz determinants from positive to negative and *vice versa*. For example, let us assume that in an economy called Alpha, consumer (y_h, e_h) and producer (y_b, e_b) revenues and outlays are apportioned as shown in Table (4.7).

(4.7) Dynamic stability of economy Alpha

A statistical survey has estimated the income source and expenditure destination ratios of accepting sectors as follows:		The steady-state of economy Alpha is stable when issuing sector payments are allocated within the following limits:
$\alpha_{hw} = 8/9$ $\beta_{hu} = 7/8$ $\alpha_{hs} = 1/9$ $\beta_{hs} = 1/8$	$\alpha_{bu} = 6/7$ $\beta_{bw} = 5/6$ $\alpha_{bs} = 1/7$ $\beta_{bs} = 1/6$	$0 < \beta_{fw} < 1$, $0 < \beta_{fs} < 0.870249$

Alpha's Routh-Hurwitz conditions given by (4.6) comprise a system of six inequalities in terms of the three (linearly dependent) bank expenditure coefficients (β_{fw}, β_{fu}, β_{fs}). With *Mathematica* or other software, it can be confirmed that whenever bankers raise the share of outlay (β_{fs}) on financial paper above 87.02% of current payments and, thus, lower the share of outlay (β_{fw} + β_{fu}) on real goods below 12.98% of current payments, Alpha is destabilised or thrown into crisis. This result deserves further interpretation and exploration in the context of larger, more diversified and, therefore, more realistic models.

4.3. Monetary flows of institutional sectors

When all Routh-Hurwitz conditions are fulfilled, prototype Euclidean model (4.1) has a unique solution $\mathbf{X}^*$ given by time invariant system (4.2) because $rh_5 > 0$, $rh_6 = a_6 \cdot rh_5 > 0$ and, therefore, $a_6 = |\mathbf{A}| > 0$. Using Cramer's rule, we obtain equations (4.8) to (4.13), which reproduce the equilibrium flows $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ of exchange network (4.1). From these equations, we deduce that the steady-state incomes and expenditures of all institutional sectors are functions of the behaviour coefficients of consumers, producers and bankers as well as of the annual increase and sector allocation of bank credit:

$$y_h^* = k_f \cdot \begin{vmatrix} 0 & 0 & -\alpha_{bs} & \beta_{bw} & 0 & \beta_{fw}+\beta_{fs} \\ \kappa_h & -1 & 0 & 0 & 0 & 0 \\ 0 & \beta_{hu} & -1 & 0 & 0 & \beta_{fu}+\beta_{fs} \\ \kappa_b & 0 & 1 & -1 & 0 & 0 \\ 0 & \beta_{hs} & 0 & \beta_{bs} & -1 & 0 \\ -1 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \Big/ |\mathbf{A}| \gtrless 0 \qquad (4.8)$$

$$e_h^* = k_f \cdot \begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bw} & 0 & \beta_{fw}+\beta_{fs} \\ 1 & \kappa_h & 0 & 0 & 0 & 0 \\ -\alpha_{hs} & 0 & -1 & 0 & 0 & \beta_{fu}+\beta_{fs} \\ 0 & \kappa_b & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & \beta_{bs} & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \end{vmatrix} \Big/ |\mathbf{A}| \gtrless 0 \qquad (4.9)$$

$$y_b^* = k_f \cdot \begin{vmatrix} -1 & 0 & 0 & \beta_{bw} & 0 & \beta_{fw}+\beta_{fs} \\ 1 & -1 & \kappa_h & 0 & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & 0 & 0 & 0 & \beta_{fu}+\beta_{fs} \\ 0 & 0 & \kappa_b & -1 & 0 & 0 \\ 0 & \beta_{hs} & 0 & \beta_{bs} & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 \end{vmatrix} \Big/ |\mathbf{A}| \gtrless 0 \qquad (4.10)$$

$$e_b^* \quad = \quad k_f \cdot \begin{vmatrix} -1 & 0 & -\alpha_{bs} & 0 & 0 & \beta_{fw} + \beta_{fs} \\ 1 & -1 & 0 & \kappa_h & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 & 0 & \beta_{fu} + \beta_{fs} \\ 0 & 0 & 1 & \kappa_b & 0 & 0 \\ 0 & \beta_{hs} & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{vmatrix} / |\mathbf{A}| \gtrless 0 \qquad (4.11)$$

$$y_f^* \quad = \quad k_f \cdot \begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bw} & 0 & \beta_{fw} + \beta_{fs} \\ 1 & -1 & 0 & 0 & \kappa_h & 0 \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 & 0 & \beta_{fu} + \beta_{fs} \\ 0 & 0 & 1 & -1 & \kappa_b & 0 \\ 0 & \beta_{hs} & 0 & \beta_{bs} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{vmatrix} / |\mathbf{A}| \gtrless 0 \qquad (4.12)$$

$$e_f^* \quad = \quad k_f \cdot \begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bw} & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & \kappa_h \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \kappa_b \\ 0 & \beta_{hs} & 0 & \beta_{bs} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} / |\mathbf{A}| \gtrless 0 \qquad (4.13)$$

It is highlighted that a dynamic network such as (4.1) is controllable only when its equilibrium vector $\mathbf{X}^*$ is unique, stable and consists of flows (or elements) which possess the same sign. Equilibrium flows (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*), being linear functions of the liquidity increment k_f, will change in the same direction as k_f if, and only if, they possess the same sign as k_f. In physics and engineering, circuits (or networks) consisting of elements (or flows) having the same sign (or direction) are viewed as turning 'counter-clockwise' and are thus classified as 'right-handed screw' systems. For obvious reasons, sector receipts and payments generated by (4.1) should be positive under all circumstances. Provided this is so, monetary policy will have a uniformly contractionary or restraining impact on the revenues and outlays of households, businesses and financiers.

When the statistical data for behaviour coefficients (α_{hj}, β_{hj} ; α_{bj}, β_{bj} ; α_{fj}, β_{fj}) yield $|\mathbf{A}| \neq 0$, steady state $\mathbf{X}^*$ of differential system (4.1) is expressible in terms of policy parameters (k_f;

κ_h, κ_b). In Alpha's case (where $\alpha_{hw} = {}^8/_9$, $\alpha_{hs} = {}^1/_9$, $\beta_{hu} = {}^7/_8$, $\beta_{hs} = {}^1/_8$; $\alpha_{bu} = {}^6/_7$, $\alpha_{bs} = {}^1/_7$, $\beta_{bw} = {}^5/_6$, $\beta_{bs} = {}^1/_6$; $\alpha_{fs} = 1$, $\beta_{fw} = {}^1/_2$, $\beta_{fu} = {}^{1145}/_{3168}$, $\beta_{fs} = {}^{439}/_{3168}$) *Mathematica* generates equations (4.14), which duplicate the incomes and expenditures of consumers, producers and bankers as functions of annual expansion k_f and allocation ($1 = \kappa_h + \kappa_b$) of new loans / deposits among accepting sectors.

$$\begin{aligned}
y_h^* &= 9\cdot(59\,560+114\,523\cdot \kappa_h)\cdot k_f/1\,771\,763\\
e_h^* &= 8\cdot(67\,005-92\,632\cdot \kappa_h)\cdot k_f/1\,771\,763\\
y_b^* &= (184\,085-103\,109\cdot \kappa_h)\cdot k_f/253\,109\\
e_b^* &= 48\cdot(1\,438-3\,125\cdot \kappa_h)\cdot k_f/253\,109\\
y_f^* &= (-13\,523+82\,368\cdot \kappa_h)\cdot k_f/1\,771\,763\\
e_f^* &= 3168\cdot(555+26\cdot \kappa_h)\cdot k_f/1\,771\,763
\end{aligned} \tag{4.14}$$

Normally, the banking system balance sheet should be expanded every year (i.e. $k_f > 0$) because labour and other resources grow naturally while salaries and prices are inflexible downwards. Equations (4.15) show the partial derivatives of sector flows (4.14) with respect to the allocation of new loans.

$$\begin{aligned}
&\partial y_h^*/\partial\kappa_h = +\,0.581741\cdot k_f \quad \partial y_b^*/\partial\kappa_h = -\,0.40737\cdot k_f \quad \partial y_f^*/\partial\kappa_h = +\,0.0464893\cdot k_f\\
&\partial e_h^*/\partial\kappa_h = -\,0.418259\cdot k_f \quad \partial e_b^*/\partial\kappa_h = +\,0.59263\cdot k_f \quad \partial e_f^*/\partial\kappa_h = +\,0.0464893\cdot k_f
\end{aligned} \tag{4.15}$$

Partial derivatives (4.15) imply that redistribution of credit in favour of consumers (against producers) affects positively four, and negatively two, of the six endogenous streams. It will subsequently become evident that, from a policy standpoint, annual sales (ρ_w^*, ρ_u^*, ρ_s^*) of market goods are more important than annual flows (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*) of institutional sectors, although the former depend on the latter.

4.4 Preconditions for a dirigible dynamic system

The elements of vector $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ may be greater or smaller than zero, depending on sector behaviour coefficients (α_{hj}, β_{hj} ; α_{bj}, β_{bj} ; α_{fj}, β_{fj}) and credit policy parameters (k_f; κ_h, κ_b). Democratically elected governments view the likelihood of insufficient household, business and financier revenue and outlay with great apprehension. As a result, they consider the capability of modifying economic equilibrium as *sine qua non*. In most economies, the central bank has less influence over (β_{fw}, β_{fu} or β_{fs}) and more influence over (k_f, κ_h or κ_b and r). According to dynamic control theory, steady-state vector $\mathbf{X}^*$ is 'steerable' if, and only if, it consists of i) unique and ii) stable elements, which iii) possess the same sign. If, in addition, the intervening authority has iv) power over a number of tools that is equal to the number of targets, the dynamic system becomes 'perfectly controllable'. Below, in the context of model (4.1), we explain the necessity for a dynamic system to be characterised by all four qualities in order to be 'perfectly dirigible'.

i) According to Euclidean methodology, closed exchange networks are reproduced by systems of first-order non-homogeneous linear differential equations of the general type $\mathbf{dX/dt} = \mathbf{A{\cdot}X} - \mathbf{K}$. When $|\mathbf{A}| = 0$, the mathematical model is homogeneous and, consequently, solved by an indefinite number of vectors. In this case, at least one Routh-Hurwitz condition, i.e. $rh_6 = a_6{\cdot}(rh_5) = 0$, is not positive. It follows that an exchange network with multiple equilibriums is unstable and uncontrollable. No one is sure where such an economy will 'end up' or 'nest' because the movement of flows – toward the stable and away from the unstable positions – depends on initial flows $\mathbf{X^0} = (y_h^0, e_h^0, y_b^0, e_b^0, y_f^0, e_f^0)$. Any attempt to modify policy **K** so as to achieve target $\mathbf{X}^{\#}$ is doomed to failure because of perverse sector and market interaction (captured by structural matrix **A**).

ii) In an unstable model, pondering over policy options is futile because equilibrium vectors are temporary and variable. Irrespective of premeditated or accidental jolts, such an economic system does not home-in on any steady state. Exchange networks with volatile flows are beyond official or unofficial control. By being unpredictable, they do not mirror economic life which, more or less, repeats itself. According to our premises, the monetary authority may influence banks to confine current outlay within proportions ($\beta_{fw} + \beta_{fu} + \beta_{fs} = 1$) that turn all Routh-Hurwitz determinants (4.6) positive and the system's equilibrium vector $\mathbf{X}^*$ stable and unique at the same time. In order to play such a stabilizing role, the monetary authority should:

- possess the data about each sector's income sources and expenditure destinations;
- estimate the bank spending allocations outside which interaction of sectors and markets becomes unstable;
- ensure that banks stay within the outlay limits that keep the system stable.

Alternatively, the monetary authority may engage in a protracted campaign to convince consumers and producers to reapportion their revenue sources and outlay destinations within ranges of (α_{hj}, β_{hj}) and (α_{bj}, β_{bj}), which would stabilise the exchange network. However, in doing so, the central bank should bear in mind that old habits die hard. If, contrary to common sense, households, businesses and financiers continue to behave in ways which destabilise the economy, then everyone – the innocent as well as the guilty – will eventually pay for the system's collapse. Sometimes, the consequences emanating from ignoring the laws of economics are as harsh as the consequences emanating from disregarding the laws of nature.

iii) Let us assume that exchange network (4.1) possesses a stable ($rh_1 > 0, \ldots, rh_6 > 0$) and thus unique ($|\mathbf{A}| \neq 0$) steady-state $\mathbf{X}^*$. Two of the four controllability prerequisites are met. In addition, dynamic control theory demands that vector elements $y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*$ have the same sign. Exchange networks should result in positive household, business and financier incomes and expenditures in order to ensure, firstly, survival of economic actors and, secondly, adjustment of sector flows in line with the monetary financial balance sheet. In engineering dynamics, endogenous variables that are always greater than zero comprise 'right-turning' circuits or 'right-handed screw' systems. In the event a twin pair (y_i^*, e_i^*) had opposite signs, receipts and payments would change contrary to experience. Equations (4.8) to (4.13) show that the annual flows of consumers (y_h^*, e_h^*), producers (y_b^*, e_b^*) and bankers (y_f^*, e_f^*) are linear functions of the yearly addition k_f and sector allocation (κ_h, κ_b) of bank credit. Due to natural growth of resources and downward inflexibility of wages, we have concluded that liquidity increment k_f should always be positive and, by all means, allocated among households and businesses in proportions (κ_h, κ_b) which would keep sector revenues and outlays greater than zero. To this end, banks should consult with the monetary authority regarding the magnitude and allocation of new loans. The exchange network welcomes and respects central bankers who are capable of regulating the monetary financial balance sheet according to well understood mathematical models of economic reality and widely accepted economic, political and social principles.

iv) Elementary model (4.1) incorporates six endogenous variables ($y_h, e_h, y_b, e_b, y_f, e_f$), six behaviour coefficients (α_{hj}, β_{hj} ; α_{bj}, β_{bj} ; α_{fj}, β_{fj}) and three credit instruments (k_f, κ_h or κ_b, r). *Prima facie*, the economic system does not appear fully dirigible because it includes more endogenous variables (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*) than policy tools (k_f, κ_h or κ_b, r). However, once we take into account that households, businesses and financiers focus on the gross turnover (ρ_w^*, ρ_u^*, ρ_s^*) of their services, products and securities relative to the available quantities ($q_w^\#$, $q_u^\#$, $q_s^\#$), the exchange network becomes perfectly controllable.

$$\rho_w^{\,*} = y_w^{\,*} = \beta_{\mathrm{bw}}\cdot e_b^{\,*} + \beta_{\mathrm{fw}}\cdot e_f^{\,*} = \alpha_{\mathrm{hw}}\cdot y_h^{\,*} = e_w^{\,*}$$
$$\rho_u^{\,*} = y_u^{\,*} = \beta_{\mathrm{hu}}\cdot e_h^{\,*} + \beta_{\mathrm{fu}}\cdot e_f^{\,*} = \alpha_{\mathrm{bu}}\cdot y_b^{\,*} = e_u^{\,*} \qquad (4.16)$$
$$\rho_s^{\,*} = y_s^{\,*} = \beta_{\mathrm{hs}}\cdot e_h^{\,*} + \beta_{\mathrm{bs}}\cdot e_b^{\,*} + \beta_{\mathrm{fs}}\cdot e_f^{\,*} = \alpha_{\mathrm{hs}}\cdot y_h^{\,*} + \alpha_{\mathrm{bs}}\cdot y_b^{\,*} + \alpha_{\mathrm{fs}}\cdot y_f^{\,*} = e_s^{\,*}$$

Relations (4.16) reproduce equilibrium revenue (y_w^*, y_u^*, y_s^*) from and equilibrium outlay (e_w^*, e_u^*, e_s^*) on market goods as reflected by cells A12, B12 and C12 of Table (3.1). Dividing annual sales (ρ_w^*, ρ_u^*, ρ_s^*) by:

- the applicable salary p_w, price p_u and value p_s, we obtain the years of labour q_w^*, pieces of output q_u^* and number of consols q_s^* traded in the corresponding market during the relevant period.
- the available man years $q_w^\#$, output pieces $q_u^\#$ and liquidity units $q_s^\#$, we obtain the salary $p_w^\#$, price $p_u^\#$ and value $p_s^\#$ that would clear the corresponding market during the relevant period.

Demanded quantities (q_w^*, q_u^*, q_s^*) may be excessive or insufficient in relation to available quantities ($q_w^\#$, $q_u^\#$, $q_s^\#$) of services, products and securities, respectively. The monetary authority has the option of narrowing deficits and / or surpluses by varying the supply k_f, the allocation (κ_h or κ_b) and the cost (reward) r of bank loans (deposits). A concrete example is presented in the next chapter.

4.5. Tug of war between owners

Most economic agents, sectors, etc., – being endowed with limited years of labour, pieces of output and units of liquidity – are tormented by scarcity. Salaries, prices and values are the outcome of intricate games between opposed groups of owners within each sector. The one faction encompasses those who insist upon high quotations because they desire relatively high stocks of the economic good under their control. The other faction includes those who acquiesce to low quotations because they are happy with relatively low stocks of the economic good under their control. Normally, market participants bargain so as to earn the maximum revenue (that is consistent with their resource endowment) irrespective of whether it emanates from high quotation (and thus high unsold reserve) or low quotation (and thus low unsold reserve) of the service, product or security traded.

Our approach allows for numerous quotations per year of labour, per piece of output, per unit of liquidity, etc. Salaries, prices and values can range from the maximum quotation, at which only one ‘atom’ of an economic good is bought and sold, to the clearance quotation

($p_w^{\#}$, $p_u^{\#}$, $p_s^{\#}$) at which all 'atoms' of an economic good are bought and sold. It should always be remembered that, due to grouping of agents into sectors, countries, zones, etc., only average salaries, prices and values are relevant to, compatible with and utilized by the Euclidean method. Granted inflexibility of salaries, prices and values, quantity gaps of market goods are not automatically cleared within the time unit. It follows that during any year consumers, enterprises and financiers are, respectively, divided among owners of:

- employed and wasted labour,
- utilised and stockpiled output,
- transacted and dormant liquidity.

Central banks and national governments, which manipulate policy instruments without clear guidance from a comprehensive mathematical model of the closed exchange network, should be ready to deal with results that are considered haphazard or objectionable by affected parties.

4.6. Computations for policy formulation

The order of computations – to be accomplished by means of *Mathematica* or other software – is presented below:

i) The dynamic properties of a mathematical model that duplicates the economic system should be studied for every year in respect of which there are authentic or hypothetical data. To this end, we should substitute the statistics for symbols (α_{hj}, β_{hj} ; α_{bj}, β_{bj} ; α_{fj}, β_{fj}) in exchange network (4.1) in order to assess the Routh-Hurwitz conditions (4.6), which define stability of monetary flows. If we wish to focus on the dynamic impact of one or two behaviour coefficients, we should let the corresponding symbols in the differential equation system and the Routh-Hurwitz determinants. For example, we may substitute the data for the behaviour coefficients of accepting sectors and leave the issuing sector's outlay percentages (β_{fw}, β_{fu} or β_{fs}) as control variables. The resulting Routh-Hurwitz determinants rh_n, $n = 1, \dots, 6$, form a system of polynomial inequalities that can be solved by *Mathematica* or other software in terms of β_{fw}, β_{fu} or β_{fs}. Before and during a financial crisis, such an exercise would enable the monetary authority to advise banking institutions how to apportion their annual expenditure so that the exchange network's Routh-Hurwitz determinants stay or become greater than zero. Monetary authorities, by securing dynamic stability, also ensure uniqueness of the economy's equilibrium vector $\mathbf{X}^*$

$= (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ because, as we have already seen, polynomial coefficient $a_6 = |\mathbf{A}|$, $rh_5 > 0$, $rh_6 = a_6 \cdot rh_5 > 0$ and, therefore, $a_6 = |\mathbf{A}| > 0$.

ii) At the next stage, by substituting in algebraic system (4.2) the data for:

- the income source and expenditure destination ratios of households (α_{hj}, β_{hj}), businesses (α_{bj}, β_{bj}) and financiers (α_{fj}, β_{fj})
- the credit policy parameters, i.e. the annual rise k_f and percent allocation of bank liabilities among consumers (κ_h) and producers (κ_b),

we get in position to obtain the exchange network's steady-state receipts and payments as well as other interesting details. Specifically:

(1) by employing Cramer's rule, we may compute the equilibrium nominal flows $\mathbf{X}^* = (y_i^*, e_i^*)$ of institutional sectors $i = h, b, f$, given by (4.8 to 4.13);

(2) by substituting the equilibrium nominal flows $(y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ – obtained via Cramer's rule – in equations (4.16), we may calculate annual sales (gross turnover) concerning labour ρ_w^*, output ρ_u^* and liquidity ρ_s^*;

(3) by dividing the gross turnover of services ρ_w^*, products ρ_u^* and securities ρ_s^* by the corresponding salary, price and value (p_w, p_u, p_s), we may derive the years of labour q_w^*, pieces of output q_u^* and number of consols q_s^* demanded in accordance with the particular economy's profile;

(4) by comparing demanded (q_w^*, q_u^*, q_s^*) with available ($q_w^\#$, $q_u^\#$, $q_s^\#$) years of labour, pieces of output and units of liquidity, we may assess the quantitative deficits or surpluses ($q_w^* - q_w^\#$, $q_u^* - q_u^\#$, $q_s^* - q_s^\#$) of market goods and thus the potential rise or fall of market quotations;

(5) by dividing gross sales (ρ_w^*, ρ_u^*, ρ_s^*) by the available years of labour $q_w^\#$, pieces of output $q_u^\#$ and number of consols $q_s^\#$, we may compute the average salary $p_w^\#$, price $p_u^\#$ and value $p_s^\#$ that would equalise the demanded with the available amount.

It is emphasised that, in the short run, differences between current and clearance quotations of services ($p_w - p_w^\#$), products ($p_u - p_u^\#$) and securities ($p_s - p_s^\#$) are sustained by myopic attitudes and slow reactions of consumers, entrepreneurs and bankers to excess or deficient reserves of the market good under their respective control.

iii) At a third stage, we should explore the limits imposed on credit policy by the structure of the network. To this end, we may substitute the data for behaviour coefficients (α_{hj}, β_{hj} ; α_{bj}, β_{bj} ; α_{fj}, β_{fj}) and monetary expansion k_f in equilibrium flows (4.8) to (4.13) but leave credit shares (κ_h or κ_b) as unknown parameters. The resulting revenue and outlay functions (4.17) may be solved as a system of six linear inequalities in terms of κ_h or κ_b.

$$\begin{aligned} &y_h^*(\kappa_h \text{ or } \kappa_b) > 0,\; y_b^*(\kappa_h \text{ or } \kappa_b) > 0,\; y_f^*(\kappa_h \text{ or } \kappa_b) > 0 \\ &e_h^*(\kappa_h \text{ or } \kappa_b) > 0,\; e_b^*(\kappa_h \text{ or } \kappa_b) > 0,\; e_f^*(\kappa_h \text{ or } \kappa_b) > 0 \end{aligned} \tag{4.17}$$

If steady state $\mathbf{X}^*$ is stable and thus unique, as previously discussed, the monetary authority may forestall economic collapse by guiding banking institutions to allocate new loans between consumers and producers in proportions (κ_h, κ_b) that generate positive sector flows ($y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*$). *Mathematica* and other software can easily solve system (4.17) for the maximum and minimum shares of annual credit to households κ_h and businesses κ_b, which keep sector receipts and payments greater than zero.

iv) Equations (4.8) to (4.13) reproduce institutional sector incomes and expenditures as functions of behaviour coefficients and credit policies. By substituting i) the statistics for behaviour coefficients (α_{ij}, β_{ij}) in (4.8) to (4.13) and ii) the resulting equations into system (4.16), we derive equilibrium sales of services, products and securities as functions of annual increment k_f and allocation (κ_h or κ_b) of liquidity. Subsequently, we may rewrite (4.16) so as to relate annual turnovers with market quotations and available quantities:

$$\begin{aligned} &\rho_w^*/p_w = \rho_w^*(k_f, \kappa_h \text{ or } \kappa_b)/p_w = q_w^\# \\ &\rho_u^*/p_u = \rho_u^*(k_f, \kappa_h \text{ or } \kappa_b)/p_u = q_u^\# \\ &\rho_s^*/p_s = \rho_s^*(k_f, \kappa_h \text{ or } \kappa_b)/p_s = q_s^\# \end{aligned} \tag{4.18}$$

Equations (4.18) show that steady-state orders for labour years ($e_w^*/p_w = q_w^*$), output pieces ($e_u^*/p_u = q_u^*$) and liquidity units ($e_s^*/p_s = q_s^*$) depend on market quotations but also on the increase and allocation of credit. Differences between demanded q_j^* and available $q_j^\#$ amounts of services $q_w^\#$, products $q_u^\#$ and securities $q_s^\#$ are not bridged automatically because salaries, prices and values tend to be inflexible. Gaps ($q_j^* - q_j^\#$), j = w, u, s, between desired and accessible quantities of labour, output and liquidity comprise a system of three equations in three unknowns (k_f, κ_h or κ_b, r) as reflected by (4.19):

$$\textbf{\textit{Labour gap}} \quad (q_w^* - q_w^\#) = \{\rho_w^*(k_f, \kappa_h \text{ or } \kappa_b) / p_w\} - q_w^\#$$

$$\textbf{\textit{Output gap}} \quad (q_u^* - q_u^\#) = \{\rho_u^*(k_f, \kappa_h \text{ or } \kappa_b) / p_u\} - q_u^\# \tag{4.19}$$

$$\textbf{\textit{Liquidity gap}} \quad (q_s^* - q_s^\#) = \{\rho_s^*(k_f, \kappa_h \text{ or } \kappa_b) / p_s\} - q_s^\#$$

The above deficits and / or surpluses can be bridged through appropriate modifications of the policy parameters. With the assistance of mathematical software, system (4.19) can be put in the form $\mathbf{\Psi} \cdot \mathbf{\Pi} = \mathbf{Q}$ and solved for the combination $\mathbf{\Pi}(k_f, \kappa_h$ or κ_b and r) that will zero quantity gaps during the next time unit. While applying Euclidean modelling methodology to periods longer than a calendar year, two distinct spreadsheets should be compiled:

- The first should record the behaviour coefficients, policy parameters, market quotations and available quantities likely to prevail during the forecast period.
- The second spreadsheet should record the sector and market flows as well as the financial stocks which result from the above mentioned sets of parameters.

Projections covering more than a handful of years may be easily questioned because they involve too many guesses about important factors (such as climatic conditions, product preferences, international relations, competition regulations, technology shocks, etc.), which affect economic, political and social developments. Our mathematical and programmable approach enables examination of numerous scenarios and alternative policies at minimum cost. Euclidean models are built and applied in accordance with the principles of simplicity, uniformity and generality.

Part II. Elementary Examples and Applications

Chapter 5. Building and Utilising an Example

In this chapter, further mathematical and economic aspects of the modular exchange network are explored in preparation for more far-reaching, more complex and more realistic simulations. Our methodology has made possible mathematical replication of any economic system, given the data concerning sector behaviour, official policy, market quotations and available quantities. In the following pages, we will deal with an elementary network called Alpha, the numerical facts of which are listed below.

5.1. The statistical profile

As a result of habit, biological necessity and other exogenous factors, consumers distribute their demand for revenue among available sources and allocate their supply of outlay among pertinent destinations as follows:

Income share from:	Services $\alpha_{hw} = 8/9$	Securities $\alpha_{hs} = 1/9$
Expenditure share for:	Products $\beta_{hu} = 7/8$	Securities $\beta_{hs} = 1/8$

As a result of management practices in relation to employment, technology, liquidity, etc., producers distribute their demand for revenue among available sources and allocate their supply of outlay among applicable destinations as shown below:

Income share from:	Products $\alpha_{bu} = 6/7$	Securities $\alpha_{bs} = 1/7$
Expenditure share for:	Services $\beta_{bw} = 5/6$	Securities $\beta_{bs} = 1/6$

Due to constitutional and legal arrangements, 100% of the revenue of bankers is derived from perpetual bonds. In order to serve economic, political and other needs, they allocate their expenditure as follows:

Outlay share for: Services $\beta_{fw} = ½$ Products $\beta_{fu} = {}^{1145}/_{3168}$ Securities $\beta_{fs} = {}^{439}/_{3168}$

The Dollar ($) is the accounting and monetary unit used by Alpha's agents and sectors for computation of current transactions and capital stocks. Annual liquidity increment $k_f = \$10^9$ is apportioned as additional credit between consumers and entrepreneurs:

Credit Share: Households $\kappa_h = ½$ Businesses $\kappa_b = ½$

With the above data, it is possible to construct the dynamic mathematical model and to evaluate the stability, steady-state and circulation properties of economy Alpha. To this end, each algebraic symbol (α_{hj}, β_{hj} ; α_{bj}, β_{bj} ; α_{fj}, β_{fj}) in (4.1) is replaced by the respective statistic to obtain system (5.1) consisting of six linear differential equations:

$$
\begin{array}{lll}
(1) & dy_h/dt = -y_h - (1/7)\cdot y_b + (5/6)\cdot e_b + ({}^{2023}/_{3168})\cdot e_f & \\
(2) & de_h/dt = y_h - \mathrm{k_h} - e_h & \\
(3) & dy_b/dt = -(1/9)\cdot y_h + (7/8)\cdot e_h - y_b + (1/2)\cdot e_f & \\
(4) & de_b/dt = y_b - \mathrm{k_b} - e_b & (5.1)\\
(5) & dy_f/dt = (1/8)\cdot e_h + (1/6)\cdot e_b - y_f & \\
(6) & de_f/dt = y_f + \mathrm{k_f} - e_f &
\end{array}
$$

5.2. Stability conditions

Alpha's monetary flows make up a system of six first-order non-homogeneous linear differential equations that possesses the general form $\mathbf{dX/dt} = \mathbf{A}\cdot\mathbf{X} - \mathbf{K}$. Discussions about alternative equilibriums and policies are meaningful if, and only if, interaction of sectors and markets is globally stable. The homogeneous system of differential equations that corresponds to the non-homogeneous system (5.1) is given below:

$$
\begin{vmatrix} dy_h/dt \\ de_h/dt \\ dy_b/dt \\ de_b/dt \\ dy_f/dt \\ de_f/dt \end{vmatrix}
=
\begin{vmatrix}
-1 & 0 & -{}^1/_7 & {}^5/_6 & 0 & {}^{2023}/_{3168} \\
1 & -1 & 0 & 0 & 0 & 0 \\
-{}^1/_9 & {}^7/_8 & -1 & 0 & 0 & {}^1/_2 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & {}^1/_8 & 0 & {}^1/_6 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1
\end{vmatrix}
\cdot
\begin{vmatrix} y_h \\ e_h \\ y_b \\ e_b \\ y_f \\ e_f \end{vmatrix}
= \mathbf{A}\cdot\mathbf{X} = \boldsymbol{\lambda}\cdot\mathbf{X} = \mathbf{0} \quad (5.2)
$$

Non-homogeneous system (5.1) is globally stable if, and only if, the Routh-Hurwitz determinants – which consist of coefficients a_i, $i = 0, \ldots, 6$ of the homogeneous system's characteristic equation $|\mathbf{A} - \boldsymbol{\lambda}\cdot\mathbf{I}|$ – are positive:

$$
|\ \mathbf{A} - \boldsymbol{\lambda}\cdot\mathbf{I}\ | =
\begin{vmatrix}
-1-\lambda & 0 & -{}^1/_7 & {}^5/_6 & 0 & {}^{2023}/_{3168} \\
1 & -1-\lambda & 0 & 0 & 0 & 0 \\
-{}^1/_9 & {}^7/_8 & -1-\lambda & 0 & 0 & {}^1/_2 \\
0 & 0 & 1 & -1-\lambda & 0 & 0 \\
0 & {}^1/_8 & 0 & {}^1/_6 & -1-\lambda & 0 \\
0 & 0 & 0 & 0 & 1 & -1-\lambda
\end{vmatrix}
= \quad (5.3)
$$

$= \lambda^6 + 6\cdot\lambda^5 + ({}^{944}/_{63})\cdot\lambda^4 + ({}^{30473}/_{1512})\cdot\lambda^3 + ({}^{2601727}/_{177408})\cdot\lambda^2 + ({}^{11556863}/_{2395008})\cdot\lambda + ({}^{253109}/_{1368576}) =$

$$= a_0\cdot\lambda^6 + a_1\cdot\lambda^5 + a_2\cdot\lambda^4 + a_3\cdot\lambda^3 + a_4\cdot\lambda^2 + a_5\cdot\lambda + a_6 = 0$$

With *Mathematica*'s help, we substitute coefficients a_i, $i = 0,\dots, 6$ of polynomial (5.3) in the principle minors of matrix (4.5) in order to compute Routh-Hurwitz conditions rh_n, $n = 1, \dots, 6$. The said determinants are equal to $rh_1 = 6$, $rh_2 = 69.751$, $rh_3 = 906.766$, $rh_4 = 8733.36$, $rh_5 = 42141.9$ and $rh_6 = 0.184943$. All Routh-Hurwitz determinants of Alpha, are greater than zero, and thus consumer, producer and bank incomes and expenditures converge toward $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$, irrespective of initial vector $\mathbf{X}^0 = (y_h^0, e_h^0, y_b^0, e_b^0, y_f^0, e_f^0)$. From equations (4.16), it can be inferred that annual sales of labour, output and liquidity tend toward steady state $\mathbf{R}^* = (\rho_w^*, \rho_u^*, \rho_s^*)$ – corresponding to $\mathbf{X}^*$ – regardless of starting combination $\mathbf{R}^0 = (\rho_w^0, \rho_u^0, \rho_s^0)$.

5.3. The equilibrium solution

Alpha's equilibrium position $\mathbf{X}^*$ is the solution of the synchronous (time-invariant) non-homogeneous system of linear equations:

$$\begin{vmatrix} -1 & 0 & -{}^1/_7 & {}^5/_6 & 0 & {}^{2023}/_{3168} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -{}^1/_9 & {}^7/_8 & -1 & 0 & 0 & {}^1/_2 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & {}^1/_8 & 0 & {}^1/_6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h^* \\ e_h^* \\ y_b^* \\ e_b^* \\ y_f^* \\ e_f^* \end{vmatrix} = \begin{vmatrix} 0 \\ k_h \\ 0 \\ k_b \\ 0 \\ -k_f \end{vmatrix} = \begin{vmatrix} 0 \\ \kappa_h \\ 0 \\ \kappa_b \\ 0 \\ -1 \end{vmatrix} \cdot \mathbf{k_f} = \mathbf{A}\cdot\mathbf{X}^* = \mathbf{K} \qquad (5.4)$$

Bank balance-sheet increment k_f and credit share κ_h of consumers or κ_b of producers are the control parameters which shape nominal flows. According to Cramer's rule, $\mathbf{X}^*$ may be derived in terms of $\mathbf{A}$ and $\mathbf{K}$ because determinant $|\mathbf{A}| \neq 0$. Annual credit $k_f = \$10^9$ is equally divided ($\kappa_h = \kappa_b = ½$) between households and businesses. Table (5.5) presents the incomes and expenditures of Alpha's sectors as functions of annual liquidity supply k_f.

(5.5) Sector flows of economy Alpha

	Income	**Expenditure**
Households	$y_h^* = 0.59342\cdot k_f = y_h \cdot k_f$	$e_h^* = 0.093412\cdot k_f = e_h \cdot k_f$
Entrepreneurs	$y_b^* = 0.52360\cdot k_f = y_b \cdot k_f$	$e_b^* = 0.02360\cdot k_f = e_b \cdot k_f$
Financiers	$y_f^* = 0.01561\cdot k_f = y_f \cdot k_f$	$e_f^* = 1.01561\cdot k_f = e_f \cdot k_f$

Factors (y_i, e_i) comprise circulation vector $\mathbf{U} = (1/k_f)\cdot\mathbf{X}^*$. Adding sector receipts and payments separately, we obtain total revenue and outlay as functions of credit increment k_f:

$$Y^* = (y_h^* + y_b^* + y_f^*) = (0.59342 + 0.52360 + 0.01561)\cdot k_f = c\cdot k_f = \$1.13263\cdot 10^9$$
$$E^* = (e_h^* + e_b^* + e_f^*) = (0.09342 + 0.02360 + 1.01561)\cdot k_f = c\cdot k_f = \$1.13263\cdot 10^9 \tag{5.6}$$

Analytic geometry and Chapters **9** and **10** explain that c = 1.13264 is the scale factor applied to the triangle or polygon that reflects the form, structure and circulation of the specific economy. Table (5.5) and equations (5.6) prove that takings Y^* and outgoings E^* are distributed among households, businesses and financiers in the following ratios:

$$(y_h + y_b + y_f) = (52.4\% + 46.2\% + 1.4\%) = 100\%$$
$$(e_h + e_b + e_f) = (8.2\% + 2.1\% + 89.7\%) = 100\% \tag{5.7}$$

From Table (3.1) or equations (4.16), we derive functions (5.8), which duplicate annual sales of labour ρ_w^*, output ρ_u^* and securities ρ_s^*:

$$\rho_w^* = {}^5/_6\cdot e_b^* + {}^1/_2\cdot e_f^* \quad ; \quad \rho_u^* = {}^7/_8\cdot e_h^* + {}^{1145}/_{3168}\cdot e_f^* \quad ; \quad \rho_s^* = {}^1/_8\cdot e_h^* + {}^1/_6\cdot e_b^* + {}^{439}/_{3168}\cdot e_f^* \tag{5.8}$$

Copying values e_h^*, e_b^* and e_f^* from Table (5.5) into set (5.8) and then dividing by c, we find that Alpha's turnover R^* is distributed among traded goods as follows:

$$(\rho_w + \rho_u + \rho_s) = (46.6\% + 39.6\% + 13.8\%) = 100\% \tag{5.9}$$

Annual flows $Y^* = E^* = R^*$ may be likened to circles with radius $r = k_f$; their circumferences ($2\pi r$) are split into fractions $(y_h + y_b + y_f)\,({}^1/_c) = (e_h + e_b + e_f)\,({}^1/_c) = (\rho_w + \rho_u + \rho_s)\,({}^1/_c)$. With a little imagination, more correspondences between Euclidean economics and Euclidean geometry can be identified. Using *Mathematica* or other software, it can be shown that, if the liquidity increment k_f were allocated in ratios $\kappa_h = 53.02\%$ and $\kappa_b = 46.98\%$ – instead of $\kappa_h = \kappa_b = ½$ – the division of Alpha's yearly flows among sectors and markets would be:

$$(y_h + y_b + y_f) = (53.6\% + 44.9\% + 1.5\%) \approx 100\%$$
$$(e_h + e_b + e_f) = (7.1\% + 3.6\% + 89.3\%) \approx 100\% \tag{5.10}$$
$$(\rho_w + \rho_u + \rho_s) = (47.7\% + 38.4\% + 13.9\%) \approx 100\%$$

– instead of (5.7) and (5.9). The above equations illustrate how bank lending policies affect sector $\mathbf{X}^*(y_h^*, e_h^*, y_b, e_b^*, y_f^*, e_f^*)$ and market $\mathbf{R}^*(\rho_w^*, \rho_u^*, \rho_s^*)$ receipts and payments. Conclusions drawn from the elementary model are valid for economic systems of any size.

5.4. Quotation or quantity gaps?

Table (5.11) presents the average quotations p_j and the available quantities $q_j^{\#}$ of services w, products u and securities s recorded by the latest statistical survey of Alpha.

(5.11) Quotations and quantities of market goods

Market Good	Average Quotation	Available Quantity
Year of Labour	$p_w = \$470.90$	$q_w^{\#} = 10^6$ years
Piece of Output	$p_u = \$0.38$	$q_u^{\#} = 10^9$ pieces
Unit of Consol	$p_s = \$13.60$	$q_s^{\#} = 10^7$ units

Each issued (sold) and accepted (purchased) consol is guaranteed to pay the *numeraire* unit in perpetuity. The present value of a bond that annually earns the unit of liquidity *ad infinitum* is equal to the inverse of the discount rate. Substituting monetary balance sheet increment $k_f = \$10^9$, first in Table (5.5) and then in equations (5.8), we obtain steady-state income y_j^* from and expenditure e_j^* on services, products and securities:

$$\begin{aligned} \rho_w^* &= p_w \cdot q_w^* = \$5.27484 \cdot 10^8 \\ \rho_u^* &= p_u \cdot q_u^* = \$4.48800 \cdot 10^8 \\ \rho_s^* &= p_s \cdot q_s^* = \$1.56346 \cdot 10^8 \end{aligned} \tag{5.12}$$

Dividing the gross turnover ρ_j^* of market good j by its average quotation p_j, we figure the quantity traded q_j^* during the time unit:

$$\begin{aligned} q_w^* &= \$5.27484 \cdot 10^8 / \$470.90 \approx 1.12016 \cdot 10^6 \text{ years of labour;} \\ q_u^* &= \$4.48800 \cdot 10^8 / \$0.38 \approx 1.18108 \cdot 10^9 \text{ pieces of output;} \\ q_s^* &= \$1.56346 \cdot 10^8 / \$13.60 \approx 1.14962 \cdot 10^7 \text{ number of consols.} \end{aligned} \tag{5.13}$$

Subtracting the demanded from the available quantity of services, products and securities, we reveal the short supply that is prevailing in each market:

$$\begin{aligned} q_w^{\#} - q_w^* &\approx -0.120156 \cdot 10^6 \text{ years of labour;} \\ q_u^{\#} - q_u^* &\approx -0.181076 \cdot 10^9 \text{ pieces of output;} \\ q_s^{\#} - q_s^* &\approx -0.149623 \cdot 10^7 \text{ number of consols.} \end{aligned} \tag{5.14}$$

Human instincts motivate each economic agent to attain the mixture of revenue sources and outlay destinations that maximises net worth, given the corresponding budget constraint. Within the time unit, households, businesses and financiers are not driven to raise or lower salaries, prices and values in order to eliminate any deficit or excess of the good under their respective control. Nevertheless, under all circumstances, money paid for (e_j^*) is *de facto* equal to money received from (y_j^*) any trade deal. Division of the annual turnover of services ρ_w^*, products ρ_u^* and securities ρ_s^* by the available amount $q_j^\#$, j = w, u, s, yields the notional quotation $p_j^\#$ that would balance quantitative demand with quantitative supply:

$$p_w^\# = \$5.27484 \cdot 10^8 / 10^6 \approx \$527.48 \text{ per year of labour;}$$

$$p_u^\# = \$4.48800 \cdot 10^8 / 10^9 \approx \$0.45 \text{ per piece of output;} \qquad (5.15)$$

$$p_s^\# = \$1.56346 \cdot 10^8 / 10^8 \approx \$15.63 \text{ per unit of consol.}$$

Surpluses and shortages of labour years, output pieces and liquidity units are brought to zero by the set of clearing quotations $p_j^\#$, j = w, u, s. However, this set may not be immediately achievable. From one year to the next, salaries, prices and values p_j tend to change proportionally to the difference between clearing and prevailing quotations:

$$\begin{aligned}
\pi_w &= (p_w^\# - p_w) / p_w = (\$527.48 - \$470.90) / \$470.90 = +\ 12.01\% \\
\pi_u &= (p_u^\# - p_u) / p_u = (\$0.45 - \$0.38) / \$0.38 = +\ 18.42\% \qquad (5.16) \\
\pi_s &= (p_s^\# - p_s) / p_s = (\$15.63 - \$13.60) / \$13.60 = +\ 14.93\%
\end{aligned}$$

Reported inflation (or deflation) π is a function of the weights assigned to traded goods by the statistical authority. Most consumer / cost-of-living and business / cost-of-operating indices exclude financial securities: The first justification emphasises distinction between real goods and promissory notes. Labour salaries and output prices decide the critical purchasing power of current flows, while the discount rate alters the uncritical present value of future wealth. A second but equally important reason is that the discount rate is the main, if not the only, means for bridging demand and supply of bank money or perpetual bonds. It is an indispensable policy tool that must be varied as required. The last but not least motive for exclusion is prevention of compensation claims by households and businesses for cost of living and cost of operating changes induced by official decisions regarding interest rates. For economic efficiency and financial transparency, unhindered flexibility of values is more essential than flexibility of salaries and prices. Turnover distribution reflected by (5.9) is one of many weight sets eligible for indicating latent inflation pressures. From this all-inclusive angle, Alpha's imminent inflation rate is:

$$\pi = (46.6 \text{ x } 12.01\%) + (39.6 \text{ x } 18.42\%) + (13.8 \text{ x } 14.93\%) = 14.95\% \qquad (5.17)$$

Beyond some point, available amounts of services $q_w^\#$, products $q_u^\#$ and consols $q_s^\#$ are practically fixed within the time unit, i.e. $t_o \leq t_1 \leq t_o+1$. The authorities of Alpha may wish to eliminate quantity gaps in a way that would preclude quotations rising by an average of *14.95%* within a year. Acting together with other monetary institutions, the central bank is in position to regulate the increment of liquidity, the allocation of loans and the rate of discount (which drive demand for labour years q_w^*, output pieces q_u^* and liquidity units q_s^*) so as to bridge the three gaps during the next period (i.e. $t_1 \le t_2 \le t_1+1$). It should be remembered that the share of loans destined for one sector may rise if the share of loans destined for another sector may fall. No modern economy seeking stability and growth can afford to overlook the bone of contention called 'reallocation of liquid resources'.

5.5. Design and implementation of policy

Alpha's shortage of traded goods and potential rise of quotations may be eliminated within a year, provided the banking authority utilizes a mathematical model, such as (5.1), to guide the exchange network. Given that $\kappa_h + \kappa_b = 1$ and $|\mathbf{A}| \neq 0$, linear system (5.4) may be solved (using *Mathematica* or other software) to yield sector revenues and outlays (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*) as functions of k_f and κ_h.

$$
\begin{array}{ll}
y_h^* = (9\, k_f/1771763)\cdot(59560+114523\cdot \kappa_h) & e_h^* = (8\, k_f/1771763)\cdot(67005-92632\cdot \kappa_h) \\
y_b^* = (k_f/253109)\cdot(184085-103109\cdot \kappa_h) & e_b^* = (48\, k_f/253109)\cdot(-1438+3125\cdot \kappa_h) \\
y_f^* = (k_f/1771763)\cdot(-13523+82368\cdot \kappa_h) & e_f^* = (3168\, k_f/1771763)\cdot(555+26\cdot \kappa_h)
\end{array}
\qquad (5.18)
$$

Looking at solutions (5.18), it may be observed that:

- steady-state flows (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*) vary along with the annual change of the banking system balance sheet k_f;
- ratios of steady-state flows (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*) are unaffected by the annual money supply increment k_f;
- the rise of issuing sector liabilities ($e_f^* - y_f^*$) is equal to the rise of accepting sector assets $(y_h^* + y_b^*) - (e_h^* + e_b^*)$.

Replacing k_f = $\$10^9$ and κ_h = ½ in (5.18), we compute the steady-state receipts and payments of consumers ($\$5\ 934\cdot10^5$, $\$934\cdot10^5$), producers ($\$5\ 236\cdot10^5$, $\$236\cdot10^5$) and financiers ($\$156\cdot10^5$, $\$1\ 0156\cdot10^5$) of economy Alpha. With mathematical software, we can verify that $Y^* = (y_h^* + y_b^* + y_f^*) = E^* = (e_h^* + e_b^* + e_f^*) = \$11\ 326\cdot10^5$. Market equations (5.8) and (5.9) show labour, output and liquidity sales to be $\mathbf{R}^* = (\rho_w^*, \rho_u^*, \rho_s^*) = (\$5\,275\cdot10^5$, $\$4\,488\cdot10^5$, $\$1\,563\cdot10^5)$ during the same year. Hence, gross turnover equals $R^* = (\rho_w^* + \rho_u^* + \rho_s^*) = \$11\ 326\cdot10^5 = Y^* = E^*$. It is obvious that economy Alpha and its Euclidean model (5.1) are free from aggregation problems. By substituting sector flows (5.18) into market flows (5.8), we derive annual sales of services ρ_w^*, products ρ_u^* and securities ρ_s^* as functions of liquidity increment k_f and credit allocation κ_h:

$$\rho_w^* = (8/_{1771763}) \cdot (59560+114523 \cdot \kappa_h) \cdot k_f$$
$$\rho_u^* = (6/_{1771763}) \cdot (184085-103109 \cdot \kappa_h) \cdot k_f \qquad (5.19)$$
$$\rho_s^* = (2/_{1771763}) \cdot (115061+46891 \cdot \kappa_h) \cdot k_f$$

Dividing both sides of relations (5.19) by the corresponding quotation and subtracting from both sides the available quantity, we derive the prevailing gaps of labour years ($q_w^* - q_w^\#$), output pieces ($q_u^* - q_u^\#$) and consol numbers ($q_s^* - q_s^\#$) as functions of k_f, κ_h and $p_s = 1/r$.

$$\{\rho_w^*/p_w\} - q_w^\# = \{(8/_{1771763}) \cdot (59560+114523 \cdot \kappa_h) \cdot k_f/p_w\} - q_w^\#$$
$$\{\rho_u^*/p_u\} - q_u^\# = \{(6/_{1771763}) \cdot (184085-103109 \cdot \kappa_h) \cdot k_f/p_u\} - q_u^\# \qquad (5.20)$$
$$\{\rho_s^*/p_s\} - q_s^\# = \{(2/_{1771763}) \cdot (115061+46891 \cdot \kappa_h) \cdot k_f/p_s\} - q_s^\#$$

Within the time unit, average quotations (p_w, p_u, p_s) and obtainable quantities ($q_w^\#$, $q_u^\#$, $q_s^\#$) are determined exogenously. With help from *Mathematica* or other software, system (5.20) may be rearranged in the form $\mathbf{\Psi} \times \mathbf{\Pi} = \mathbf{Q}^\#$ shown by (5.21) and solved for vector $\mathbf{\Pi}$ because determinant $|\mathbf{\Psi}| = 4.16263\text{x}10^6 \neq 0$.

$$\begin{vmatrix} \frac{476480}{1771763} & \frac{916184}{1771763} & 0 \\ \frac{1104510}{1771763} & \frac{-618654}{1771763} & 0 \\ \frac{230122}{1771763} & \frac{93782}{1771763} & -10^7 \end{vmatrix} \cdot \begin{vmatrix} k_f \\ \kappa_h \cdot k_f \\ p_s \end{vmatrix} = \begin{vmatrix} 470.9 \cdot 10^6 \\ 0.38 \cdot 10^9 \\ 0 \end{vmatrix} = \mathbf{\Psi} \times \mathbf{\Pi} = \mathbf{Q}^\# \qquad (5.21)$$

From (5.21) we derive credit policy set $\mathbf{\Pi}(k_f = \$0.86706\text{x}10^9,\ \kappa_h = 0.530203,\ p_s = \$15.6349$ or $r = 6.39596\%)$ that would eliminate all quantity gaps during the following period. Alpha's central bank may clear all markets by guiding monetary institutions to:

- reduce the annual liquidity supply to $k_f \approx \$0.87 \text{ x } 10^9$ from $k_f = \$1.00 \text{ x } 10^9$;
- increase the credit share of consumers to $\kappa_h \approx 53.02\%$ from $\kappa_h \approx 50.00\%$;
- reduce the credit share of producers to $\kappa_b \approx 46.98\%$ from $\kappa_b \approx 50.00\%$;
- lower the annual discount rate to $r \approx 6.40$ per cent from $r \approx 7.30$ per cent.

It has already been highlighted that it is impossible to solve econometric, DSGE and other models as mathematical systems, in order to derive the policy parameters that would bridge deficits and surpluses of market goods.

(5.22) Steady state of Alpha when $\kappa_h = ½$ and $\kappa_b = ½$

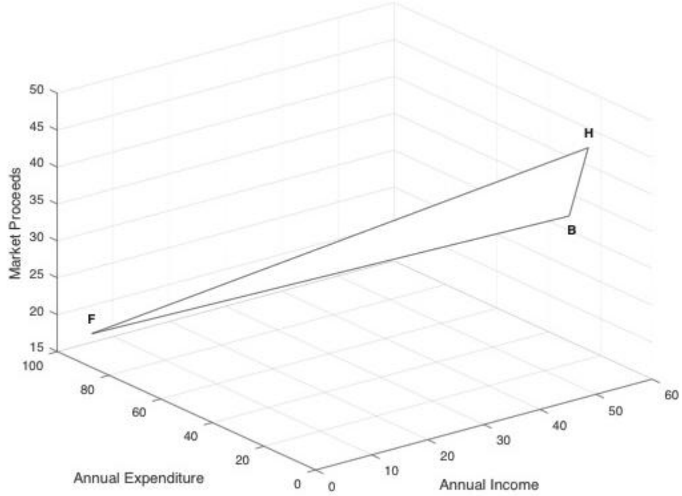


Figure (5.22) illustrates the distribution of Alpha's equilibrium income, expenditure and sales – among households **H**(52.4%, 8.2%, 46.6%), enterprises **B**(46.2%, 2.1%, 39.6%) and bankers **F**(1.4%, 89.7%, 18.8.6%) – when κ_h = 50% and κ_b = 50%. Beyond the triangle's vertices, there exist forces along the three axes of coordinates which compel dependent variables y, e, and ρ to return to their steady-state combinations.

(5.23) Steady state of Alpha when $\kappa_h = 0.530203$ and $\kappa_b = 0.469797$

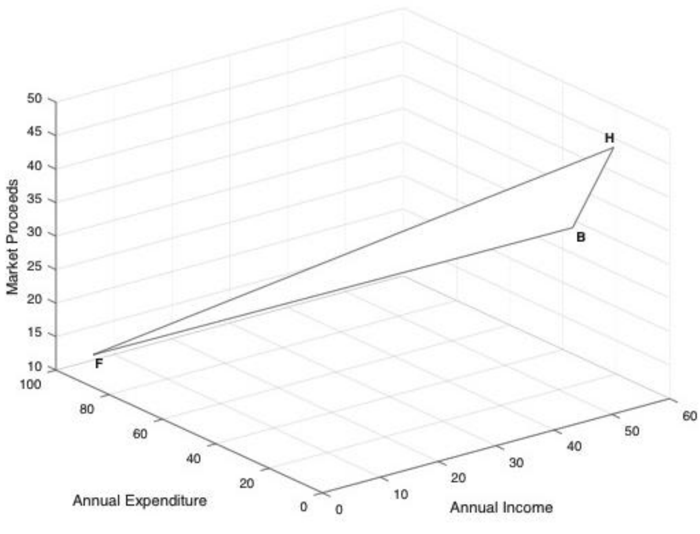


Figure (5.23) reflects the reallocation of Alpha's revenue, outlay and turnover – among consumers **H**(53.6%, 7.1%, 47.7%), producers **B**(44.9%, 3.6%, 38.4%) and financiers **F**(1.5%, 89.3%, 13.9%) – as a result of bank credit reapportionment in favour of households (+3.02%) and against businesses (–3.02%). *Mathematica* and other software offer numerous options for quickly i) redrawing plots by changing independent parameters and ii) combining plots to depict the system's progression or regression. Without a Euclidean replica of the exchange network there is no alternative but to uphold flexibility of quotations as the only means for balancing quantities demanded with those supplied.

However, governments having at their disposal a mathematical model of the closed economic system are able to weigh the effects of alternative policies and take pre-emptive action as befits each case. Monetary authorities of such countries are in position to employ scientific arguments in order to persuade banking institutions to follow their guidance. For example, equations (5.18) and (5.19) make possible estimation of partial derivatives (5.24), which predict flow changes emanating from marginally higher household loans and lower businesses loans.

$$
\begin{array}{lll}
\partial y_h^* / \partial \kappa_h = +\,0.5817\cdot k_f & \partial y_b^* / \partial \kappa_h = -\,0.4073\cdot k_f & \partial y_f^* / \partial \kappa_h = +\,0.0468\cdot k_f \\
\partial e_h^* / \partial \kappa_h = -0.4182\cdot k_f & \partial e_b^* / \partial \kappa_h = +\,0.5926\cdot k_f & \partial e_f^* / \partial \kappa_h = +\,0.\,0464\cdot k_f \\
\partial \rho_w^* / \partial \kappa_h = +\,0.5171\cdot k_f & \partial \rho_u^* / \partial \kappa_h = -\,0.3491\cdot k_f & \partial \rho_s^* / \partial \kappa_h = +\,0.0529\cdot k_f
\end{array}
\tag{5.24}
$$

It has also been mentioned that some credit apportionments among sectors, countries or zones are unacceptable because they induce left-turning or counterintuitive spirals of receipts and payments. Considering (5.18) as a system of inequalities, while bearing in mind that $k_f > 0$, let us derive accepting sector credit share limits within which Alpha's monetary flows remain positive. Table (5.25) displays the aforesaid ranges of κ_h and κ_b.

(5.25) Sector credit shares within which Alpha's monetary flows stay positive

Consumers (κ_h)		**Producers (κ_b)**	
Min	Max	Min	Max
0.46016	0.72334	0.276654	0.55984

Other things being equal, Alpha's institutional sectors enjoy positive revenues and outlays only as long as households receive between 46.02% and 72.33%, while businesses receive more than 27.67% but less than 55.98%, of loan / deposit supply k_f during the year. It follows that one of the reasons for the financial disasters that started in 2007 – with the collapse of Northern Rock, continued with the Lehman Brothers debacle and peaked with the bail-outs of Portugal, Italy, Spain and Greece – was misallocation of bank credit within and outside national borders. Given the requisite exogenous variables, Euclidean modelling methodology enables exploration of policy scenarios that neutralise causes and effects of the economic crisis and restore stability and prosperity to the exchange network.

5.6 Gross turnover versus value added

Banks keep liquid securities in order to meet their current expenses. At an aggregate level, monetary financial institutions issue loans to and accept deposits from each other.

Therefore, they receive interest on perpetual bonds bought from and pay interest on perpetual bonds sold to other banks. Within this framework, the monetary sector's value added v_f^* is smaller than its gross turnover ρ_s^* per time period. In order to estimate value added by banks – while avoiding double counting – it is essential to i) take into account only interest received from third parties, i.e. consumers and producers, or ii) deduct from sector throughput interest paid to monetary financial institutions:

$$\rho_s^* = (\beta_{hs} \cdot e_h^* + \beta_{bs} \cdot e_b^*) + \beta_{fs} \cdot e_f^* \quad => \quad v_f^* = \beta_{hs} \cdot e_h^* + \beta_{bs} \cdot e_b^* = \rho_s^* - \beta_{fs} \cdot e_f^* \qquad (5.26)$$

Table (5.27) confirms that Alpha's gross turnover R^* is \$1 132 630·10^3 while Alpha's value added $V^* = (v_h^* + v_b^* + v_f^*)$ is \$991 894·10^3. Analytically:

- services generate 46.6% of R^* compared to 53.2% of V^*;
- products generate 39.6% of R^* compared to 45.2% of V^*;
- securities generate 13.8% of R^* in contrast to 1.6% of V^*.

Distinction between gross turnover and value added is especially important when considering the banking sector's contribution and role in modern economies.

5.7. Diverse effects of market intervention

Relative stability of living, operating and financing costs is among the primary objectives of most governments. Throughout an economic cycle – caused by climatic, psychological or other exogenous factors – some goods may be in excess demand while others may be in excess supply. Equations (5.14) show prevailing gaps in Alpha. Specifically:

- the services deficit amounts to 12.01% of available labour;
- the products deficit amounts to 18.42% of annual output;
- the securities deficit amounts to 14.93% of consol supply.

Equations (5.16) reveal how inflationary (deflationary) tendencies are related to shortages (surpluses) of market goods. With reference to economy Alpha, three seller and three buyer basket indices as well as a gross turnover index can be easily identified. Table (5.28) demonstrates that inflationary (deflationary) trends affect households, businesses and financiers differently, depending on whether they are acting as suppliers or customers.

(5.27) Sector and market takings and outgoings

	COUNTRY	MARKET						
	SECTOR	**Labour**		**Output**		**Finance**		**Annual Stream**
	Consumer Household	Supply	Demand	Supply	Demand	Supply	Demand	
1	Income (Sources) $x10^3$	$ 527 484				$ 65 936		$ 593 420
2	Expenditure (Destinations) $x10^3$				$ 81 743		$ 11 677	$ 93 420
3	+ Surplus or Net Deposits $x10^3$							$ 500 000
	Industrial Corporation							
4)	Income (Sources) $x10^3$			$ 448 800		$ 74 800		$ 523 600
5	Expenditure (Destinations) $x10^3$		$ 19 667				$ 3 933	$ 23 600
6	+ Surplus or Net Deposits $x10^3$							$ 500 000
	Monetary Institution							
7	Income (Sources) $x10^3$					$ 15 610		$ 15 610
8	Expenditure (Destinations) $x10^3$		$ 507 817		$ 367 057		$ 140 736	$ 1 015 610
9	– Deficit or Net Loans $x10^3$							–$ 1 000 000
10	***Aggregate Demand*** $x10^3$		$ 527 484		$ 448 800		$ 156 346	$ 1 132 630
11	***Aggregate Supply*** $x10^3$	$ 527 484		$ 448 800		$ 156 346		$ 1 132 630
12	***GROSS TURNOVER*** $x10^3$	**$ 527 484**		**$ 448 800**		**$ 156 346**		**$ 1 132 630**
13	***Own Expenditure*** $x10^3$					– $ 140 736		– $140 736
14	***VALUE ADDED*** $x10^3$	***$ 527 484***		***$ 448 800***		***$ 15 610***		***$ 991 894***
		A		B		C		D

Subject to minor rounding errors

(5.28) Indices of living, operating and financing costs

POINT OF VIEW		(Market weight) x *Quotation rate of change*			**INDEX Rate of Change**
↓ SECTOR		**Services**	**Products**	**Securities**	
Household	Revenue sources	(8/9)x*12.01*	–	(1/9)x*14.93*	*12.33*
	Outlay targets	–	(7/8)x*18.42*	(1/8)x*14.93*	*17.98*
Business	Revenue sources	–	(6/7)x*18.42*	(1/7)x*14.93*	*17.92*
	Outlay targets	(5/6)x*12.01*	–	(1/6)x*14.93*	*12.50*
Financial	Revenue sources	–	–	(1)x*14.93*	*14.93*
	Outlay targets	(1/2)x*12.01*	($^{1145}/_{3168}$)x*18.42*	($^{439}/_{3168}$)x*14.93*	*14.73*
Gross Turnover		46.6%x*12.01*	39.6%x*18.42*	13.8%x*14.93*	*14.95*

Government officials monitor market developments with a view to keeping gaps of services, products and securities within acceptable limits. For political, social or other reasons, the authorities may wish to prevent salaries and prices rising by *12.01%* and *18.42%*, respectively, within a year. It is reiterated that official cost-of-living and cost-of-operating indices neglect financial securities for reasons explained in Section 5.4. Supposing that the relative throughputs of market goods reflected by (5.9) or (5.12) are the appropriate weights, equation (5.29) shows that shift from **Π1**(k_f = \$$10^9$, κ_h = ½, r = 7.35294%) to **Π2**(k_f = \$0.86706x10^9, κ_h = 0.530203, r = 6.39596%) will lower Alpha's inflation rate from *14.95%* to *2.06%* during the subsequent period.

$$\pi = (46.6 \text{ x } \mathit{0.0\%}) + (39.6 \text{ x } \mathit{0.0\%}) + (13.8 \text{ x } \mathit{14.93\%}) = \mathit{2.06\%} \qquad (5.29)$$

Let us suppose that Alpha's central bank has persuaded other monetary financial institutions to move from policy vector **Π1**(k_f = \$1x10^9, κ_h = 0.50, p_s = \$13.60 or r = 7.35294%) to **Π2**(k_f = \$0.86706x10^9, κ_h = 0.530203, p_s = \$15.6349 or r = 6.39596%) on the basis of a Euclidean model such as (5.1) and its offshoot (5.20). *Ceteris paribus*, within a year from their decision to:

- lower the annual expansion of their balance sheet by 13 percent,
- redistribute 3.2 percent of loans from producers to consumers and
- reduce the annual discount rate by approximately 95 basis points,

banks will succeed in eliminating quantity gaps and minimizing potential inflation. It should be noted that excess demand for securities will be zeroed by the inevitable *14.93%* rise of consol values. In order to compare the new credit policy's impact on Alpha's sectors and markets, we should substitute **Π1**($k_f = \$10^9$, $\kappa_h = ½$, r = 7.35294%) and **Π2**($k_f = \$0.86706\text{x}10^9$, $\kappa_h = 0.530203$, r = 6.39596%) in equations (5.18) and (5.19); the ensuing Tables (5.30) and (5.31) demonstrate that the policy shift will lessen:

- business takings and bank outgoings by 4.20% and 13.59%, respectively;
- receipts and payments of all institutional sectors by 12.78% on average;
- sales of services, products and consols by 10.73%, 15.33% and 12.41%, respectively.

(5.30) Absolute change of nominal flows caused by shift from **Π1** to **Π2**

Households	Businesses	Financiers
$y_{ht+1} - y_{ht} = -\ \$63\,654.3 \cdot 10^3$	$y_{bt+1} - y_{bt} = -\ \$80\,276.9 \cdot 10^3$	$y_{ft+1} - y_{ft} = -\ \$858.0 \cdot 10^3$
$e_{ht+1} - e_{ht} = -\ \$23\,372.1 \cdot 10^3$	$e_{bt+1} - e_{bt} = -\ \$12\,380.9 \cdot 10^3$	$e_{ft+1} - e_{ft} = -\$133\,798.0 \cdot 10^3$
$\rho_{wt+1} - \rho_{wt} = -\$56\,581.6 \cdot 10^3$	$\rho_{ut+1} - \rho_{ut} = -\ \$68\,808.8 \cdot 10^3$	$\rho_{st+1} - \rho_{st} = -\ \$19\,398.8 \cdot 10^3$

(5.31) Relative change of nominal flows caused by shift from **Π1** to **Π2**

Households	Businesses	Financiers
$(y_{ht+1} - y_{ht})\,/\,y_{ht} = -\ 3.777\%$	$(y_{bt+1} - y_{bt})\,/\,y_{bt} = -\ 4.203\%$	$(y_{ft+1} - y_{ft})\,/\,y_{ft} = -\ 0.001\%$
$(e_{ht+1} - e_{ht})\,/\,e_{ht} = -\ 0.218\%$	$(e_{bt+1} - e_{bt})\,/\,e_{bt} = -\ 0.029\%$	$(e_{ft+1} - e_{ft})\,/\,e_{ft} = -\ 13.588\%$
$(\rho_{wt+1} - \rho_{wt})\,/\,\rho_{wt} = -\ 10.727\%$	$(\rho_{ut+1} - \rho_{ut})\,/\,\rho_{ut} = -\ 15.331\%$	$(\rho_{st+1} - \rho_{st})\,/\,\rho_{st} = -\ 12.407\%$

Apparently, policy shift from **Π1** to **Π2** will decrease the revenue and outlay of consumers (–3.78%, –0.22%), producers (–4.20%, –0.03%) and bankers (0.00%, –13.59%) and thus eliminate the tendency of wages and prices to rise. Aggregate demand and average quotations are inextricably linked like two sides of the same coin. The preceding examples and exercises have illustrated the wealth of information that radiates from every Euclidean economic model.

5.8 Financial accounts of institutional sectors

Most consumers, producers, bankers and officials have little scientific knowledge about exchange networks. They do not understand how to link behaviour propensities, economic policies and resource constraints in order to forecast nominal flows, market sales and inflation trends. From their myopic standpoint, the future is a projection or repetition of the present, while the consol yield is the sole factor connecting future flows with present values. Using this rule of thumb, let us examine the certainty weighted balance sheets of Alpha's sectors at year end. From Table (5.27), we infer that households, businesses and bankers, respectively, held 52.4%, 46.2% and 1.4% of financial rights (present value of aggregate revenue) and bore 8.2%, 2.1% and 89.7% of financial obligations (present value of aggregate outlay). At the market clearing price of \$15.63, consumers and producers owned \$2 200 389$\cdot 10^3$ worth of consols (deposits), while bankers owned only \$244 060$\cdot 10^3$ worth of consols (loans). As a result, monetary financial institutions of Alpha:

- paid \$65 936$\cdot 10^3$ to consumers and \$74 800$\cdot 10^3$ to producers as interest on their bank deposits or security holdings;
- earned \$11 677$\cdot 10^3$ from households and \$3 933$\cdot 10^3$ from businesses as interest on bank loans or securities purchased;
- covered their deficit by issuing \$1$\cdot 10^6$ worth of consols, which were fully accepted because of the legal framework and the yield adjustment.

Consumers and producers are the owners of Alpha's net liquid wealth while bankers are burdened with the associated payments. Legal arrangements and operational realities regarding money and finance constitute the driving and balancing forces of modern economies. Incomes and expenditures as well as assets and liabilities may be temporarily altered by a natural disaster, a technological discovery or some other development. Every year, sector flows and market stocks adjust according to endogenous dynamics and exogenous pressures ruling the exchange network. Provided certain well-defined conditions are fulfilled, monetary, fiscal and other authorities may attain their common targets during subsequent periods. Given time series of eligible behaviour coefficients, policy parameters, market quotations and available quantities, Euclidean modelling methodology makes possible multi-annual forecasts regarding equilibrium, stability and controllability of:

- receipts and payments of institutional sectors;
- quotation and quantity gaps of market goods.

Our analysis and synthesis have confirmed that even elementary models of the general type $\mathbf{dX/dt} = \mathbf{A}\cdot\mathbf{X} - \mathbf{K}$ are capable of depicting economic interaction among institutional sectors and market platforms in meaningful and realistic ways.

Chapter 6. Simulation of Fiscal Policy

While pursuing economic optimisation, wealth preservation and power politics, consumers, producers, bankers, etc., may behave emotionally and irrationally. All aspects of behaviour are reflected by the distributions of sector receipts (α_{ij}) and payments (β_{ij}). The extent to which households, businesses, financiers, etc., engage in transactions is constrained by their annual budgets. The banking sector is the only one constitutionally entitled to issue consols (deposits) for the settlement of debts. All other sectors are legally obliged to accept consols (deposits) for the settlement of debts. In order to maintain their privileged role, financiers are known to yield to economic, political and social pressures. As a result, they may:

- apportion expenditure within certain minimum and maximum ratios per accepting sector for the sake of the economic system's stability;
- allocate credit to consumers, producers, etc., within certain minimum and maximum shares with a view to preventing sector flows turning negative;
- purchase consols from – and thus redistribute loans / deposits among – accepting sectors so as to minimize quantity and quotation gaps.

Bank freedom is circumscribed by scarcity of real resources for exchange with book entries. In this context, we will integrate general government in the economic network for the purpose of studying its impact on sectors and markets. The executive and legislative authorities are responsible for preparing and approving a country's yearly budget in line with the expectations of active voters and the demands of dominant labour, business and other groups. Fiscal policy − expressed by taxes and subsidies − reflects the rational optimisation, instinctual self-preservation and political ambition of those in power. The twelve premises of Euclidean methodology apply to all economic systems and their mathematical models, including those incorporating a public sector. In this regard, the following propositions are considered self-evident:

- Governments annually revise their market behaviour in accordance with the prevailing situation and the approved budget.
- Governments find it easier to raise public revenue by imposing taxes on outlay because private incomes may be successfully concealed.
- Governments derive revenue from and effect outlay for market goods which have at least one vendor and at least one buyer.

6.1. Behaviour of public sector income

Allotment of sector flows among real goods and financial paper is the first step toward building and applying economic models according to Euclidean methodology. The relevant shares can be estimated by means of annual statistical or banking surveys. Let us assume that a particular government draws $\alpha_{g\tau}$ percent of income y_g from withholding taxes (τ) and another α_{gs} percent from bank deposits or liquid bonds (s) that it inevitably keeps:

$$y_g = \alpha_{g\tau} \cdot y_g + \alpha_{gs} \cdot y_g = y_{g\tau} + y_{gs} \qquad (\alpha_{g\tau} + \alpha_{gs}) = 1 \tag{6.1}$$

Tax is the public sector's 'characteristic good' because it is solely 'bought' by the other sectors, which pay for it a portion of their respective expenditure. Given household e_h, business e_b and financier e_f outgoings, government tax revenue $y_{g\tau}$ is a function of private sector outlay coefficients (β_{hj}, β_{bj}, β_{fj}) as well as official withholding tax rates (τ_w, τ_u, τ_s) on labour, output and capital, as depicted by formula (6.2).

$$\begin{aligned} y_{g\tau} &= \tau_w \cdot (\beta_{bw} \cdot e_b + \beta_{fw} \cdot e_f) + \tau_u \cdot (\beta_{hu} \cdot e_h + \beta_{fu} \cdot e_f) + \tau_s \cdot (\beta_{hs} \cdot e_h + \beta_{bs} \cdot e_b + \beta_{fs} \cdot e_f) = \\ &= (\tau_u \cdot \beta_{hu} + \tau_s \cdot \beta_{hs}) \cdot e_h + (\tau_w \cdot \beta_{bw} + \tau_s \cdot \beta_{bs}) \cdot e_b + (\tau_w \cdot \beta_{fw} + \tau_u \cdot \beta_{fu} + \tau_s \cdot \beta_{fs}) \cdot e_f = \\ &= \tau_h \cdot e_h + \tau_b \cdot e_b + \tau_f \cdot e_f = \alpha_{g\tau} \cdot y_g \end{aligned} \tag{6.2}$$

Upon enactment, withholding tax coefficients (τ_w, τ_u, τ_s) are internalised by consumers, producers, bankers, etc., and incorporated in the economy's mathematical structure. Private sectors pay the total levy $y_{g\tau}$ through the turnover of traded goods. Government, by fixing the withholding rates on services τ_w, products τ_u and securities τ_s, also determines the percentages τ_h, τ_b, τ_f paid by households, businesses and financiers as taxes on their respective expenditures, since the one set of coefficients is derived from the other:

$$\begin{aligned} e_{h\tau} &= (\tau_u \cdot \beta_{hu} + \tau_s \cdot \beta_{hs}) \cdot e_h = \tau_h \cdot e_h \\ e_{b\tau} &= (\tau_w \cdot \beta_{bw} + \tau_s \cdot \beta_{bs}) \cdot e_b = \tau_b \cdot e_b \\ e_{f\tau} &= (\tau_w \cdot \beta_{fw} + \tau_u \cdot \beta_{fu} + \tau_s \cdot \beta_{fs}) \cdot e_f = \tau_f \cdot e_f \end{aligned} \tag{6.3}$$

In conformity with our premises, institutional sectors except banks spend less than their annual earnings in order to accumulate the monetary increment k_f. It follows that the same tax rates (τ_w, τ_u, τ_s) would have yielded higher public revenue had they been applied to takings instead of outgoings. Up to the present, no one has found a privacy-respecting, cost-effective and problem-free way of collecting income taxes. According to Euclidean methodology, most questions relating to taxation theory and practice can be dealt with by means of sets (τ_w, τ_u, τ_s) or (τ_h, τ_b, τ_f) of fiscal policy parameters. The fact that taxes are imposed on gross sales does not justify reaction or rejection on ethical or other grounds. All levies should be judged mainly by their contribution to wider economic objectives, such as fairer distribution and better utilisation of available resources. Since the 2007 debt crisis, the so called Tobin tax has been brought to the forefront again. The exchange network approach is eminently suitable for examining proposals regarding improvement of public finances and prevention of financial disasters. At this stage, our overwhelming goal is to keep the prototype fiscal model as simple as possible and thus we assume that interest and security transactions are exempt from tax. Replacing $\tau_s = 0$ in equations (6.3), we obtain:

$$\tau_h = \tau_u \cdot \beta_{hu}$$

$$\tau_b = \tau_w \cdot \beta_{bw} \qquad (6.4)$$

$$\tau_f = \tau_w \cdot \beta_{fw} + \tau_u \cdot \beta_{fu}$$

Notwithstanding the above assumption, monetary financial institutions pay tax at the rate $\tau_f = (\tau_w \cdot \beta_{fw} + \tau_u \cdot \beta_{fu})$ through their purchases of labour and output.

6.2. Behaviour of public sector expenditure

Government supplies expenditure by means of purchases of various goods as long as households, businesses and financiers demand income by means of sales of the identical goods. In economic reality and its Euclidean replica, public spending on labour, output and liquidity is a form of state aid or subsidy (σ) to the sector selling the particular good. Although revenue fractions (α_{iw}, α_{iu}, α_{is}) – sought by consumers, producers and bankers, $i = h, b, f$, respectively, from services w, products u and securities s – are relevant, government allots outlay e_g among the aforementioned goods as it deems fit, i.e. in accordance with its own economic, political or social priorities:

$$e_g = (\sigma_w + \sigma_u + \sigma_s) \cdot e_g \quad ; \quad \sigma_w + \sigma_u + \sigma_s = 1 \qquad (6.5)$$

Parenthetically, it should be noted that coefficient σ_s is the percentage of government expenditure allocated to public debt service. If public debt $\Pi\Delta$ consisted exclusively of variable interest consols, the annual amount required for debt service would be equal to:

$$\sigma_s \cdot e_g = r \cdot \Pi\Delta \qquad (6.6)$$

During the yearly budget process, government officials forecast receipts and plan payments so as to predetermine the fiscal gap:

$$\begin{aligned} d\Pi\Delta/dt = k_g = (y_g - e_g) &= (\alpha_{g\tau} + \alpha_{gs}) \cdot y_g - (\sigma_w + \sigma_u + \sigma_s) \cdot e_g =\,_{>}^{<}\, 0 \\ &= \{\alpha_{g\tau} \cdot y_g - (\sigma_w + \sigma_u) \cdot e_g\} + \{\alpha_{gs} \cdot y_g - \sigma_s \cdot e_g\} \end{aligned} \tag{6.7}$$

Equation (6.7) shows that the annual surplus (or deficit) is equal to the sum of *primary balance* – i.e. tax revenue minus public outlay on labour and output – and *secondary balance* – i.e. investment receipts minus debt payments. Sovereign countries may be classified under four groups according to their primary and secondary surpluses or deficits. Given that it is impossible for institutional sectors to be either all borrowers or all lenders concurrently, and that banks constitute the only sector entitled to expand outlay beyond revenue indefinitely, the annual increments of household k_h, business k_b and government k_g liquid assets are related to the expansion of monetary financial liabilities k_f as follows:

$$\begin{aligned} &dM/dt = k_f = (e_f - y_f) = (y_h - e_h) + (y_b - e_b) + (y_g - e_g) > 0 \\ &k_f = k_h + k_b + k_g = \kappa_h \cdot k_f + \kappa_b \cdot k_f + \kappa_g \cdot k_f > 0 \quad ; \quad (\kappa_h + \kappa_b + \kappa_g) = 1. \end{aligned} \tag{6.8}$$

Per time unit, the rise of bank assets (loans) is equal to the rise of bank liabilities (deposits) allocated to consumers, producers and the public sector as described by (6.8).

6.3. Construction of the exchange network

It has been assumed that banks record all transactions among institutional sectors as debits and credits to their monetary financial accounts. According to Euclidean methodology, income and expenditure flows – connecting supply of and demand for traded goods – are functions of behaviour coefficients and policy parameters. As already discussed, payments for and receipts from services, products, securities, taxes, etc., are equal tautologically. Equations (6.9) stem from Table (6.10), which summarises circulation of takings and outgoings among households, businesses, banks and the government.

$$\begin{aligned} &\alpha_{hw} \cdot y_h = \beta_{bw} \cdot e_b + \beta_{fw} \cdot e_f + \sigma_w \cdot e_g - (\tau_w \cdot \beta_{bw} \cdot e_b + \tau_w \cdot \beta_{fw} \cdot e_f) = \rho_w && \text{(a)} \\ &\alpha_{bu} \cdot y_b = \beta_{hu} \cdot e_h + \beta_{fu} \cdot e_f + \sigma_u \cdot e_g - (\tau_u \cdot \beta_{hu} \cdot e_h + \tau_u \cdot \beta_{fu} \cdot e_f) = \rho_u && \text{(b)} \\ &\alpha_{hs} \cdot y_h + \alpha_{bs} \cdot y_b + y_f + \alpha_{gs} \cdot y_g = \beta_{hs} \cdot e_h + \beta_{bs} \cdot e_b + \beta_{fs} \cdot e_f + \sigma_s \cdot e_g = \rho_s && \text{(c)} \\ &\alpha_{g\tau} \cdot y_g = \tau_u \cdot \beta_{hu} \cdot e_h + \tau_w \cdot \beta_{bw} \cdot e_b + (\tau_w \cdot \beta_{fw} + \tau_u \cdot \beta_{fu}) \cdot e_f = \rho_\tau && \text{(d)} \end{aligned} \tag{6.9}$$

(6.10) Circulation of receipts and payments

COUNTRY	**MARKET**							
SECTOR	**Labour**		**Output**		**Securities**		**Taxes**	
Consumer Household	Demand for revenue	Supply of outlay	Demand for revenue	Supply of outlay	Demand for revenue	Supply of outlay	Demand for revenue	Supply of outlay
Income sources	$\alpha_{hw}\cdot y_h$		-		$\alpha_{hs}\cdot y_h$			
Expenditure destinations		-		$(1-\tau_u)\cdot\beta_{hu}\cdot e_h$		$\beta_{hs}\cdot e_h$		$\tau_u\cdot\beta_{hu}\cdot e_h$
+ Surplus or net deposits								
Business Corporation								
Income sources	-		$\alpha_{bu}\cdot y_b$		$\alpha_{bs}\cdot y_b$			
Expenditure destinations		$(1-\tau_w)\cdot\beta_{bw}\cdot e_b$		-		$\beta_{bs}\cdot e_b$		$\tau_w\cdot\beta_{bw}\cdot e_b$
+ Surplus or net deposits								
General Government								
Income Sources					$\alpha_{gs}\cdot y_g$		$\alpha_{g\tau}\cdot y_g$	
Expenditure Destinations		$\sigma_w\cdot e_g$		$\sigma_u\cdot e_g$		$\sigma_s\cdot e_g$		
+ Surplus or net deposits								
Monetary Institution								
Income Sources	-		-		$\alpha_{fs}\cdot y_f$			
Expenditure Destinations		$(1-\tau_w)\cdot\beta_{fw}\cdot e_f$		$(1-\tau_u)\cdot\beta_{fu}\cdot e_f$		$\beta_{fs}\cdot e_f$		$(\tau_w\cdot\beta_{fw}+\tau_u\cdot\beta_{fu})\cdot e_f$
– Deficit or net loans								

Aggregate Demand for	$(1-\tau_w)\cdot(\beta_{bw}\cdot e_b+\beta_{fw}\cdot e_f)+\sigma_w\cdot e_g$	$(1-\tau_u)\cdot(\beta_{hu}\cdot e_h+\beta_{fu}\cdot e_f)+\sigma_u\cdot e_g$	$\beta_{hs}\cdot e_h+\beta_{bs}\cdot e_b+\beta_{fs}\cdot e_f+\sigma_s\cdot e_g$	$\alpha_{g\tau}\cdot y_g$
Aggregate Supply of	$\alpha_{hw}\cdot y_h$	$\alpha_{bu}\cdot y_b$	$\alpha_{hs}\cdot y_h+\alpha_{bs}\cdot y_b+\alpha_{fs}\cdot y_f+\alpha_{gs}\cdot y_g$	$\tau_u\cdot\beta_{hu}\cdot e_h+\tau_w\cdot\beta_{bw}\cdot e_b+(\tau_w\cdot\beta_{fw}+\tau_u\cdot\beta_{fu})\cdot e_f$

System (6.9) is easily converted into a dynamic economic model where sector monetary flows interact with official policy tools over time. The algebraic operations described below may be carried out conventionally or with *Mathematica*'s assistance. Adding together (6.9) (a)+(c), we derive the income of households:

$$y_h = \beta_{hs} \cdot e_h - \alpha_{bs} \cdot y_b + (1-\tau_w)\cdot\beta_{bw} \cdot e_b + \beta_{bs} \cdot e_b - y_f + (1-\tau_w)\cdot\beta_{fw} \cdot e_f + \beta_{fs} \cdot e_f + \sigma_w \cdot e_g \quad (6.11)$$

Adding together (6.9) (b)+(c), we derive the income of businesses:

$$y_b = -\alpha_{hs} \cdot y_h + (1-\tau_u)\cdot\beta_{hu} \cdot e_h + \beta_{hs} \cdot e_h + \beta_{bs} \cdot e_b - y_f + (1-\tau_u)\cdot\beta_{fu} \cdot e_f + \beta_{fs} \cdot e_f + \sigma_u \cdot e_g \quad (6.12)$$

Rearranging equation (6.9) (c), we derive the income of financiers:

$$y_f = -\alpha_{hs} \cdot y_h + \beta_{hs} \cdot e_h - \alpha_{bs} \cdot y_b + \beta_{bs} \cdot e_b + \beta_{fs} \cdot e_f - \alpha_{gs}\cdot y_g + \sigma_s \cdot e_g \quad (6.13)$$

Adding together (6.9) (c)+(d), we derive the income of government:

$$y_g = -\alpha_{hs} \cdot y_h + (\tau_u\cdot\beta_{hu} + \beta_{hs})\cdot e_h - \alpha_{bs} \cdot y_b + (\tau_w\cdot\beta_{bw} + \beta_{bs})\cdot e_b - y_f + (\tau_w\cdot\beta_{fw} + \tau_u\cdot\beta_{fu} + \beta_{fs})\cdot e_f + \sigma_s\cdot e_g \quad (6.14)$$

Per calendar year, every sector's revenue fluctuates according to the other sectors' outlay on services, products, securities and taxes. Simultaneously, consumers, entrepreneurs and officials are obliged to keep their outgoings lower than their takings in order to accumulate their share of the monetary increment. On the contrary, banks are entitled to expand their spending up to their endogenous receipts plus the (authorised) increase (issue) of bank liabilities (money). Non-homogeneous linear differential equation system (6.15) – derived from the above reasoning in accordance with the laws of mathematics – replicates the dynamic interaction of sector incomes and expenditures:

$$\begin{aligned}
&dy_h/dt = -y_h + \beta_{hs}\cdot e_h - \alpha_{bs}\cdot y_b + \{(1-\tau_w)\cdot\beta_{bw} + \beta_{bs}\}\cdot e_b - y_f + \{(1-\tau_w)\cdot\beta_{fw} + \beta_{fs}\}\cdot e_f + \sigma_w\cdot e_g \\
&de_h/dt = +y_h - \mathrm{k_h} - e_h = y_h - \kappa_h\cdot \mathrm{k_f} - e_h \\
&dy_b/dt = -\alpha_{hs}\cdot y_h + \{(1-\tau_u)\cdot\beta_{hu} + \beta_{hs}\}\cdot e_h - y_b + \beta_{bs}\cdot e_b - y_f + \{(1-\tau_u)\cdot\beta_{fu} + \beta_{fs}\}\cdot e_f + \sigma_u\cdot e_g \\
&de_b/dt = +y_b - \mathrm{k_b} - e_b = y_b - \kappa_b\cdot \mathrm{k_f} - e_b \\
&dy_f/dt = -\alpha_{hs}\cdot y_h + \beta_{hs}\cdot e_h - \alpha_{bs}\cdot y_b + \beta_{bs}\cdot e_b - y_f + \beta_{fs}\cdot e_f - \alpha_{gs}\cdot y_g + \sigma_s\cdot e_g \qquad (6.15) \\
&de_f/dt = +y_f + \mathrm{k_f} - e_f = y_f + \mathrm{k_f} - e_f \\
&dy_g/dt = -\alpha_{hs}\cdot y_h + (\tau_u\cdot\beta_{hu} + \beta_{hs})\cdot e_h - \alpha_{bs}\cdot y_b + (\tau_w\cdot\beta_{bw} + \beta_{bs})\cdot e_b - y_f + (\tau_w\cdot\beta_{fw} + \tau_u\cdot\beta_{fu} + \beta_{fs})\cdot e_f - y_g + \sigma_s\cdot e_g \\
&de_g/dt = +y_g - \mathrm{k_g} - e_g = y_g - \kappa_g\cdot \mathrm{k_f} - e_g
\end{aligned}$$

Elementary fiscal model (6.15), like other exchange networks founded on the twelve major premises, assumes the general form **dX/dt = A·X – K**. Column **X** consists of the eight endogenous variables (y_h, e_h, y_b, e_b, y_f, e_f, y_g, e_g). Table (6.16) reproduces structural matrix **A** that duplicates the interface of household, business, bank and government receipts and payments. Column **K**(0, k_h, 0, k_b, 0, $-k_f$, 0, k_g) consists of household, business and government ($\kappa_h + \kappa_b + \kappa_g = 1$) portions of yearly liquidity increment k_f.

(6.16) Matrix **A** describing the network of monetary flows

$$
\begin{vmatrix}
-1 & \beta_{hs} & -\alpha_{bs} & (1-\tau_w)\cdot\beta_{bw}+\beta_{bs} & -1 & (1-\tau_w)\cdot\beta_{fw}+\beta_{fs} & 0 & \sigma_w \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\alpha_{hs} & (1-\tau_u)\cdot\beta_{hu}+\beta_{hs} & -1 & \beta_{bs} & -1 & (1-\tau_u)\cdot\beta_{fu}+\beta_{fs} & 0 & \sigma_u \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
-\alpha_{hs} & \beta_{hs} & -\alpha_{bs} & \beta_{bs} & -1 & \beta_{fs} & -\alpha_{gs} & \sigma_s \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
-\alpha_{hs} & \tau_u\cdot\beta_{hu}+\beta_{hs} & -\alpha_{bs} & \tau_w\cdot\beta_{bw}+\beta_{bs} & -1 & \tau_w\cdot\beta_{fw}+\tau_u\cdot\beta_{fu}+\beta_{fs} & -1 & \sigma_s \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{vmatrix}
$$

Elementary fiscal model (6.15), like other Euclidean exchange networks, can be depicted in terms of i) differential equations or matrices, ii) moving polygons or polyhedrons, iii) flow circuits or networks, iv) dynamic planes or spaces as well as v) static or animated graphics. Whereas prototype model (4.1) encompasses six endogenous variables (y_h, e_h, y_b, e_b, y_f, e_f) and three monetary policy parameters (k_f; κ_h or κ_h; r), elementary system (6.15) encompasses eight endogenous variables (y_h, e_h, y_b, e_b, y_f, e_f, y_g, e_g), four monetary policy parameters (k_f; κ_h, κ_b or κ_g; r) and five fiscal policy parameters ($\alpha_{g\tau}$ or α_{gs}; τ_w and τ_u; σ_w, σ_u or σ_s). One of these parameters (k_f) determines the sizes, while all the rest determine the ratios of monetary flows. With inclusion of government, the model's magnitude has increased to eight from six first-order non-homogeneous linear differential equations.

Irrespective of a closed network's dimension, the same mathematical methods are applicable to examination of its dynamic, static and controllability properties. Sector incomes and expenditures, being functions of market conduct and economic policy, vary in coordinated ways. Granted the annual behaviour coefficients (α_{ij} ; β_{ij}), the monetary (k_f; κ_h, κ_b or κ_g; r) and fiscal ($\alpha_{g\tau}$ or α_{gs}; τ_w and τ_u; σ_w, σ_u or σ_s) instruments as well as the average quotations (p_1, ... , p_j) and available quantities (q_1, ... ,q_j) of traded goods, mathematical software enables us to formulate the exchange system's dynamic stability situation and steady state solution in partially symbolic or algebraic manner so as to identify the most appropriate modifications of policy parameters. A Euclidean economic model, which includes monetary and fiscal tools, incorporates as many endogenous flows as control variables and thus may satisfy all prerequisites of a dirigible dynamic system.

6.4. Are systemic properties mathematical or economic?

i) According to Cramer's rule, time-invariant system $\mathbf{A} \cdot \mathbf{X}^* = \mathbf{K}$ corresponding to model (6.15) can be solved with regard to $\mathbf{X}^*$ in terms of $\mathbf{A}$ and $\mathbf{K}$, provided matrix $\mathbf{A}$ is non-singular. When $|\mathbf{A}| \neq 0$, equilibrium vector $\mathbf{X}^*$ consists of the annual revenues and outlays (y_i^*, e_i^*) of institutional sectors $i = h, b, f, g$. It addition, it may be seen that:

- The numerator of each element y_i^* or e_i^* is a polynomial consisting of a unique combination of consumer (α_{hw}, α_{hs}; β_{hu}, β_{hs}), producer (α_{bu}, α_{bs}; β_{bw}, β_{bs}) and banker (β_{fw}, β_{fu}, β_{fs}) behaviour coefficients as well as of a unique combination of fiscal ($\alpha_{g\tau}$ or α_{gs}; τ_w, τ_u; σ_w, σ_u or σ_s) and monetary (k_f; κ_h, κ_b or κ_g) policy tools.
- The denominator of every element y_i^* or e_i^* is determinant $|\mathbf{A}|$ formed by sector behaviour coefficients and fiscal policy parameters only. It is highlighted that monetary policy parameters (k_f; κ_h, κ_b or κ_g) are not part of matrix $\mathbf{A}$ and therefore do not affect either the economy's or the model's dynamic stability.

ii) To every system of linear differential equations $\mathbf{dX/dt} = \mathbf{A \cdot X} - \mathbf{K}$ corresponds a characteristic polynomial $|\mathbf{A} - \boldsymbol{\lambda} \cdot \mathbf{I}|$. A model of this general type is globally stable if, and only if, Routh-Hurwitz determinants rh_n, which consist of the coefficients of polynomial $|\mathbf{A} - \boldsymbol{\lambda} \cdot \mathbf{I}|$, are greater than zero. Given the fractions α_{ij} and β_{ij} of institutional sector flows derived from and destined for market goods as well as the set of fiscal policy parameters $\mathbf{\Pi}$ ($\alpha_{g\tau}$ or α_{gs}; τ_w, τ_u; σ_w, σ_u or σ_s), it is possible to identify – in the neighbourhood of fiscal policy vector $\mathbf{\Pi}$ – the fiscal parameter limits within which all Routh-Hurwitz conditions are satisfied. The aforementioned boundaries may contain taxation and subsidy coefficients which may not be regarded as logical, feasible or acceptable by affected parties.

iii) Euclidean exchange networks consist of vendors and buyers who also act as borrowers and lenders. Inevitably, each sector (and market) is associated with an income vector (y_i, 0) and an expenditure vector (0, e_i). In the two-dimensional Euclidean plane, the y_i are measured on the axis of abscissas and the e_i on the axis of ordinates. The sums of receipts $Y = (y_1 + ... + y_n, 0)$ and payments $E = (0, e_1 + ... + e_f)$ of all sectors (or markets) mirror the entire economy or country concerned. From the standpoint of applied mathematics, income and expenditure vectors are perpendicular and bound together. According to Euclidean methodology, the angle between any pair of revenue and outlay vectors is either $+90^0$ or -90^0. The cross product of two such vectors, e.g. $\boldsymbol{\gamma} = (y_i, 0)$ and $\boldsymbol{\varepsilon} = (0, e_i)$, yields a new vector $\boldsymbol{\delta}$ that is perpendicular to the plane of $\boldsymbol{\gamma}$ and $\boldsymbol{\varepsilon}$. The length of $\boldsymbol{\delta}$ is the product of the length of $\boldsymbol{\gamma}$ by the length of $\boldsymbol{\varepsilon}$ by the sine of the angle between them i.e.

$$\boldsymbol{\delta} = |\boldsymbol{\gamma} \times \boldsymbol{\varepsilon}| = |\boldsymbol{\gamma}| \cdot |\boldsymbol{\varepsilon}| \cdot \text{sine of angle } (\boldsymbol{\gamma}, \boldsymbol{\varepsilon}) \tag{6.17}$$

It is understood that $|\boldsymbol{\gamma}| = (y_i^2 + 0^2)^{½} = y_i$ is the length of $\boldsymbol{\gamma}$ and $|\boldsymbol{\varepsilon}| = (0^2 + e_i^2)^{½} = e_i$ is the length of $\boldsymbol{\varepsilon}$. The magnitude of an exchange network is better reflected by expression (6.17) or the rectangular area having $\boldsymbol{\gamma}$ and $\boldsymbol{\varepsilon}$ as adjacent edges.

iv) If the sine of the angle between vectors $\boldsymbol{\gamma} = (y_i, 0)$ and $\boldsymbol{\varepsilon} = (0, e_i)$ is:

- positive, the direction of vector $\boldsymbol{\delta}$ is ascending and the system formed by the sector's (or market's) income and expenditure is a 'right-handed screw' system where money circulates in an anti-clockwise manner;
- negative, the direction of vector $\boldsymbol{\delta}$ is descending and the system formed by sector (or market) receipts and payments is a 'left-handed screw' system where money circulates in a clockwise manner.

In every Euclidean economic model with right-handed circuits, incomes and expenditures (y_i, e_i) vary along with the annual supply of liquidity k_f. If a sector's (or market's) twin flows happen to have opposite signs, then one is a positive while the other is a negative function of bank balance sheet increment k_f. Receipts and payments in 'left-handed screw' systems (i.e. exchange networks) change in opposite directions contrary to experience. This illogical reaction renders pursuit of purely expansionary or purely restrictive monetary policies impossible or meaningless. Incidence of left-handed circuits in economic systems may be prevented through appropriate allocation of credit among consumers, producers, governments, etc., per time unit.

v) Most households, businessmen and financiers believe, *inter alia*, that:

- prevention of sector, market and general economic crises must rank high among the priorities of every government;

- economic life must always lead to positive income and expenditure for all irrespective of their own mistakes;
- the monetary flows of sectors and markets must be commensurate to the exogenous quotations and the available quantities.

Given the applicable behaviour, taxation, subsidization and credit coefficients, mathematical software can easily calculate – in the neighbourhood of current policy vector **Π** – the ranges of κ_h, κ_b or κ_g, $\alpha_{g\tau}$, τ_w and τ_u, σ_w, σ_u or σ_s within which:

- steady-state $\mathbf{X}^*$ consists of stable, unique and positive monetary flows;
- government may alter the steady-state $\mathbf{X}^*$ so as to attain policy targets.

6.5. Prerequisites for a controllable economic system

A dynamically stable system of the general type $\mathbf{dX/dt = A{\cdot}X - K}$ may be guided to any chosen steady-state **X**, provided it incorporates as many control instruments as targeted variables. Hence, technocrats who administer credit policy and bureaucrats who administer budget policy should harmonise their decisions. Only by acting together can they ensure that all sectors and markets will enjoy stable, unique and positive receipts and payments analogous to the quantities or quotations of services, products and securities. By coordinating monetary and fiscal policies, the authorities may direct the exchange network to achieve a number of flow targets that is equal to the number of control tools. Increase and allocation of credit as well as taxation and subsidisation rates are complementary instruments because they affect endogenous flows in completely different ways. It is widely recognised that the 2007 – 2015 economic crisis worsened due to insufficient provisions for coordination of monetary and fiscal policies in the Eurozone. Our methodology classifies government tools under two groups:

- The first group includes the share $\alpha_{g\tau}$ of public revenue y_g derived from taxes as well as the withholding rates on labour salaries τ_w and output sales τ_u.
- The second group includes the percentages of government payments e_g directed to services σ_w, products σ_u and securities σ_s.

Distribution of public income among sources ($\alpha_{g\tau}$, α_{gs}) and allotment of public expenditure among destinations (σ_w, σ_u, σ_s) are of paramount importance because – in addition to expressing the concept, design and implementation of fiscal policy – they shape:

- the annual receipts and payments of sectors and markets;
- the dynamic stability of flows among sectors and markets.

In the medium term, economic policy is the art of achieving what is feasible in a world of erratic preferences, rigid quotations and scarce resources. Governments alter the sources of public revenue and the destinations of public outlay in accordance with the rational optimisation, survival instinct and political ambition that guide their actions. Most agents are apprehensive of fiscal measures which may structurally reform the economic system against their interests. As a rule, market participants are willing to accept reasonable variations of withholding tax and state subsidy rates to which they have been accustomed. Public officials should primarily ensure that the chosen tax and subsidy rates, combined with other factors, drive the exchange network to an equilibrium solution **X** consisting of stable and positive monetary flows. National authorities, which make use of Euclidean models and appropriate software, may test plausible policy scenarios regarding public income sources and public expenditure destinations. In this way, they may consider in advance the effects of likely fiscal packages on the economic system's stability conditions rh_i, equilibrium flows (y_i^*, e_i^*) and quantity gaps ($q_j^* - q_j^\#$). The difficulty of each experiment is directly related to the number of fiscal parameters remaining as control variables in the exchange network model.

6.6. Planning the stages of policy intervention

Average quotations tend to rise (fall) in line with scarcity (plethora) of labour, output and liquidity. Before corrective action, officials are expected to take into account domestic and world standards regarding budget deficits or surpluses. Under all circumstances, they bear in mind that what ultimately matters is the purchasing power of incomes and expenditures. With reference to model (6.15), one may easily ask ''*Is it possible to figure out the magnitudes of eight independent parameters* **Π**(k_f, κ_h, κ_b or κ_g; τ_w and τ_u; σ_w, σ_u or σ_s; r), *which together ensure that the eight endogenous flows* **X**(y_i, e_i) $i = h, b, f, g$: (i) *are stable and thus unique;* (ii) *circulate anti clock-wise being greater than zero*; (iii) *attain target sales* **R**$^\#$($\rho_j^\#$) $j = w, u, s, \tau$, *which are commensurate with current quotations and available quantities*?'' The above mathematical problem is composed of three interrelated systems:

(i) eight polynomial inequalities comprising the Routh-Hurwitz conditions rh_n, i = 1,…, 8, which should be greater than zero;

(ii) four pairs of sector flows (y_i^*, e_i^*) $i = h, b, f, g$ – originating from Cramer's rule – which must compose a right-handed screw system;

(iii) four functions $(\rho_j^* - \rho_j^\#)$ defining the differences between equilibrium and target turnovers of services w, products u, securities s and taxes τ.

Serious computational and other obstacles prevent solution of the general mathematical problem. The three interdependent systems encompass 20 mathematical relations with only eight independent parameters $\mathbf{\Pi}(k_f, \kappa_h, \kappa_b$ or κ_g; τ_w and τ_u; σ_w, σ_u or σ_s; r). Moreover, systems of polynomial inequalities $rh_n > 0$, $i = 1, \ldots, n$, which contain more than two variables in algebraic form cannot be solved meaningfully. This conclusion is derived from the fact that the limits of each variable – within which the system of polynomial inequalities $rh_n > 0$ is solved – are functions of the other variables which have remained as algebraic symbols. The problem may be simplified by solving system of inequalities $rh_n > 0$ – in the neighbourhood of current policy vector $\mathbf{\Pi}$ – for the upper and lower limits of one policy variable at a time.

Initially, we should substitute the actual or hypothetical data for the behaviour coefficients of consumers (α_{hw}, α_{hs}; β_{hu}, β_{hs}), producers (α_{bu}, α_{bs}; β_{bw}, β_{bs}) and financiers (β_{fw}, β_{fu}, β_{fs}) in general model (6.15). Subsequently, we may concentrate on combinations of public revenue sources ($\alpha_{g\tau}$ or α_{gs}, τ_w and τ_u), public outlay destinations (σ_w, σ_u or σ_s) as well as on credit expansion (k_f) and allocation (κ_h, κ_b or κ_g) in order to reveal their effects on the exchange network's static, dynamic and controllability properties. It should be noted that:

- rates of taxation ($\alpha_{g\tau}$, τ_w and τ_u) and subsidisation (σ_w, σ_u or σ_s) influence stability conditions rh_n and annual flows (y_h^*, e_h^*), (y_b^*, e_b^*), (y_f^*, e_f^*), (y_g^*, e_g^*) of the economic system.
- rise (k_f) and division (κ_h, κ_b or κ_g) of bank credit – although instrumental in deciding sizes and signs of sector flows $\mathbf{X}^*$ – do not touch stability conditions $rh_n > 0$ of the economic system.

Consequently, as policy planners, we should focus on i) fiscal parameters as means to securing dynamic stability and ii) credit allocations as means to securing positive flows. Remember that k_f determines the magnitudes while the other policy tools determine the proportions of receipts and payments. Official objectives are achievable as long as the exchange network remains controllable. Dirigibility presupposes that general government and central bank choose policy parameters within the boundaries that generate stable and, therefore, unique equilibrium vectors, which consist of positive monetary flows for all concerned. With reference to elementary fiscal model (6.15), we intend to illustrate the proposed methodology and to compute with *Mathematica* the:

- ranges of tax ($\alpha_{g\tau}$; τ_w, τ_u) and subsidy (σ_w, σ_u or σ_s) rates that yield positive Routh-Hurwitz determinants $rh_i > 0$, $i = 1, \ldots, n$, and thus stable and unique steady-states;
- ranges of household, business or government credit shares ($\kappa_h + \kappa_b + \kappa_g = 1$) where all enjoy positive incomes and expenditures $y_i^*(\kappa_h, \kappa_g) > 0$, $e_i^*(\kappa_h, \kappa_g) > 0$, while tax ($\alpha_{g\tau}$; τ_w, τ_u) and subsidy (σ_w, σ_u or σ_s) rates are within the above-mentioned limits;
- percentage responses (elasticities) of consumer, producer, banker and government flows to one percent differentiation of fiscal and monetary policy parameters.

6.7. Moving from theory to practice

We will clarify the policy exploration process through a number of practical examples. Specifically, let us apply the Euclidean method to an elementary economy called Beta, the non-government sectors of which behave in accordance with the following data:

On account of biological, psychological and other exogenous factors, consumers distribute their annual income among applicable sources and allocate their annual expenditure among pertinent destinations as follows:

Fraction of income from:	Services $\alpha_{hw} = 8/9$	Securities $\alpha_{hs} = 1/9$
Fraction of expenditure on:	Products $\beta_{hu} = 7/8$	Securities $\beta_{hs} = 1/8$

As a result of management choices regarding technology, liquidity, etc., producers derive their annual receipts from feasible sources and direct their annual payments to justified destinations as shown below:

Fraction of income from:	Products $\alpha_{bu} = 6/7$	Securities $\alpha_{bs} = 1/7$
Fraction of expenditure on:	Services $\beta_{bw} = 5/6$	Securities $\beta_{bs} = 1/6$

Government income originates $\alpha_{g\tau} = {}^{99}/_{100}$ from withholding taxes and $\alpha_{gs} = {}^{1}/_{100}$ from bank deposits or liquid securities. The withholding rates on the turnovers of services and products are $\tau_w = {}^{4}/_{20}$ and $\tau_u = {}^{3}/_{20}$, respectively. The shares of public expenditure allocated to labour, output and liquidity, respectively, are $\sigma_w = {}^{3}/_{6}$, $\sigma_u = {}^{2}/_{6}$ and $\sigma_s = {}^{1}/_{6}$.

The yearly revenue of monetary financial institutions comes from interest on secured loans. With a view to satisfying their own operational requirements while yielding to third-party demands, banks allocate annual outlay among market goods as follows:

Fraction of expenditure on: Services $\beta_{fw} = {}^{10}/_{20}$ Products $\beta_{fu} = {}^{3}/_{20}$ Securities $\beta_{fs} = {}^{7}/_{20}$

The monetary financial system provides and absorbs liquidity by accepting and issuing guaranteed consols according to economic, political or other criteria. Annual liquidity increment k_f = €10^9 is distributed as additional credit to households, businesses and the government in proportions $\kappa_h = {}^5/_{10}$, $\kappa_b = {}^4/_{10}$ and $\kappa_g = {}^1/_{10}$, respectively. Beta's exchange network is identical to model (6.15) that assumes general form $\mathbf{dX/dt = B{\cdot}X - K}$. Structural matrix **B**, duplicated by (6.18), has been derived from the statistical survey of Beta.

(6.18) Structural matrix **B** reflecting the interaction of sectors and markets

$$\begin{vmatrix}
-1 & {}^1/_8 & -{}^1/_7 & (1-\tau_w)\cdot{}^5/_6+{}^1/_6 & -1 & (1-\tau_w)\cdot\tfrac{1}{2}+{}^7/_{20} & 0 & \sigma_w \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-{}^1/_9 & (1-\tau_u)\cdot{}^7/_8+{}^1/_8 & -1 & {}^1/_6 & -1 & (1-\tau_u)\cdot{}^3/_{20}+{}^7/_{20} & 0 & \sigma_u \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
-{}^1/_9 & {}^1/_8 & -{}^1/_7 & {}^1/_6 & -1 & {}^7/_{20} & -\alpha_{gs} & \sigma_s \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
-{}^1/_9 & \tau_u\cdot{}^7/_8+{}^1/_8 & 0 & \tau_w\cdot{}^5/_6+{}^1/_6 & -1 & \tau_w\cdot\tfrac{1}{2}+\tau_u\cdot{}^3/_{20}+{}^7/_{20} & -1 & \sigma_s \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{vmatrix}$$

Corresponding to matrix **B** there is a characteristic equation or eigenvalue polynomial that is equal to determinant (6.19) of matrix (6.20).

$$|\mathbf{B} - \lambda{\cdot}\mathbf{I}| = b_0{\cdot}\lambda^8 + b_1{\cdot}\lambda^7 + b_2{\cdot}\lambda^6 + b_3{\cdot}\lambda^5 + b_4{\cdot}\lambda^4 + b_5{\cdot}\lambda^3 + b_6{\cdot}\lambda^2 + b_7{\cdot}\lambda + b_8 = 0 \qquad (6.19)$$

(6.20) Characteristic equation of matrix **B**

$$\begin{vmatrix}
-1-\lambda & {}^1/_8 & -{}^1/_7 & (1-\tau_w)\cdot{}^5/_6+{}^1/_6 & -1 & (1-\tau_w)\cdot\tfrac{1}{2}+{}^7/_{20} & 0 & \sigma_w \\
1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
-{}^1/_9 & (1-\tau_u)\cdot{}^7/_8+{}^1/_8 & -1-\lambda & {}^1/_6 & -1 & (1-\tau_u)\cdot{}^3/_{20}+{}^7/_{20} & 0 & \sigma_u \\
0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 \\
-{}^1/_9 & {}^1/_8 & -{}^1/_7 & {}^1/_6 & -1-\lambda & {}^7/_{20} & -\alpha_{gs} & \sigma_s \\
0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 \\
-{}^1/_9 & \tau_u\cdot{}^7/_8+{}^1/_8 & 0 & \tau_w\cdot{}^5/_6+{}^1/_6 & -1 & \tau_w\cdot\tfrac{1}{2}+\tau_u\cdot{}^3/_{20}+{}^7/_{20} & -1-\lambda & \sigma_s \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda
\end{vmatrix}$$

Evidently, it is very difficult to solve systems of non-linear inequalities in terms of more than two unknowns. As a result, during policy exploration exercises, Routh-Hurwitz determinants rh_n and equilibrium flows $\mathbf{X}^*(y_i^*, e_i^*)$ are successively expressed as functions of one or two parameters at a time and the related inequalities solved in the neighbourhood of the current policy vector $\mathbf{\Pi}(\alpha_{g\tau}$, τ_w and τ_u; σ_w, σ_u or σ_s; k_f, κ_h, κ_b or $\kappa_g)$.

6.8. Taxes as a fraction of government income

Let us suppose that government wishes to modify the percentage of public revenue derived from taxes $\alpha_{g\tau}$, while keeping other policy parameters constant. With *Mathematica*'s assistance, we have computed coefficients b_i, $i = 0, \dots, 8$, of polynomial (6.19) or (6.20) as functions of $\alpha_{g\tau}$; the coefficients are presented by equations (6.21).

$$\begin{aligned}
&b_0 = 1 \quad ; \quad b_1 = 8 \quad ; \quad b_2 = (23516280+907200\cdot\alpha_{g\tau})/907200 \\
&b_3 = (40849806+4913748\cdot\alpha_{g\tau})/907200 \quad ; \quad b_4 = (40493364+11306610\cdot\alpha_{g\tau})/907200 \\
&b_5 = (21107036+14101743\cdot\alpha_{g\tau})/907200 \quad ; \quad b_6 = (3413681+9712743\cdot\alpha_{g\tau})/907200 \\
&b_7 = (-1259223+3280125\cdot\alpha_{g\tau})/907200 \quad ; \quad b_8 = (-363714+369063\cdot\alpha_{g\tau})/907200
\end{aligned} \tag{6.21}$$

The eight minors of matrix (6.22) comprise Routh-Hurwitz conditions rh_n, $n = 1, \dots, 8$.

$$\begin{vmatrix}
b_1 & b_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b_3 & b_2 & b_1 & b_0 & 0 & 0 & 0 & 0 \\
b_5 & b_4 & b_3 & b_2 & b_1 & b_0 & 0 & 0 \\
b_7 & b_6 & b_5 & b_4 & b_3 & b_2 & b_1 & b_0 \\
0 & b_8 & b_7 & b_6 & b_5 & b_4 & b_3 & b_2 \\
0 & 0 & 0 & b_8 & b_7 & b_6 & b_5 & b_4 \\
0 & 0 & 0 & 0 & 0 & b_8 & b_7 & b_6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & b_8
\end{vmatrix} \tag{6.22}$$

Economic system $\mathbf{dX/dt = B\cdot X - K}$ is globally stable if, and only if, the eight Routh-Hurwitz determinants rh_n are greater than zero. Using *Mathematica*, we substitute the computed values of coefficients b_i, $i = 0,\dots, 8$, in the minors of matrix (6.22) to obtain the eight Routh-Hurwitz determinants, when government varies percentage $\alpha_{g\tau}$ (and thus α_{gs}) of revenue derived from tax levies (from liquid bonds) while everything else stays the same. With *Mathematica*'s help, it can be proven that determinants rh_n, $n = 1,\dots, 8$, are positive as long as government receives more than 98.55 percent of income from taxes and, therefore, less than 1.45 percent of income from interest on bank deposits or guaranteed securities.

When $0.9855 < \alpha_{g\tau} < 1$, all Routh-Hurwitz conditions are fulfilled and thus consumer, producer, bank and public receipts and payments converge to $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*, y_g^*, e_g^*)$ irrespective of the original combination $\mathbf{X}^0$ of sector flows. In this case, gross proceeds from services, products, securities and taxes – reflected by equations (6.9) – also tend toward sustainable flows $\mathbf{R}^*(\rho_w^*, \rho_u^*, \rho_s^*, \rho_\tau^*)$ in accordance with $\mathbf{X}^*$ of $\mathbf{dX/dt = B \cdot X - K}$. Using Beta's statistical data, we have applied Cramer's rule to system $\mathbf{B \cdot X^* = K}$ to derive household (y_h^*, e_h^*), business (y_b^*, e_b^*), financier (y_f^*, e_f^*) and government (y_g^*, e_g^*) takings and outgoings as functions of $\alpha_{g\tau}$ and k_f. System (6.23) consists of eight non-linear equations in terms of two algebraic variables.

$$\begin{aligned}
y_h^* &= (-1374+1553\cdot\alpha_{g\tau})\cdot 117\cdot k_f/(-1212380+1230210\cdot\alpha_{g\tau})\\
e_h^* &= (222716-216702\cdot\alpha_{g\tau})\cdot k_f/(-606190+615105\cdot\alpha_{g\tau})\\
y_b^* &= 7\cdot(26221-23977\cdot\alpha_{g\tau})\cdot k_f/(-1212380+1230210\cdot\alpha_{g\tau})\\
e_b^* &= (668499-659923\cdot\alpha_{g\tau})\cdot k_f/(-1212380+1230210\cdot\alpha_{g\tau})\\
y_f^* &= 2\cdot(-58266+59587\cdot\alpha_{g\tau})\cdot k_f/(-363714+369063\cdot\alpha_{g\tau})\\
e_f^* &= 131\cdot(-3666+3727\cdot\alpha_{g\tau})\cdot k_f/(-363714+369063\cdot\alpha_{g\tau})\\
y_g^* &= 1703\cdot k_f/(-242476+246042\cdot\alpha_{g\tau})\\
e_g^* &= 9\cdot(-14417+13669\cdot\alpha_{g\tau})\cdot k_f/(1212380-1230210\cdot\alpha_{g\tau})
\end{aligned} \tag{6.23}$$

While other parameters remain as given by the survey, let us find the ranges of k_f and $\alpha_{g\tau}$ within which Beta's institutional sectors enjoy positive monetary flows. By considering (6.23) as a system of eight inequalities $y_i^*(k_f, \alpha_{g\tau}) > 0$ and $e_i^*(k_f, \alpha_{g\tau}) > 0$, $i = h, b, g, f$, it can be proven that Beta's consumers, producers, bankers and government enjoy positive income and expenditure as long as $k_f > 0$ and $0.9855 < \alpha_{g\tau} < 1$. When institutional sectors enjoy stable (thus, unique) and positive revenues and outlays, the authorities are able to regulate a number of market flows with an equal number of policy tools. Most economies perform like Beta; they remain stable as long as annual budgets are predominantly funded through taxes. This deduction has implications regarding the 2007 – 2015 Eurozone crisis. For years, a number of countries, including Portugal, Italy, Spain and Greece, financed large percentages of government spending by means of loans instead of taxes. The said countries have been forced to drastically reform their public sectors for the sake of preventing collapse of their economies and the entire Euro system. Let us now suppose that the fiscal authority has increased the fraction of public revenue $\alpha_{g\tau}$ derived from taxes by one percent, while everything else has stayed the same. In this case, receipts and payments of Beta's institutional sectors would fall in accordance with the elasticities shown below:

$$dy_h^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/y_h^*) = -210.91\% \qquad de_h^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/e_h^*) = -246.54\%$$
$$dy_b^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/y_b^*) = -229.88\% \qquad de_b^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/e_b^*) = -263.37\% \qquad (6.24)$$
$$dy_f^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/y_f^*) = -138.97\% \qquad de_f^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/e_f^*) = -64.83\%$$
$$dy_g^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/y_g^*) = -220.32\% \qquad de_g^*/d\alpha_{g\tau}\cdot(\alpha_{g\tau}/e_g^*) = -235.62\%$$

Modifications of $\alpha_{g\tau}$ alter consumer, producer, financier and government incomes and expenditures in the opposite direction. Due to significant and contrary responses of sector flows to variations of $\alpha_{g\tau}$, fiscal authorities rarely increase the share of public revenue derived from taxes without having already expanded public outlay. By substituting $\alpha_{g\tau} = {}^{99}/_{100}$ in (6.23), we obtain equations (6.25) which show the absolute and relative flows of Beta's households, businesses, financiers and government:

$$y_h^* = 3.4599\cdot k_f \Rightarrow 38.4\% \qquad e_h^* = 2.9599\cdot k_f \Rightarrow 32.8\%$$
$$y_b^* = 3.14521\cdot k_f \Rightarrow 34.9\% \qquad e_b^* = 2.74521\cdot k_f \Rightarrow 30.5\%$$
$$y_f^* = 0.874509\cdot k_f \Rightarrow 9.6\% \qquad e_f^* = 1.87451\cdot k_f \Rightarrow 20.7\% \qquad (6.25)$$
$$y_g^* = 1.54037\cdot k_f \Rightarrow 17.1\% \qquad e_g^* = 1.44037\cdot k_f \Rightarrow 16.0\%$$
$$Y^* = 9.01999\cdot k_f \Rightarrow 100.0\% \qquad E^* = 9.01999\cdot k_f \Rightarrow 100.0\%$$

At this point, it can be confirmed that the sum of sector receipts $Y^* = (y_h^* + y_b^* + y_f^* + y_g^*)$ is equal to the sum of sector payments $E^* = (e_h^* + e_b^* + e_f^* + e_g^*)$. Substituting sector flows (6.25) in market turnovers (6.9), we find Beta's sustainable throughputs of services, products, securities and taxes as functions of annual liquidity increment k_f:

$$y_w^* = {}^4/_5\cdot{}^5/_6\cdot 2.745\cdot k_f + {}^4/_5\cdot{}^1/_2\cdot 1.874\cdot k_f + {}^1/_2\cdot 1.440\cdot k_f = €3.30013\cdot k_f = e_w^* = \rho_w^*$$
$$y_u^* = {}^{17}/_{20}\cdot{}^7/_8\cdot 2.959\cdot k_f + {}^{17}/_{20}\cdot{}^3/_{20}\cdot 1.874\cdot k_f + {}^1/_3\cdot 1.440\cdot k_f = €2.92052\cdot k_f = e_u^* = \rho_u^*$$
$$y_s^* = {}^1/_8\cdot 2.959\cdot k_f + {}^1/_6\cdot 2.745\cdot k_f + {}^7/_{20}\cdot 1.874\cdot k_f + {}^1/_6\cdot 1.440\cdot k_f = €1.72366\cdot k_f = e_s^* = \rho_s^* \qquad (6.26)$$
$$y_\tau^* = {}^3/_{20}\cdot{}^7/_8\cdot 2.959\cdot k_f + {}^1/_5\cdot{}^5/_6\cdot 2.745\cdot k_f + ({}^3/_{20}\cdot{}^3/_{20} + {}^1/_5\cdot{}^1/_2)\cdot 1.874\cdot k_f = €1.07566\cdot k_f = e_\tau^* = \rho_\tau^*$$

The above mathematical relations reveal that transactions concerning labour, output, liquidity and taxation, respectively, make up 36.6%, 32.4%, 19.1% and 11.9% of Beta's monetary circulation. Income demand, expenditure supply and gross turnover are perfectly allocated among institutional sectors and traded goods. It can be verified that $9.0199 \cdot k_f = (y_h + y_b + y_g + y_f) \cdot k_f = (e_h + e_b + e_g + e_f) \cdot k_f = (\rho_w + \rho_u + \rho_\tau + \rho_s) \cdot k = c \cdot k$ where c is the scale factor (Euclidean multiplier) of economy Beta. The impressive result $Y^* = E^* = R^*$ is the logical consequence of the twelve premises of Euclidean modelling methodology.

6.9. Adjusting the withholding tax rates

As shown by equations (6.2), withholding tax rates (τ_w, τ_u, τ_s) determine i) the percentages of private sector outlay (τ_h, τ_b, τ_f) paid as well as ii) the portion of government revenue collected as taxes ($y_{g\tau}^* = \alpha_{g\tau} \cdot y_g^*$). They also affect the exchange network's stability conditions rh_n, n = 1, … , n, and equilibrium flows (y_i^*, e_i^*), $i = 1, \dots, n$. When withholding tax rates (τ_w, τ_u) are modified in accordance with public policy, while other parameters stay as measured by the survey, determinant (6.27) duplicates Beta's characteristic polynomial.

(6.27) Characteristic equation of matrix **B**

$$\begin{vmatrix}
-1-\lambda & \frac{1}{8} & -\frac{1}{7} & (1-\tau_w)\cdot\frac{5}{6}+\frac{1}{6} & -1 & (1-\tau_w)\cdot\frac{1}{2}+\frac{7}{20} & 0 & \frac{3}{6} \\
1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{9} & (1-\tau_u)\cdot\frac{7}{8}+\frac{1}{8} & -1-\lambda & \frac{1}{6} & -1 & (1-\tau_u)\cdot\frac{3}{20}+\frac{7}{20} & 0 & \frac{2}{6} \\
0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 \\
-\frac{1}{9} & \frac{1}{8} & -\frac{1}{7} & \frac{1}{6} & -1-\lambda & \frac{7}{20} & -\frac{1}{99} & \frac{1}{6} \\
0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 \\
-\frac{1}{9} & \tau_u\cdot\frac{7}{8}+\frac{1}{8} & 0 & \tau_w\cdot\frac{5}{6}+\frac{1}{6} & -1 & \tau_w\cdot\frac{1}{2}+\tau_u\cdot\frac{3}{20}+\frac{7}{20} & -1-\lambda & \frac{1}{6} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda
\end{vmatrix}$$

$$= \left| \mathbf{B} - \lambda \cdot \mathbf{I} \right| = b_0 \cdot \lambda^8 + b_1 \cdot \lambda^7 + b_2 \cdot \lambda^6 + b_3 \cdot \lambda^5 + b_4 \cdot \lambda^4 + b_5 \cdot \lambda^3 + b_6 \cdot \lambda^2 + b_7 \cdot \lambda + b_8 = 0 \qquad (6.27)$$

Equations (6.28) present polynomial coefficients b_i, i = 0, … , 8, as functions of τ_w and τ_u:

$$b_0 = 1 \qquad b_1 = 8 \qquad b_2 = 339089/12600$$

$$b_3 = (228800556 - 657396 \cdot \tau_u + 657396 \cdot \tau_w)/4536000$$

$$b_4 = (258728496 - 764370 \cdot \tau_u + 400380 \cdot \tau_w - 3307500 \cdot \tau_u \cdot \tau_w) / 4536000$$

$$b_5 = (174180831 + 3569022 \cdot \tau_u + 5049470 \cdot \tau_w - 12909015 \cdot \tau_u \cdot \tau_w) / 4536000 \qquad (6.28)$$

$$b_6 = (62981859 + 6738912 \cdot \tau_u + 7913660 \cdot \tau_w - 14298165 \cdot \tau_u \cdot \tau_w) / 4536000$$

$$b_7 = (8798709 + 3817278 \cdot \tau_u + 3604110 \cdot \tau_w - 5053965 \cdot \tau_u \cdot \tau_w) / 4536000$$

$$b_8 = (-250299 + 880362 \cdot \tau_u + 742980 \cdot \tau_w - 735315 \cdot \tau_u \cdot \tau_w) / 4536000$$

As mathematical economists, we are interested in isolating the impact of withholding tax rates τ_w and τ_u on the economic system's dynamic stability, steady state and gross turnover, so as to base policy recommendations on incontestable ground. Using *Mathematica*, we substitute coefficient values (6.28) in the minors of matrix (6.22) to compute the eight Routh-Hurwitz determinants as functions of τ_w and τ_u. Among other things, it can be shown that rh_n, $n = 1,.., 8$, are positive when the withholding rate τ_u on output sales is $^{15}/_{100}$ while the withholding rate τ_w on labour salaries is $^{20}/_{100}$. For policy design purposes, it is also advisable to find the withholding rate boundaries within which Beta's equilibrium flows remain greater than zero. With *Mathematica*'s assistance, we apply Cramer's rule to $\mathbf{B}\cdot\mathbf{X}^* = \mathbf{K}$ in order to obtain solution (6.29) that expresses household, business, financier and government incomes and expenditures as functions of withholding tax rates (τ_w, τ_u) and annual money supply k_f.

$$
\begin{aligned}
y_h^* &= 9\cdot(2449 + \tau_u\cdot(88598 - 75905\cdot\tau_w) - 14180\cdot\tau_w)\cdot k_f / 70\cdot(-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
e_h^* &= (219603 + \tau_u\cdot(-334944 + 271190\cdot\tau_w) - 682960\cdot\tau_w)\cdot k_f / 35\cdot(-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
y_b^* &= (40689 + \tau_u\cdot(-13289 + 43710\cdot\tau_w) - 131120\cdot\tau_w)\cdot k_f / 10\cdot(11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
e_b^* &= \cdot(17673 + \tau_u\cdot(-41511 + 54238\cdot\tau_w) - 54528\cdot\tau_w)\cdot k_f / 2\cdot(-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
y_f^* &= (-20679 + \tau_u\cdot(83412 - 61100\cdot\tau_w) + 60360\cdot\tau_w)\cdot k_f / 6\cdot(-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
e_f^* &= (-92193 + \tau_u\cdot(334944 - 2711905\cdot\tau_w) + 272640\cdot\tau_w)\cdot k_f / (-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
y_g^* &= 5\cdot(822 + \tau_u\cdot(3699 + 3155\cdot\tau_w) - 3100\cdot\tau_w)\cdot k_f / 7\cdot(-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)\\
e_g^* &= 3\cdot(41511 + \tau_u\cdot(-36168 + 134285\cdot\tau_w) - 134220\cdot\tau_w)\cdot k_f / 7\cdot(-11919 + 41922\cdot\tau_u + 35380\cdot\tau_w - 35015\cdot\tau_u\cdot\tau_w)
\end{aligned}
\tag{6.29}
$$

Given $k_f > 0$, system (6.29) comprises eight non-linear functions of two unknown variables, τ_w and τ_u. It can be proven that Beta's institutional sectors enjoy positive flows, i.e. $y_i^* > 0$ and $e_i^* > 0$, $i = h, b, g, f$, as long as $18.69\% < \tau_w < 24.67\%$ and $13.87\% < \tau_u < 22.07\%$. Concomitantly, small variations of τ_w and τ_u cause significant and opposite changes in sector revenues and outlays. In consequence, governments rarely increase withholding taxes without having already increased public outlay. Using mathematical software, the following conclusions may also be drawn from equations (6.29):

- When tax rate τ_w on salaries is raised by one percent, consumer, producer, bank and government receipts and payments fall by:

$$dy_h^*/d\tau_w\cdot(\tau_w/y_h^*) = -15.74\% \qquad de_h^*/d\tau_w\cdot(\tau_w/e_h^*) = -18.40\%$$
$$dy_b^*/d\tau_w\cdot(\tau_w/y_b^*) = -17.05\% \qquad de_b^*/d\tau_w\cdot(\tau_w/e_b^*) = -19.54\%$$
$$dy_f^*/d\tau_w\cdot(\tau_w/y_f^*) = -10.32\% \qquad de_f^*/d\tau_w\cdot(\tau_w/e_f^*) = -4.81\% \qquad (6.30)$$
$$dy_g^*/d\tau_w\cdot(\tau_w/y_g^*) = -15.88\% \qquad de_g^*/d\tau_w\cdot(\tau_w/e_g^*) = -16.99\%$$

- When tax rate τ_u on sales is increased by one percent, consumer, producer, bank and government receipts and payments decrease by:

$$dy_h^*/d\tau_u\cdot(\tau_u/y_h^*) = -12.22\% \qquad de_h^*/d\tau_u\cdot(\tau_u/e_h^*) = -14.29\%$$
$$dy_b^*/d\tau_u\cdot(\tau_u/y_b^*) = -13.43\% \qquad de_b^*/d\tau_u\cdot(\tau_u/e_b^*) = -15.38\%$$
$$dy_f^*/d\tau_u\cdot(\tau_u/y_f^*) = -8.11\% \qquad de_f^*/d\tau_u\cdot(\tau_u/e_f^*) = -3.78\% \qquad (6.31)$$
$$dy_g^*/d\tau_u\cdot(\tau_u/y_g^*) = -12.50\% \qquad de_g^*/d\tau_u\cdot(\tau_u/e_g^*) = -13.37\%$$

Withholding tax rates τ_i on the outgoings of institutional sectors (i = h, b, f) are linear combinations of withholding tax rates τ_j on the turnovers of market goods (j = w, u, s) and *vice versa*. Substituting Beta's survey data for the algebraic symbols of (6.4), we derive:

$$\tau_h = \tau_u\cdot\beta_{hu} = {}^3/_{20}\cdot{}^7/_8 = {}^{21}/_{160} = 13.125\%$$
$$\tau_b = \tau_w\cdot\beta_{bw} = {}^1/_5\cdot{}^5/_6 = {}^5/_{30} = 16.666\% \qquad (6.32)$$
$$\tau_f = \tau_w\cdot\beta_{fw} + \tau_u\cdot\beta_{fu} = ({}^1/_5\cdot{}^1/_2 + {}^3/_{20}\cdot{}^3/_{20}) = ({}^{40}/_{400} + {}^9/_{400}) = 12.25\%$$

Households and businesses pay 13.125% and 16.666% of their respective annual expenditure as tax. Although transactions concerning financial assets and liabilities are not subject to withholding tax, the outlay of banking institutions is taxed at $\tau_f = 12.25\%$.

6.10. Subsidisation of market goods

In theory, withholding tax rates τ_j on market turnovers (j = 1, … , n–1) are independent of each other. However, the shares σ_j of traded goods (j = 1, … , n–1) in public expenditure are linearly connected. Given $\sigma_1 + \ldots + \sigma_{n-1} = 1$, the budget ratio destined for one market good may be increased if, and only if, the budget ratio destined for another market good may be reduced. Most politicians tend to ignore this fine detail. Below we derive coefficients b_i, i = 0, … , 8, of Beta's characteristic polynomial $|\mathbf{B} - \lambda\cdot\mathbf{I}|$ as functions of public expenditure proportions σ_j, j = w, u, s, destined for services σ_w, products σ_u and securities σ_s, assuming that all other parameters remain as estimated by the survey:

$b_0 = 1$ $\qquad b_1 = 8$ $\qquad b_2 = 818853600\cdot\sigma_s - 30240000/30240000$

$$b_3 = (1547333884 - 151200000\cdot\sigma_s + 3360000\cdot\sigma_w)/30240000$$

$$b_4 = (1773758830 - 292484400\cdot\sigma_s - 14880000\cdot\sigma_u + 5691000\cdot\sigma_w)/30240000$$

$$b_5 = (1233178919 - 272124400\cdot\sigma_s - 47571800\cdot\sigma_u - 6084400\cdot\sigma_w)/30240000 \tag{6.33}$$

$$b_6 = (484639919 - 93849400\cdot\sigma_s - 61252100\cdot\sigma_u - 28542200\cdot\sigma_w)/30240000$$

$$b_7 = (89690925 + 15759900\cdot\sigma_s - 36391500\cdot\sigma_u - 27834100\cdot\sigma_w)/30240000$$

$$b_8 = (4631379 + 7547700\cdot\sigma_s - 7226400\cdot\sigma_u - 6850500\cdot\sigma_w)/30240000$$

Using *Mathematica*, we substitute coefficients (6.33) in the minors of matrix (6.22) in order to compute Routh-Hurwitz conditions rh_n, $n = 1, \ldots, 8$, as functions of σ_w, σ_u and σ_s. It can be proven that determinants rh_n are positive – and thus dynamic economy Beta is stable – when the budget share spent on:

- labour is $0 < \sigma_w < 0.508069$
- output is $0.270996 < \sigma_u < 0.340983$
- debt is $0.159343 < \sigma_s < 0.225774$.

The ranges of σ_w, σ_u and σ_s, within which sector flows (y_i^*, e_i^*) remain greater than zero, are also very important for policy design purposes. Using Cramer's rule, we solve system $\mathbf{dX/dt = B\cdot X - K}$ for steady-state vector $\mathbf{X^* = B^{-1}\cdot K}$ to obtain household (y_h^*, e_h^*), business (y_b^*, e_b^*), financier (y_f^*, e_f^*) and government (y_g^*, e_g^*) receipts and payments as functions of the allocation (σ_w, σ_u, σ_s) of public expenditure and the increment k_f of bank liabilities, while other exogenous variables stay constant:

$$\begin{aligned}
y_h^* &= 3\cdot(221149 + 324242\cdot\sigma_s - 316818\cdot\sigma_u - 291942\cdot\sigma_w)\cdot k_f/11200000\cdot|B| \\
e_h^* &= (-873971 - 1912483\cdot\sigma_s + 1744957\cdot\sigma_u + 1767533\cdot\sigma_w)\cdot k_f/50400000\cdot|B| \\
y_b^* &= (229223 + 215089\cdot\sigma_s - 217231\cdot\sigma_u - 219737\cdot\sigma_w)\cdot k_f/14400000\cdot|B| \\
e_b^* &= (-1523537 - 2852659\cdot\sigma_s + 2704861\cdot\sigma_u + 2531947\cdot\sigma_w)\cdot k_f/33600000\cdot|B| \\
y_f^* &= (1080693 + 1831887\cdot\sigma_s - 1677033\cdot\sigma_u - 1589537\cdot\sigma_w)\cdot k_f/20160000\cdot|B| \\
e_f^* &= (4168279 + 6863687\cdot\sigma_s - 6494633\cdot\sigma_u - 6156537\cdot\sigma_w)\cdot k_f/20160000\cdot|B| \\
y_g^* &= (18092 + 25159\cdot\sigma_s - 24088\cdot\sigma_u - 22835\cdot\sigma_w)\cdot k_f/1008000\cdot|B| \\
e_g^* &= 88469\cdot k_f/33600000\cdot|B|
\end{aligned} \tag{6.34}$$

In this case, $|B| = (1543793 + 2515900 \cdot \sigma_s - 2408800 \cdot \sigma_u - 2283500 \cdot \sigma_w) / 10080000$. Considering that $1 = \sigma_w + \sigma_u + \sigma_s$, system (6.34) comprises eight non-linear equations in three unknown variables, k_f, σ_w, σ_u or σ_s. Granted $k_f > 0$, it can be established that Beta's institutional sectors enjoy positive monetary flows, i.e. $y_i^* > 0$ and $e_i^* > 0$, $i = h, b, g, f$, as long as the portion of public expenditure allocated to:

- labour employment is $0.433406 < \sigma_w < 0.508069$
- output procurement is $0.270996 < \sigma_u < 0.340983$
- debt service is $0.159343 < \sigma_s < 0.225774$.

The budget ratio destined for one traded good may be raised if, and only if, the budget ratio destined for another traded good may be reduced.

(6.35) Reallocation of public expenditure on labour

Increase of σ_w through a reduction of σ_u	Increase of σ_w through a reduction of σ_s
$(dy_h^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -19.8824 \cdot k_f$	$(dy_h^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 810.891 \cdot k_f$
$(de_h^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -19.8824 \cdot k_f$	$(de_h^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 810.891 \cdot k_f$
$(dy_b^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -21.4828 \cdot k_f$	$(dy_b^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 802.695 \cdot k_f$
$(de_b^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -21.4828 \cdot k_f$	$(de_b^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 802.695 \cdot k_f$
$(dy_f^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -3.5727 \cdot k_f$	$(dy_f^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 134.938 \cdot k_f$
$(de_f^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -3.5727 \cdot k_f$	$(de_f^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 134.938 \cdot k_f$
$(dy_g^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -9.79458 \cdot k_f$	$(dy_g^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 375.164 \cdot k_f$
$(de_g^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_w) = -9.79458 \cdot k_f$	$(de_g^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_w) = 375.164 \cdot k_f$

Tables (6.35), (6.36) and (6.37), show the effects of opposite but simultaneous differentiations of the distribution of public expenditure. Beta's structure is such that sector flows (y_i^*, e_i^*) change inversely with the percentage of public expenditure allocated to output procurement σ_u and directly with the percentage of public expenditure allocated to debt service σ_s. The budget ratio σ_w allocated to labour employment has an expansionary impact when it is increased through reduction of public expenditure for debt service σ_s and a contractionary impact when it is increased through reduction of public expenditure for output procurement σ_u.

(6.36) Reallocation of public expenditure on output

Increase of σ_u through a reduction of σ_w	Increase of σ_u through a reduction of σ_s
$(dy_h^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 19.8824\cdot k_f$	$(dy_h^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 830.774\cdot k_f$
$(de_h^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 19.8824\cdot k_f$	$(de_h^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 830.774\cdot k_f$
$(dy_b^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 21.4828\cdot k_f$	$(dy_b^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 824.178\cdot k_f$
$(de_b^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 21.4828\cdot k_f$	$(de_b^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 824.178\cdot k_f$
$(dy_f^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 3.5727\cdot k_f$	$(dy_f^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 138.51\cdot k_f$
$(de_f^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 3.5727\cdot k_f$	$(de_f^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 138.51\cdot k_f$
$(dy_g^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 9.79458\cdot k_f$	$(dy_g^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 384.959\cdot k_f$
$(de_g^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_u) = 9.79458\cdot k_f$	$(de_g^*/d\sigma_s)\cdot(d\sigma_s/d\sigma_u) = 384.959\cdot k_f$

(6.37) Reallocation of public expenditure on securities

Increase of σ_s through a reduction of σ_u	Increase of σ_s through a reduction of σ_w
$(dy_h^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -830.774\cdot k_f$	$(dy_h^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -810.891\cdot k_f$
$(de_h^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -830.774\cdot k_f$	$(de_h^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -810.891\cdot k_f$
$(dy_b^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -824.178\cdot k_f$	$(dy_b^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -802.695\cdot k_f$
$(de_b^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -824.178\cdot k_f$	$(de_b^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -802.695\cdot k_f$
$(dy_f^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -138.51\cdot k_f$	$(dy_f^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -134.938\cdot k_f$
$(de_f^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -138.51\cdot k_f$	$(de_f^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -134.938\cdot k_f$
$(dy_g^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -384.959\cdot k_f$	$(dy_g^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -375.164\cdot k_f$
$(de_g^*/d\sigma_u)\cdot(d\sigma_u/d\sigma_s) = -384.959\cdot k_f$	$(de_g^*/d\sigma_w)\cdot(d\sigma_w/d\sigma_s) = -375.164\cdot k_f$

Finally, given the survey data regarding Beta, equations (6.9) enable us to calculate the portions of market proceeds (ρ_w^*, ρ_u^*, ρ_s^*) which stem from public spending, while equations (6.11) to (6.14) enable us to calculate the portions of private incomes (y_h^*, y_b^*, y_f^*) which stem from public spending. Specifically, in economy Beta:

- 21.8% of labour sales, 16.4% of output sales and 13.9% of bond sales are generated by government purchases;

- 27.5% of household income, 15.3% of businesses income and 20.8% of financier income derive from public payments.

6.11. Sector allocation of credit

Polynomial coefficients b_i, $i=0, \dots, 8$, and thus Routh-Hurwitz determinants rh_n, $n=1, \dots, 8$, are functions of consumer, producer and financier behaviour coefficients (α_{ij}, β_{ij}), $i=h, b, f$, as well as of fiscal policy parameters ($\alpha_{g\tau}$, τ_j, σ_j), $j=w, u, s$. Annual expansion (k_f) and sector distribution ($\kappa_h + \kappa_b + \kappa_g = 1$) of bank credit influence neither the polynomial coefficients b_i nor the Routh-Hurwitz conditions rh_n. However, monetary policy vector $\mathbf{K}(\kappa_h \cdot k_f, 0, \kappa_b \cdot k_f, 0, -k_f, 0, \kappa_g \cdot k_f)$ is decisive regarding the magnitudes and directions of sector flows $\mathbf{X}^*$ and market proceeds $\mathbf{R}^*$. Assuming that all other exogenous variables stay constant, we apply Cramer's rule to $\mathbf{B} \cdot \mathbf{X}^* = \mathbf{K}$ in order to compute household, business, bank and government annual revenues and outlays as functions of k_f, κ_h, κ_b and κ_g:

$$
\begin{aligned}
y_h^* &= -9 \cdot (-1564862 + 1546016\kappa_b + 1097930\kappa_g + 1630823\kappa_h) \cdot k_f / 55279 \\
e_h^* &= -2 \cdot (-7041879 + 6957072\kappa_b + 4940685\kappa_g + 7366343\kappa_h) \cdot k_f / 55279 \\
y_b^* &= -(-1990537 + 1964969\kappa_b + 1396992\kappa_g + 2080025\kappa_h) \cdot k_f / 7897 \\
e_b^* &= -(-1990537 + 1972866\kappa_b + 1396992\kappa_g + 2080025\kappa_h) \cdot k_f / 7897 \\
y_f^* &= -(-1034914 + 1015440\kappa_b + 722775\kappa g + 1071485\kappa h) \cdot k_f / 23691 \\
e_f^* &= -5 \cdot (-211721 + 203088\kappa_b + 144555\kappa_g + 214297\kappa_h) \cdot k_f / 23691 \\
y_g^* &= -100 \cdot (-65292 + 64476\kappa_b + 45761\kappa_g + 68148\kappa_h) \cdot k_f / 55279 \\
e_g^* &= -3 \cdot (-2176400 + 2149200\kappa_b + 1543793\kappa_g + 2271600\kappa_h) \cdot k_f / 55279
\end{aligned}
\tag{6.38}
$$

With a view to regulating receipts and payments, the banking authority needs to know the current liquidity shares of consumers, producers and the government as well as the liquidity share ranges within which institutional sector flows remain greater than zero. Granted that $k_f > 0$ and $\kappa_h + \kappa_b + \kappa_g = 1$, system (6.38) encompasses eight linear equations in terms of two unknowns. With *Mathematica's* help, it can be shown that Beta's institutional sectors enjoy positive incomes and expenditures, i.e. $y_i^* > 0$ and $e_i^* > 0$, $i = h, b, f, g$, as long as the portion of new loans allocated to:

- consumer households is $0 < \kappa_h < 0.510422$
- business enterprises is $0 < \kappa_b < 0.410989$

- government organisations is $0 < \kappa_g < 0.115518$.

Given that κ_h should not exceed 51.04% and that κ_b should not exceed 41.10%, it is deduced that κ_g should not be less than 7.86%. Similarly, we can show that the minimum share κ_h is 47.35% and the minimum share κ_b is 37.41% of Beta's annual credit. With constant liquidity expansion, the amount of credit destined for one sector may be raised if, and only if, the amount of credit destined for another sector may be reduced.

(6.39) Reallocation of bank credit to the household sector

Increase of κ_h through reduction of κ_b	Increase of κ_h through reduction of κ_g
$(dy_h^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -13.8075\cdot k_f$	$(dy_h^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -86.7606\cdot k_f$
$(de_h^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -14.8075\cdot k_f$	$(de_h^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -87.7606\cdot k_f$
$(dy_b^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -14.5696\cdot k_f$	$(dy_b^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -86.4927\cdot k_f$
$(de_b^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -13.5696\cdot k_f$	$(de_b^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -86.4927\cdot k_f$
$(dy_f^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -2.36567\cdot k_f$	$(dy_f^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -14.7191\cdot k_f$
$(de_f^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -2.36567\cdot k_f$	$(de_f^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -14.7191\cdot k_f$
$(dy_g^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -6.64267\cdot k_f$	$(dy_g^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -40.4982\cdot k_f$
$(de_g^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_h) = -6.64267\cdot k_f$	$(de_g^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_h) = -39.4982\cdot k_f$

(6.40) Reallocation of bank credit to the business sector

Increase of κ_b through reduction of κ_h	Increase of κ_b through reduction of κ_g
$(dy_h^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 13.8075\cdot k_f$	$(dy_h^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -72.9531\cdot k_f$
$(de_h^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 14.8075\cdot k_f$	$(de_h^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -72.9531\cdot k_f$
$(dy_b^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 14.5696\cdot k_f$	$(dy_b^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -71.9231\cdot k_f$
$(de_b^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 13.5696\cdot k_f$	$(de_b^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -72.9231\cdot k_f$
$(dy_f^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 2.36567\cdot k_f$	$(dy_f^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -12.\ 3534\cdot k_f$
$(de_f^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 2.36567\cdot k_f$	$(de_f^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -12.\ 3534\cdot k_f$
$(dy_g^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 6.64267\cdot k_f$	$(dy_g^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -33.8555\cdot k_f$
$(de_g^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_b) = 6.64267\cdot k_f$	$(de_g^*/d\kappa_g)\cdot(d\kappa_g/d\kappa_b) = -32.8555\cdot k_f$

(6.41) Reallocation of bank credit to the government sector

Increase of κ_g through reduction of κ_b	Increase of κ_g through reduction of κ_h
$(dy_h^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 72.9531\cdot k_f$	$(dy_h^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 86.7606\cdot k_f$
$(de_h^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 72.9531\cdot k_f$	$(de_h^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 87.7606\cdot k_f$
$(dy_b^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 71.9231\cdot k_f$	$(dy_b^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 86.4927\cdot k_f$
$(de_b^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 72.9231\cdot k_f$	$(de_b^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 86.4927\cdot k_f$
$(dy_f^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 12.\ 3534\cdot k_f$	$(dy_f^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 14.\ 7191\cdot k_f$
$(de_f^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 12.\ 3534\cdot k_f$	$(de_f^*/d\kappa_h)\cdot(d\mathrm{h}/d\kappa\mathrm{g}) = 14.\ 7191\cdot k_f$
$(dy_g^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 33.8555\cdot k_f$	$(dy_g^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 40.4982\cdot k_f$
$(de_g^*/d\kappa_b)\cdot(d\kappa_b/d\kappa_g) = 32.8555\cdot\ k_f$	$(de_g^*/d\kappa_h)\cdot(d\kappa_h/d\kappa_g) = 39.4982\cdot k_f$

In Tables (6.39), (6.40) and (6.41) above, we compare the changes of household, business, financier and government annual flows induced by simultaneous but opposite differentiations of the credit shares of two institutional sectors. Beta's circulation structure forces sector receipts and payments (y_i^*, e_i^*) to change directly with household share κ_h and inversely with government share κ_g of new bank loans. The percentage allocated to businesses κ_b influences steady-state flows positively when raised through decrease of κ_h, and negatively when raised through decrease of κ_g. It is reiterated that annual liquidity increment (k_f), credit allocation (κ_h, κ_b, κ_g) and discount rate ($r = 1/p_s$), while affecting nominal and real flows, do not touch dynamic stability. On the basis of a Euclidean model of the global economy, international financial organisations, such as the IMF, World Bank, ECB, etc., will be well-positioned to offer advice and take steps regarding optimum expansion, distribution and valuation of the universal banking balance sheet.

6.12. Example of domestic intervention

Within their respective budget constraints, consumers, producers, etc., choose the combinations of revenue sources and outlay destinations which, in their opinion, maximise their net worth. Whereas market receipts and payments are by definition equal, demanded quantities (via supply of expenditure) may differ from available quantities due to real shocks, rigid quotations and defective policies. Private sectors track market shortages and surpluses with the intention of adjusting salaries, prices or values and claiming appropriate policy changes. Households, businesses and financiers appear in favour of government involvement because they understand that on their own they would be unable to:

- fine-tune the incomes and expenditures of institutional sectors;
- ameliorate negative effects originating from external shocks;
- enforce constitutional arrangements, lawful contracts, etc.

Institutional sectors compare their current (ρ_w^*, ρ_u^*, ρ_s^*, ρ_τ^*) to target ($\rho_w^\#$, $\rho_u^\#$, $\rho_s^\#$, $\rho_\tau^\#$) proceeds that would sustain more acceptable labour ($q_w^* - q_w^\#$), output ($q_u^* - q_u^\#$), liquidity ($q_s^* - q_s^\#$) and taxation ($y_{g\tau}^* - y_{g\tau}^\#$) gaps. In particular, government is under incessant pressure to fulfil international standards regarding the primary surplus. Since 2007, Eurozone countries have tightened rules on fiscal shortfalls in order to fight the public debt crisis. To this end, they have set up a centralized mechanism that is automatically triggered when a member state registers a budget deficit that deviates from the permissible borrowing by 0.5 percent of gross domestic product. Fiscal and monetary authorities acting together may guide $\mathbf{dX/dt = B \cdot X - K}$ to steady state $\mathbf{X}^\#$ and therefore $\mathbf{R}^\#(\rho_w^\#, \rho_u^\#, \rho_s^\#, \rho_\tau^\#)$ that is characterised by tolerable shortages and surpluses. Needless to reiterate that any four independent parameters (k_f; κ_h, κ_b or κ_g; τ_w, τ_u; σ_w, σ_u or σ_s), which solve four equation system (6.9) or (6.42) according to the agreed target $\mathbf{R}^\#$, should also yield positive Routh - Hurwitz determinants (rh_n, $n = 1, \ldots, 8$) and positive sector flows ($y_i^* > 0$, $e_i^* > 0$, $i = h, b, f, g$).

$$\begin{aligned}
&\rho_w = \beta_{bw} \cdot e_b + \beta_{fw} \cdot e_f + \sigma_w \cdot e_g - (\tau_w \cdot \beta_{bw} \cdot e_b + \tau_w \cdot \beta_{fw} \cdot e_f) = \rho_w = p_w \cdot q_w \\
&\rho_u = \beta_{hu} \cdot e_h + \beta_{fu} \cdot e_f + \sigma_u \cdot e_g - (\tau_u \cdot \beta_{hu} \cdot e_h + \tau_u \cdot \beta_{fu} \cdot e_f) = \rho_u = p_u \cdot q_w \\
&\rho_s = \beta_{hs} \cdot e_h + \beta_{bs} \cdot e_b + \beta_{fs} \cdot e_f + \sigma_s \cdot e_g = \rho_s = p_s \cdot q_s \\
&\rho_\tau = \tau_u \cdot \beta_{hu} \cdot e_h + \tau_w \cdot \beta_{bw} \cdot e_b + (\tau_w \cdot \beta_{fw} + \tau_u \cdot \beta_{fu}) \cdot e_f = \rho_\tau = \alpha_{g\tau} \cdot y_g
\end{aligned} \tag{6.42}$$

The following observations concern possible choices and combinations of instruments:

- One should always bear in mind linear dependence among tools belonging to the same class and the unique role of k_f regarding nominal magnitudes.
- Ideally, any four linearly independent tools should be selected among (k_f; κ_h, κ_b or κ_g; τ_w, τ_u; σ_w, σ_u or σ_s) for solving system (6.42).
- Bank balance sheet increment (k_f) and sector credit shares (κ_h, κ_b or κ_g) enter the numerators but not the denominator $|\mathbf{B}|$ of sector and market flows.

- Monetary policy instruments affect sector and market flows but neither the structure nor the stability of the exchange network.
- To steer clear of massive non-linearities and multiple solutions, it is advisable to allow only one of the fiscal tools (τ_w, τ_u; σ_w, σ_u or σ_s) among the four unknowns of (6.42).

Aiming to exemplify a method for bridging quantity deficits and surpluses within a year, we have chosen to solve system (6.42) in terms of τ_u, k_f, κ_h and κ_g. The recommended steps toward minimisation of market and budget gaps are outlined in the following scenario:

1) Using *Mathematica*, all symbols of behaviour coefficients and policy parameters in $\mathbf{B}^{-1} \cdot \mathbf{K}$ should be replaced with the matching data to compute Beta's nominal equilibrium $\mathbf{X}^*$. The figures for (y_i^*, e_i^*) should be placed in (6.42) to yield the proceeds from services ρ_w^*, products ρ_u^*, securities ρ_s^* and taxes ρ_τ^*, listed in Table (6.43).

(6.43) Sector and market flows of economy Beta

$y_h^* = 3.45990 \cdot k_f^*$	$e_h^* = 2.959 \cdot k_f^*$	$\rho_w^* = 3.30013 \cdot k_f^*$
$y_b^* = 3.145207 \cdot k_f^*$	$e_b^* = 2.745 \cdot k_f^*$	$\rho_u^* = 2.92052 \cdot k_f^*$
$y_f^* = 0.874 \cdot k_f^*$	$e_f^* = 1.874 \cdot k_f^*$	$\rho_s^* = 1.72366 \cdot k_f^*$
$y_g^* = 1.540 \cdot k_f^*$	$e_g^* = 1.440 \cdot k_f^*$	$\rho_\tau^* = 1.07566 \cdot k_f^*$
$Y^* = 9.0199 \cdot k_f^*$	$E^* = 9.0199 \cdot k_f^*$	$R^* = 9.0199 \cdot k_f^*$

Given that Euclidean systems (6.9), (6.15) and (6.42) have been derived according to the laws of economics and mathematics, it can be confirmed that $Y^* = E^* = R^* = 9.0199 \cdot k_f^*$.

2) Beta's statistical survey has revealed that scarcity of real goods, excess supply of securities and a small budget surplus occur at the same time. As a result, contractionary and expansionary policies should be pursued simultaneously. During a debate on the economic situation, experts argued in favour of reducing demand for services and products by 9.5% and 28.7%, respectively, while increasing demand for consols by 39.15%. In this connection, ECB, IMF and credit rating agencies indicated that next year's total spending should be 93.7% of the current level in order to keep quotations as stable as possible. Everyone agreed that a policy mix encompassing four tools (one fiscal and three monetary) should be immediately estimated and suitably implemented for achieving the four target turnovers listed in the middle column of Table (6.44).

(6.44) Current and target turnovers of economy Beta

$\rho_w^* = €3.30013\cdot 10^9$	$\rho_w^\# = €2.98731\cdot 10^9$	$\rho_w^\# / \rho_w^* = 0.9052$
$\rho_u^* = €2.92052\cdot 10^9$	$\rho_u^\# = €2.08099\cdot 10^9$	$\rho_u^\# / \rho_u^* = 0.7125$
$\rho_s^* = €1.72366\cdot 10^9$	$\rho_s^\# = €2.39846\cdot 10^9$	$\rho_s^\# / \rho_s^* = 1.39150$
$\rho_\tau^* = €1.07566\cdot 10^9$	$\rho_\tau^\# = €0.986286\cdot 10^9$	$\rho_\tau^\# / \rho_\tau^* = 0.9169$
$R^* = €9.01999\cdot 10^9$	$R^\# = €8.45304\cdot 10^9$	$R^\# / R^* = 0.937146$

3) Without further ado, Beta's parliament raised the withholding tax on salaries to $^{21}/_{100}$ from $^{20}/_{100}$, thus leaving four unknowns in four equation system (6.42). Next, a mathematical economist undertook to obtain sector revenues and outlays as functions of control variables τ_u, k_f, κ_h and κ_g by substituting the statistics for behaviour coefficients and other parameters in $\mathbf{X}^* = \mathbf{B}^{-1} \cdot \mathbf{K}$. Table (6.45) gives the derived expressions for (y_i^*, e_i^*).

4) Subsequently, the expert placed steady-state flows (6.45) in system (6.42) to generate target proceeds $\rho_w^\#$, $\rho_u^\#$, $\rho_s^\#$ and $\rho_\tau^\#$ as functions of τ_u, k_f, κ_h and κ_g. Table (6.46) displays the resulting system $\mathcal{R}^\# = \mathbf{\Psi} \cdot \mathbf{\Pi}^\#$ that comprises four non-linear equations in four unknowns. Solution of (6.46) with *Mathematica* confirms that the authorities will attain their annual turnover goals if they put into effect set $\mathbf{\Pi}^\#(\tau_u = {}^{16}/_{100}$, $k_f = €4\cdot 10^9$, $\kappa_h = {}^{45}/_{100}$ and $\kappa_g = {}^{7}/_{100})$, which is the more meaningful of the two solutions of (6.46). Other things being equal, all inflationary and deflationary gaps will be eliminated during the next period provided the legislative authority raises the withholding tax rate on output sales by 1%, while the banking authority quadruples provision of liquidity and reallocates 8% of annual credit to productive enterprises from consumer households (-5%) and government organisations (-3%). In this connection, the expert also verified that solution $\mathbf{\Pi}^\#$ will not upset either the stability conditions or the positive signs of Beta's monetary flows.

Groups of labour, business and political leaders, although experienced in debating public policy issues, are not capable of solving economic problems accurately. Whenever the requisite data become available, mathematical economists versed in construction and application of Euclidean models are in position to offer:

- exact solutions regarding appropriate magnitudes of monetary and fiscal tools;
- precise forecasts regarding the consequences of alternative economic policies.

(6.45) Sector flows as functions of the four policy variables

$y_h^* = 3\ k_f \cdot(-36\,400 - 444\,424\cdot\kappa_g + 163\,800\cdot\kappa_h + 117\,138\cdot\tau_u + 11\,066\cdot\kappa_g\cdot\tau_u - 527\,121\cdot\kappa_h\cdot\tau_u) \,/\, 7\cdot(-29\,928 + 230\,459\cdot\tau_u)$

$e_h^* = 2\ k_f \cdot(-54\,600 - 666\,636\cdot\kappa_g + 140\,952\cdot\kappa_h + 175\,707\cdot\tau_u + 16\,599\cdot\kappa_g\cdot\tau_u + 15\,925\cdot\kappa_h\cdot\tau_u) \,/\, 7\cdot(-29\,928 + 230\,459\cdot\tau_u)$

$y_b^* = 2\ k_f \cdot(-5\,264 - 109\,646\cdot\kappa_g + 23\,688\cdot\ \kappa_h + 5\,264\cdot\tau_u + 106\,899\cdot\kappa_g\cdot\tau_u - 23\,688\cdot\kappa_h\cdot\tau_u) \,/\, (-29\,928 + 230\,459\cdot\tau_u)$

$e_b^* = k_f \cdot(40\,456 + 189\,364\cdot\kappa_g - 77\,304\cdot\kappa_h - 240\,987\cdot\tau_u + 16\,661\cdot\kappa_g\cdot\tau_u + 277\,835\cdot\kappa_h\cdot\tau_u) \,/\, (-29\,928 + 230\,459\cdot\tau_u)$

$y_f^* = k_f \cdot(-19\,696 + 85\,580\cdot\kappa_g - 31\,080\cdot\kappa_h + 187\,158\cdot\tau_u + 82\,995\cdot\kappa_g\cdot\tau_u + 79\,625\cdot\kappa_h\cdot\tau_u) \,/\, 3\cdot(-29\,928 + 230\,459\cdot\tau_u)$

$e_f^* = 5\ k_f \cdot(-21\,896 + 17\,116\cdot\kappa_g - 6216\cdot\kappa_h + 175\,707\cdot\tau_u + 16\,599\cdot\ \kappa_g\cdot\tau_u + 15\,925\cdot\kappa_h\cdot\tau_u) \,/\, 3\cdot(-29\,928 + 230\,459\cdot\tau_u)$

$y_g^* = 50\ k_f \cdot(672 + 9\,540\cdot\kappa_g - 3\,024\cdot\kappa_h - 672\cdot\tau_u + 20\,677\cdot\kappa_g\cdot\tau_u + 3024\cdot\kappa_h\cdot\tau_u) \,/\, 7\cdot(-29\,928 + 230\,459\cdot\tau_u)$

$e_g^* = 3\ k_f \cdot(-11\,200 - 228\,832\cdot\kappa_g + 50\,400\cdot\kappa_h + 11\,200\cdot\tau_u + 193\,121\cdot\kappa_g\cdot\tau_u - 50\,400\cdot\kappa_h\cdot\tau_u) \,/\, 7\cdot(-29\,928 + 230\,459\cdot\tau_u)$

(6.46) Market sales as functions of the four policy variables

$\rho_w^{\#} = k_f\cdot(306\,992 + 3\,884\,330\cdot\kappa_g - 1\,381\,464\cdot\kappa_h - 952\,896\cdot\tau_u - 409\,225\cdot\kappa_g\cdot\tau_u + 4\,288\,032\cdot\kappa_h\cdot\tau_u) \,/\, 21\ (-29\,928 + 230\,459\cdot\tau_u)$

$\rho_u^{\#} = -13\ k_f\cdot(-5\,264 - 109\,646\cdot\kappa_g + 23\,688\cdot\kappa_h + 5\,264\cdot\tau_u + 106\,899\cdot\kappa_g\cdot\tau_u - 23\,688\cdot\kappa_h\cdot\tau_u) \,/\, 7\ (-29\,928 + 230\,459\cdot\tau_u)$

$\rho_s^{\#} = -k_f\cdot(137\,760 - 3\,431\,340\cdot\kappa_g + 1\,056\,048\cdot\kappa_h - 2\,320\,752\ \tau_u + 80\,959\cdot\kappa_g\ \tau_u - 2\,462\,320\cdot\kappa_h\ \tau_u) \,/\, 42\ (-29\,928 + 230\,459\cdot\tau_u)$

$\rho_\tau^{\#} = k_f \cdot(6\,496 + 72\,268\cdot\kappa_g - 29\,232\cdot\kappa_h - 6\,496\cdot\tau_u + 353\,517\cdot\kappa_g\cdot\tau_u + 29\,232\ \cdot\kappa_h\cdot\tau_u) \,/\, 2\ (-29\,928 + 230\,459\cdot\tau_u)$

(6.47) Beta's equilibrium at $\tau_w = {}^{20}/_{100}$, $\tau_u = {}^{15}/_{100}$, k_f=€10^9, $\kappa_h = {}^{50}/_{100}$, $\kappa_b = {}^{40}/_{100}$, $\kappa_g = {}^{10}/_{100}$

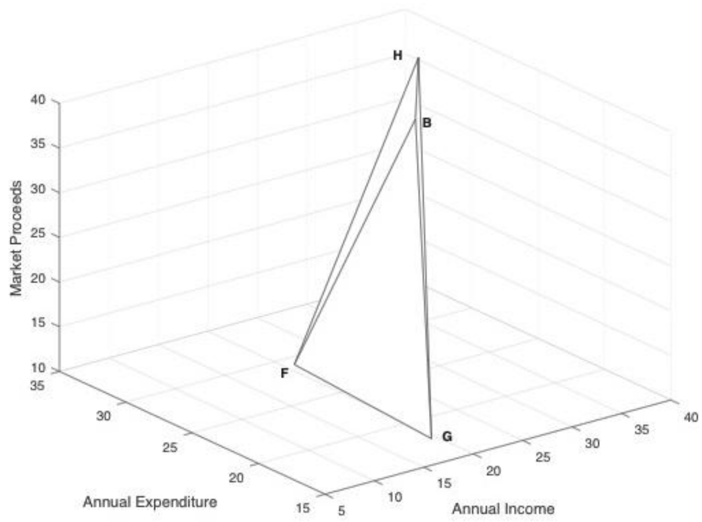


Figure (6.47) depicts the proportions of annual income, expenditure and throughput going to households **H**(38.36%, 32.8%, 36.59%), businesses **B**(34.87%, 30.43%, 32.38), financiers **F**(9.7%, 20.8%, 19.11%) and government G(17.07%, 15.97%, 11.92%). Beyond the tetrahedron's vertices there exist forces acting in both directions on each axis, which compel the dependent variables to return to their steady state combinations.

(6.48) Beta's equilibrium at $\tau_w = {}^{21}/_{100}$, $\tau_u = {}^{16}/_{100}$, k_f=€$4 \cdot 10^9$, $\kappa_h = {}^{45}/_{100}$, $\kappa_b = {}^{48}/_{100}$, $\kappa_g = {}^{7}/_{100}$

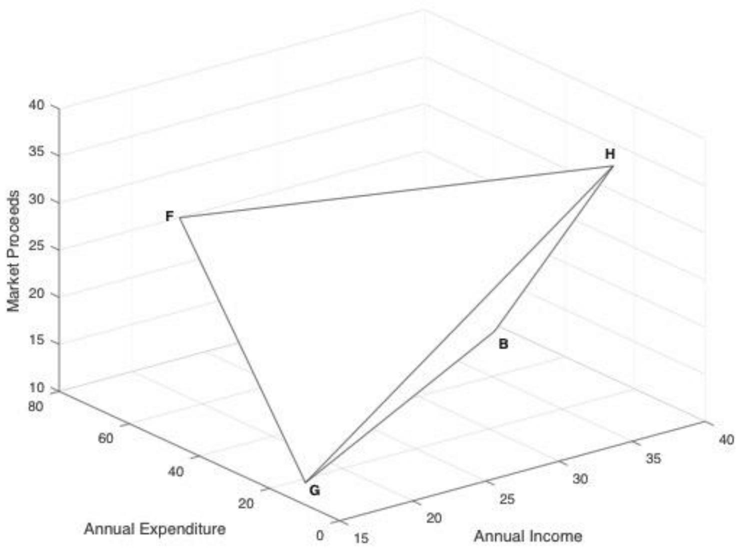


Figure (6.48) reflects the reallocation of consumer **H**(37.63%, 16.33%, 35.34%), producer **B**(26.51%, 3.8%, 24.62%), financier **F**(20.25%, 67.57%, 28.37%) and government **G**(15.61%, 12.30%, 11.67%) yearly flows triggered by higher withholding taxes on labour and output combined with redistribution of the annual credit increment. The new mix of fiscal and monetary policies has reshaped Beta's circulation structure and geometric form.

It is noted that selling the measures and their effects to the electorate is a different matter altogether. In Beta's case, transition from $\mathbf{\Pi}^*(\tau_w = {}^{20}/_{100}$ and $\tau_u = {}^{15}/_{100}$, $k_f = €10^9$, $\kappa_h = {}^{50}/_{100}$, $\kappa_b = {}^{40}/_{100}$, $\kappa_g = {}^{10}/_{100})$ to $\mathbf{\Pi}^\#(\tau_w = {}^{21}/_{100}$ and $\tau_u = {}^{16}/_{100}$, $k_f = €4 \cdot 10^9$, $\kappa_h = {}^{45}/_{100}$, $\kappa_b = {}^{48}/_{100}$, $\kappa_g = {}^{7}/_{100})$ will drive the primary surplus component of fiscal gap (6.7) to €4.40·10^8 from €3.25·10^8 during the year before. Mainly due to reduction of credit to government, public expenditure e_g will drop more than public revenue y_g. As a result, government will add €2.8·10^8 to its bank deposits (liquidity cushion) as compared to adding €10^8 during the previous year. Assuming that nominal salaries, output prices and interest rates remain steady, the purchasing power of sector receipts and payments will fall by 6.3% on average.

In Section 5.6 we explained that monetary financial institutions generate higher gross turnover than value added per time unit because they routinely settle interest coupons using newly issued consols (or money), i.e. their own characteristic good. Interest paid by banks on their outstanding securities (or deposits) should be deducted from Beta's annual turnover R^* for the purpose of estimating Beta's value added:

$$V^* = R^* - \beta_{fs} \cdot e_f^* \tag{6.49}$$

In view of the above, the move from policy set $\mathbf{\Pi}^*$ to policy set $\mathbf{\Pi}^\#$ will cause Beta's value added to drop by 22.8% from €8.364·10^9 to €6.454·10^9. This dismal projection should mark the end of the role played by mathematical economists and the beginning of the role played by labour, business and political leaders.

It this chapter, we have shown that – under certain well defined conditions – the central bank and the public sector may steer a closed economy's steady state by changing their own takings and outgoings in concerted ways. In the short run, the authorities may not always be able to guide the system as they would wish because of i) unstable dynamics generated by the exchange network and ii) limited variability of monetary and fiscal instruments. In the long run, however, quantity gaps tend to be minimised through inevitable adjustments of salaries, prices and values as well as purposeful modifications of behaviour coefficients and policy parameters. In the next chapter, two separate national economies will be combined into an international system of trade and payments. Among other things, it will be shown that central banks and sovereign governments – which harmonise their policies by means of Euclidean modelling methodology and global financial coordination – are capable of reallocating current flows and capital stocks so as to improve utilisation of available resources worldwide.

Chapter 7. International Exchange Model

The twelve major propositions are equally valid from the domestic and the world-wide point of view. Nevertheless, construction of a multi-country economic model requires additional specifications regarding the ways national currencies are related, sector flows are united and market goods are traded in a single exchange network. In this chapter, we will assemble a prototype model consisting of two elementary economies connected via the international system of trade and payments. In the process, we will focus on the global banking system and its interface with national economies. The dimension of a world-wide exchange network is determined by the numbers of interacting sectors and markets as well as countries and currencies. The simplest reproduction of international buying and selling consists of ten first-order non-homogeneous linear differential equations.

7.1. Economic and financial framework

Consumers, producers, bankers, etc., depend on each other in respect of services, products, securities and so on. Distributions of sector revenue among feasible sources and allocations of sector outlay among relevant destinations are assumed to remain constant within the time unit, although they may change from one year to the next. The elementary international economy under construction comprises two currency zones called Country One and Country Two (or the rest of the world) which operate according to the twelve fundamental premises. Households, businesses and financiers of both areas are free to buy and sell labour, output and liquidity anywhere in the world by means of bank money or perpetual bonds. The said economies are connected through their payment and saving systems, which contain the national central banks and other monetary financial institutions. An International Accounting Unit (*IAU*) is indispensable for comparisons of consumer, producer and bank income and expenditure with salaries, prices or values of traded goods. Institutional sectors do not suffer from money illusion and thus do not discriminate between nominal flows (or the corresponding present values) in accordance with the country of origin or currency of destination. In every zone, households, businesses and financiers demand and supply the same amount of certainty weighted capital in exchange for one unit of international liquidity per year. Under conditions justifying balanced portfolios (discussed in Section 7.8), institutional sectors wish to derive steady income shares from, and spend steady expenditure shares on, guaranteed securities. Takings (assets) and outgoings (liabilities) of both banking systems may be added, subtracted, compared and reconciled in *IAU*s. The *IAU* may coincide with one of the national currencies (e.g. Country One $\xi_1 = 1 IAU/\$$) because the two sovereign areas may be unequal from an economic, political or other point of view. Besides, it is understood that:

- the number of *IAUs* per national currency unit ξ_x is determined autonomously;
- buying and selling of consols between banks does not affect the money supply.

Exchange rates of national currencies as well as country shares of global liquidity are objects of discussion among monetary authorities worldwide. These consultations are aimed at promoting conditions favouring financial stability. Comprehensive notation has been introduced to facilitate analysis and synthesis as well as computations and simulations regarding the global banking system and its interaction with national economies. Capital letters signify variables measured in *IAUs*. Subscripts (taken from left to right) denote: first 'sovereign country' and then 'institutional sector'. For example, lower case k_{1f} refers to the amount of credit granted by Country One banks in the national currency. Per time unit, the universal monetary increment K_F (measured in *IAU*s) is *de facto* divided between the national currencies of the two issuing countries (or zones) as follows:

$$K_F = K_{1F} + K_{2F} = \xi_1 \cdot k_{1f} + \xi_2 \cdot k_{2f} = \varphi_1 \cdot K_F + \varphi_2 \cdot K_F \tag{7.1}$$

The portion of Country x = 1, 2, in worldwide liquidity expansion $\varphi_x \cdot K_x$ is equal to the exchange rate ξ_x (i.e. the *IAU*s per currency unit) times the national monetary increment k_{xf}. Evidently, ξ_i and φ_i are connected variables. From a mathematical point of view, the two banking sectors are linearly and otherwise dependent. Yearly, both monetary financial systems allocate their shares of world liquidity expansion K_{XF} among the global economy's households and businesses ($i = h, b$), in absolute and percentage terms as follows:

$$\begin{aligned} K_F &= \xi_1 \cdot (k_{1f1h} + k_{1f2h} + k_{1f1b} + k_{1f2b}) + \xi_2 \cdot (k_{2f1h} + k_{2f2h} + k_{2f1b} + k_{2f2b}) \\ K_F &= \varphi_1 \cdot K_F + \varphi_2 \cdot K_F = \xi_1 \cdot k_{1f} + \xi_2 \cdot k_{2f} = (\mu_{1h} + \mu_{1b} + \mu_{2h} + \mu_{2b}) \cdot K_F \end{aligned} \tag{7.2}$$

Greek letter μ_{xi} denotes the percentage of universal bank credit granted to country x sector i, as denoted by subscript xi. Identity (7.1) and its derivative (7.2) explain why most countries (except those having the *IAU* as their national money) must choose between:

- strong currency and small increases of their banking sector balance sheet or
- weak currency and large increases of their banking sector balance sheet.

In Euclidean models, receipts and payments are linear functions of the annual increment and sector allocation of bank credit. The same causal relationship holds true regarding takings and outgoings of sovereign countries and currency zones. Consequently, relative supplies of national currencies and their exchange rates are potential sources of argument and conflict, especially among economically important countries. During the 2007 – 2015 financial crises, international media occasionally reported that the United States of America recklessly allowed the Dollar to depreciate while the Peoples Republic of China purposely kept the Yuan from appreciating. The ECB was accused of fuelling inflation among commodity exporters and emerging markets because it provided cheap liquidity (via quantitative easing) to Eurozone banks. According to the twelfth major proposition, a form of financial coordination among important players is absolutely essential for smooth functioning of domestic and international networks. Without such uniting influence, beggar thy neighbour policies – including aggressive inflationary or deflationary spirals – would be everyday phenomena. Due to constitutional and legal provisions regarding safeguard of personal data, freedom of capital movement and protection of private property, national authorities often have to take the judicial route in order to obtain information about bank deposits and loans as well as interest receipts and payments. Operation of offshore banking centres and deficiency of international reporting standards have also contributed to opacity of world-wide monetary financial statistics. If the aim is to prevent economic hardship, social upheaval and political collapse, optimum utilisation of worldwide resources should be a main criterion regarding expansion and allocation of global bank credit. In the model network under construction, money issuing sectors are, supposedly, harmonized by an international financial organisation (IFO) that is very similar to the IMF. Each country's banking institutions annually:

- deal with households, business, governments, etc., in all countries by buying and selling perpetual bonds denominated in the national accounting unit;
- expand their monetary financial balance sheet (measured in *IAU*s) around the amount $\varphi_x \cdot K_F = K_{XF} = \xi_x \cdot k_{xf} = (\mu_{xh}+\mu_{xb})\cdot K_F$, agreed with the IFO.

IFO officials consult with mathematical economists of national authorities with the object of reaching agreement on universal liquidity expansion K_F and the latter's allocation among countries and sectors. National and sector shares of K_F, i.e. $\sum\xi_x\cdot k_{xf} = \sum\varphi_x\cdot K_F = \sum(\mu_{xh}+\mu_{xb})\cdot K_F = K_F$, may be decided by taking into account banking transactions, quantity gaps and quotation trends world-wide.

7.2. Mathematical analysis and synthesis of transactions

Worldwide, institutional sectors optimise their wealth according to their rational ability, survival instinct and political adeptness. Within the time unit, salaries, prices and values as well as increases of loans and allocations of deposits are exogenous. When trade and payments are conducted freely, domestic and global monetary flows as well as the respective present values are compared in *IAU*s. Through international exchange, consumers, producers and bankers distribute their incomes among the feasible sources and allocate their expenditures among the applicable destinations as outlined by Table (7.3).

(7.3) Income demand and expenditure supply by country, sector and market good

Market Goods: Supply & Demand	**COUNTRY ONE**				**COUNTRY TWO**				**BANKING INSTITUTIONS**	
	HOUSEHOLDS		**BUSINESSES**		***HOUSEHOLDS***		**BUSINESSES**			
	Y_{1H}	E_{1H}	Y_{1B}	E_{1B}	Y_{2H}	E_{2H}	Y_{2B}	E_{2B}	Y_F	E_F
	SERVICES									
Y_{1W}	Y_{1H1W}	-	-	-	-	-	-	-	-	-
E_{1W}	-	-	-	E_{1B1W}	-	E_{2H1W}	-	E_{2B1W}	-	E_{F1W}
Y_{2W}	-	-	-		Y_{2H2W}	-	-	-	-	-
E_{2W}	-	E_{1H2W}	-	E_{1B2W}	-	-	-	E_{2B2W}	-	E_{F2W}
	PRODUCTS									
Y_{1U}	-	-	Y_{1B1U}		-	-		-	-	-
E_{1U}	-	E_{1H1U}	-		-	E_{2H1U}	-	E_{2B1U}	-	E_{F1U}
Y_{2U}	-	-	-		-	-	Y_{2B2U}	-	-	-
E_{2U}	-	E_{1H2U}	-	E_{1B2W}	-	E_{2H2U}	-	-	-	E_{F2U}
	SECURITIES									
Y_{FS}	Y_{1HFS}	-	Y_{1BFS}		Y_{2HFS}	-	Y_{2BFS}	-	Y_{FS}	-
E_{FS}	-	E_{1HFS}	-	E_{1BFS}	-	E_{2HFS}	-	E_{2BFS}	-	E_{FS}

Capital letters indicate flows measured in *IAUs*. Subscripts, taken in pairs from left to right, denote 'country sector' and 'market good'. Table (7.3) is comparable to monetary model's Table (3.1) and fiscal model's Table (6.10). In its simplest form, Euclidean methodology assumes that each money-accepting sector is differentiated by supplying (i.e. demanding revenue from) a distinct good that is not bought by the sector itself and is not sold by other sectors. In contrast, national banking systems regulate the quantity, allocation and cost (remuneration) of credit (debit) by dealing in own currency denominated consols. In this context, they routinely buy and sell national currency denominated bonds from and to institutional sectors around the world. Liquidity growth, discount rate and exchange parity combinations outside the loci of balanced portfolios (see

Section 7.8) hinder monetary operations and upset financial stability by inviting speculators to carry out borrowing and lending arbitrages in different currencies, albeit temporarily. Speculative practices also take place within common currency areas, such as the Eurozone, whenever monetary operations are conducted via national securities which attract different discount rates due to changing certainty of underlying payments. This explains why during the 2007 - 2015 series of crises, managing Euro area problems seemed like building castles on shifting sand. The situation deteriorated due to delay in spreading country risk by means of mutually guaranteed Euro bonds. This type of debt may be an inevitable step toward improvement of Eurozone cohesion. Only when money-accepting sectors and countries are indifferent regarding the composition of their perpetual bond portfolios, can central banks conduct financial operations with predictable results. Payments by the universal banking system in respect of issued consols (deposits) are equal to receipts by households, businesses, etc., in respect of accepted consols (deposits) in all currencies:

$$E_{FS} = Y_{1HFS} + Y_{1BFS} + Y_{2HFS} + Y_{2BFS} = (Y_{1FFS}) + (Y_{2FFS}) \tag{7.4}$$

Identity (7.4) is fundamental for the mathematical specification of the income functions of accepting sectors worldwide. As a result of globalisation:

- Consumers derive income (in *IAU*s) from labour sold to other sectors and capital accumulated internationally. The following equations measure household revenue emanating from services *XW* rendered to other sectors and securities *FS* purchased from banks in all countries and currencies:

$$\begin{aligned} Y_{1H} &= (E_{2H1W} + E_{1B1W} + E_{2B1W} + E_{F1W}) + (Y_{1HFS}) \\ Y_{2H} &= (E_{1H2W} + E_{1B2W} + E_{2B2W} + E_{F2W}) + (Y_{2HFS}) \end{aligned} \tag{7.5}$$

By rearranging equation (7.4), we derive the proceeds of country $X = 1, 2,$ households H from global financial investments *FS* as shown below:

$$\begin{aligned} Y_{1HFS} &= E_{FS} - (Y_{2HFS} + Y_{1BFS} + Y_{2BFS}) \\ Y_{2HFS} &= E_{FS} - (Y_{1HFS} + Y_{1BFS} + Y_{2BFS}) \end{aligned} \tag{7.6}$$

Every country's consumers sell labour services and own guaranteed securities worldwide. As a result, their earnings (in *IAU*s) adjust as follows:

$$dY_{1H}/dt = (E_{2H1W} + E_{1B1W} + E_{2B1W} + E_{F1W}) + (Y_{1HFS}) - Y_{1H}$$
$$dY_{2H}/dt = (E_{1H2W} + E_{1B2W} + E_{2B2W} + E_{F2W}) + (Y_{2HFS}) - Y_{2H} \tag{7.7}$$

Households conserve their wealth by modifying their expenditure according to their income. The self-preservation instinct ensures that their payments do not exceed their endogenously determined receipts, minus the obligatory increase of their monetary assets exogenously determined by the universal banking system per time unit:

$$dE_{1H}/dt = Y_{1H} - \mathrm{K}_{1H} - E_{1H}$$
$$dE_{2H}/dt = Y_{2H} - \mathrm{K}_{2H} - E_{2H} \tag{7.8}$$

- Producers derive revenue (in *IAU*s) from output *XU* sold to other sectors and guaranteed securities *FS* accumulated internationally. The annual income of country *X* businesses (Y_{XB}) is equal to the sum of their earnings from product sales *XU* and liquid investments *FS* realised in all countries and currencies:

$$Y_{1B} = (E_{1H1U} + E_{2H1U} + E_{2B1U} + E_{F1U}) + (Y_{1BFS})$$
$$Y_{2B} = (E_{1H2U} + E_{2H2U} + E_{1B2U} + E_{F2U}) + (Y_{2BFS}) \tag{7.9}$$

By rearranging equation (7.4), we derive the proceeds of country $X = 1, 2$, producers *B* from guaranteed securities *FS* worldwide:

$$Y_{1BFS} = E_{FS} - (Y_{1HFS} + Y_{2HFS} + Y_{2BFS})$$
$$Y_{2BFS} = E_{FS} - (Y_{1HFS} + Y_{2HFS} + Y_{1BFS}) \tag{7.10}$$

The income of country *X* businesses varies according to output *XU* sold to other sectors and interest *FS* earned on liquid investments around the world:

$$dY_{1B}/dt = (E_{1H1U} + E_{2H1U} + E_{2B1U} + E_{F1U}) + (Y_{1BFS}) - Y_{1B}$$
$$dY_{2B}/dt = (E_{1H2U} + E_{2H2U} + E_{1B2U} + E_{F2U}) + (Y_{2BFS}) - Y_{2B} \tag{7.11}$$

Producers adjust their expenditure according to their endogenously determined income minus the obligatory increase of their monetary financial assets exogenously determined by the global banking system during a particular period:

$$dE_{1B}/dt = Y_{1B} - \mathrm{K}_{1B} - E_{1B}$$
$$dE_{2B}/dt = Y_{2B} - \mathrm{K}_{2B} - E_{2B} \tag{7.12}$$

- Bankers derive revenue (in *IAU*s) only from their holdings of loans / consols *FS* issued by households and businesses in both countries and currencies:

$$Y_{FS} = (E_{1HFS} + E_{2HFS} + E_{1BFS} + E_{2BFS}) \qquad (7.13)$$

Bank receipts vary in accordance with other sector payments for the capital they borrowed by means of perpetual bonds during the relevant period:

$$\mathrm{d}Y_F/\mathrm{dt} = Y_{FS} - Y_F = (E_{1HFS} + E_{2HFS} + E_{1BFS} + E_{2BFS}) - Y_F \qquad (7.14)$$

In addition to their outlay in respect of issued consols / deposits – described by equation (7.4) – monetary financial institutions purchase services and products in both countries and currencies, in accordance with their operational needs and other priorities:

$$E_F = E_{FS} + E_{F1W} + E_{F2W} + E_{F1U} + E_{F2U} \qquad (7.15)$$

The united banking system adjusts its outgoings (in *IAU*s) according to the sum of its endogenously generated takings described by equation (7.12) plus the annual increase of its liabilities exogenously determined by the IFO:

$$\mathrm{d}E_F/\mathrm{dt} = Y_F + \mathrm{K_F} - E_F \qquad (7.16)$$

Equations (7.7), (7.8), (7.11), (7.12), (7.14) and (7.16) comprise the simplest mathematical reproduction of a worldwide exchange network consisting of two currency areas:

$$\begin{aligned}
\mathrm{d}Y_{1H}/\mathrm{dt} &= (E_{2H1W} + E_{1B1W} + E_{2B1W} + E_{F1W}) + E_{FS} - (Y_{2HFS} + Y_{1BFS} + Y_{2BFS}) - Y_{1H} \\
\mathrm{d}E_{1H}/\mathrm{dt} &= Y_{1H} - E_{1H} - \mathrm{K_{1H}} \\
\mathrm{d}Y_{2H}/\mathrm{dt} &= (E_{1H2W} + E_{1B2W} + E_{2B2W} + E_{F2W}) + E_{FS} - (Y_{1HFS} + Y_{1BFS} + Y_{2BFS}) - Y_{2H} \\
\mathrm{d}E_{2H}/\mathrm{dt} &= Y_{2H} - E_{2H} - \mathrm{K_{2H}} \\
\mathrm{d}Y_{1B}/\mathrm{dt} &= (E_{1H1U} + E_{2H1U} + E_{2B1U} + E_{F1U}) + E_{FS} - (Y_{1HFS} + Y_{2HFS} + Y_{2BFS}) - Y_{1B} \\
\mathrm{d}E_{1B}/\mathrm{dt} &= Y_{1B} - E_{1B} - \mathrm{K_{1B}} \\
\mathrm{d}Y_{2B}/\mathrm{dt} &= (E_{1H2U} + E_{2H2U} + E_{1B2U} + E_{F2U}) + E_{FS} - (Y_{1HFS} + Y_{2HFS} + Y_{1BFS}) - Y_{2B} \\
\mathrm{d}E_{2B}/\mathrm{dt} &= Y_{2B} - E_{2B} - \mathrm{K_{2B}} \\
\mathrm{d}Y_F/\mathrm{dt} &= (E_{1HFS} + E_{1BFS} + E_{2HFS} + E_{2BFS}) - Y_F \\
\mathrm{d}E_F/\mathrm{dt} &= Y_F - E_F + \mathrm{K_F}
\end{aligned} \qquad (7.17)$$

Model (7.17), which assumes the general form **dX/dt = A·X – K**, is defined in the (R^{10}, t) vector space. It encompasses ten first-order non-homogeneous linear differential equations, which replicate interaction among the endogenous variables (i.e. household, business and financier income and expenditure measured in *IAUs*) given the exogenous parameters (i.e. market behaviour coefficients, credit policy tools, foreign exchange rates, etc.) prevailing in both sovereign countries and currency zones. Country / sector nominal flows (Y_{XI}, E_{XI}), which are normally measured in *IAU*s, may be expressed in any national currency (y_{xi}, e_{xi}) according to the following example:

$$Y_{1H1W} = \alpha_{1h1w} \cdot Y_{1H} = \alpha_{1h1w} \cdot \xi_1 \cdot y_{1h}$$
$$E_{2B1S} = \beta_{2b1s} \cdot E_{2B} = \beta_{2b1s} \cdot \xi_2 \cdot e_{2b} \tag{7.18}$$

Table (7.19) depicts the distributions of income demand and expenditure supply among the institutional sectors and traded goods of an elementary global economy.

(7.19) Percentage allocation of worldwide income demand and expenditure supply

Market Goods: Supply & Demand	**COUNTRY ONE**				**COUNTRY TWO**				**BANKING INSTITUTIONS**	
	HOUSEHOLDS		**BUSINESSES**		**HOUSEHOLDS**		**BUSINESSES**			
	Y_{1H}	E_{1H}	Y_{1B}	E_{1B}	Y_{2H}	E_{2H}	Y_{2B}	E_{2B}	Y_F	E_F
Labour										
Y_{1W}	α_{1h1w}	-	-	-	-	-	-	-	-	-
E_{1W}	-	-	-	β_{1b1w}	-	β_{2h1w}	-	β_{2b1w}	-	β_{f1w}
Y_{2W}	-	-	-		α_{2h2w}	-	-	-	-	-
E_{2W}	-	β_{1h2w}	-	β_{1b2w}	-	-	-	β_{2b2w}	-	β_{f2w}
Production										
Y_{1U}	-	-	α_{1b1u}		-	-		-	-	-
E_{1U}	-	β_{1h1u}	-		-	β_{2h1u}	-	β_{2b1u}	-	β_{f1u}
Y_{2U}	-	-	-		-	-	α_{2b2u}	-	-	-
E_{2U}	-	β_{1h2u}	-	β_{1b2w}	-	β_{2h2u}	-	-	-	β_{f2u}
Liquidity										
Y_{FS}	α_{1hfs}	-	α_{1bfs}		α_{2hfs}	-	α_{2bfs}	-	α_{fs}	-
E_{FS}	-	β_{1hfs}	-	β_{1bfs}	-	β_{2hfs}	-	β_{2bfs}	-	β_{fs}

Autonomy of salaries, prices and values during the time unit is the second major hypothesis of Euclidean methodology. Model (7.17) enables us to compare the equilibrium receipts and payments of institutional sectors with the average quotations of market goods (all measured in *IAUs*) in order to design policies which will ameliorate labour, output and liquidity gaps over the next period. In every exchange network there exist combinations of household, business, financier, etc., behaviour coefficients which lead to (i) positive and negative Routh-Hurwitz determinants rh_n, (ii) positive and negative country / sector flows ($^*Y_{XH}$, $^*E_{XH}$; $^*Y_{XB}$, $^*E_{XB}$; ... , $^*Y_F$, $^*E_F$) and thus to (iii) unruly situations from the standpoint of domestic and international economic policy.

7.3. Example of global trade and payments

According to their inherent abilities, instinctual needs and wider goals, institutional sectors of Country One and Country Two (or the rest of the world) derive from and spend on services, products and securities constant fractions of their incomes and expenditures per year. Consumers, producers and bankers, while engaging in trade and payments without money illusion, accept and issue perpetual bonds denominated in both currencies. A statistical survey of monetary flows has revealed that institutional sectors buy and sell real goods and financial paper worldwide in line with the proportions shown by Table (7.20).

(7.20) International apportionment of income demand and expenditure supply

Market Goods: Supply & Demand	**COUNTRY ONE**				**COUNTRY TWO**				***BANKING INSTITUTIONS***	
	HOUSEHOLDS		**BUSINESSES**		**HOUSEHOLDS**		**BUSINESSES**			
	Y_{1H}	E_{1H}	Y_{1B}	E_{1B}	Y_{2H}	E_{2H}	Y_{2B}	E_{2B}	Y_F	E_F
Labour										
Y_{1W}	$^{7}/_{10}$	-	-	-	-	-	-	-	-	-
E_{1W}	-	-	-	$^{4}/_{16}$	-	$^{5}/_{15}$	-	$^{2}/_{16}$	-	$^{1}/_{100}$
Y_{2W}	-	-	-	-	$^{5}/_{8}$	-		-	-	-
E_{2W}	-	$^{1}/_{12}$	-	$^{3}/_{16}$	-	-	-	$^{6}/_{16}$	-	$^{2}/_{100}$
Production										
Y_{1U}	-	-	$^{4}/_{7}$	-	-	-		-	-	-
E_{1U}	-	$^{4}/_{12}$	-	-	-	$^{3}/_{15}$	-	$^{5}/_{16}$	-	$^{3}/_{100}$
Y_{2U}	-	-	-	-	-	-	$^{5}/_{9}$	-	-	-
E_{2U}	-	$^{3}/_{12}$	-	$^{5}/_{16}$	-	$^{4}/_{15}$	-	-	-	$^{4}/_{100}$
Liquidity										
Y_{FS}	$^{3}/_{10}$	-	$^{3}/_{7}$	-	$^{3}/_{8}$	-	$^{4}/_{9}$	-	1	-
E_{FS}	-	$^{4}/_{12}$	-	$^{4}/_{16}$	-	$^{3}/_{15}$	-	$^{3}/_{16}$	-	$^{90}/_{100}$
Sector Total	$^{10}/_{10}$	$^{12}/_{12}$	$^{7}/_{7}$	$^{16}/_{16}$	$^{8}/_{8}$	$^{15}/_{15}$	$^{9}/_{9}$	$^{16}/_{16}$	$^{1}/_{1}$	$^{100}/_{100}$

By replacing the symbols of system (7.17) with the fractions shown in Table (7.20), we derive linear differential equation model (7.21) that depicts a global economy called Delta. Delta's exchange network (7.21) has the general form $\mathbf{dX/dt = \Delta \cdot X - K_F}$. Hence, we are in position to study Delta's equilibrium flows ($^*Y_{XH}$, $^*E_{XH}$, $^*Y_{XB}$, $^*E_{XB}$, $^*Y_F$, $^*E_F$), stability conditions ($rh_1 > 0$, $rh_2 > 0$, ... , $rh_{10} > 0$) and controllability properties ($^*Y_{XH} > 0$, $^*E_{XH} > 0$, $^*Y_{XB} > 0$, $^*E_{XB} > 0$, $^*Y_F > 0$, $^*E_F > 0$) with respect to every behaviour coefficient, policy parameter, exchange rate, etc. Following an economic jolt, country / sector receipts and payments (measured in *IAU*s) fluctuate in accordance with the first-order non-homogeneous linear differential equation system shown below:

$$dY_{1H}/dt = -Y_{1H} - {}^3/_8 \cdot Y_{2H} + {}^1/_3 \cdot E_{2H} - {}^3/_7 \cdot Y_{1B} + {}^1/_4 \cdot E_{1B} - {}^4/_9 \cdot Y_{2B} + {}^1/_8 \cdot E_{2B} + {}^{91}/_{100} \cdot E_F$$

$$dE_{1H}/dt = Y_{1H} - E_{1H} - K_{1H}$$

$$dY_{2H}/dt = -{}^3/_{10} \cdot Y_{1H} + {}^1/_{12} \cdot E_{1H} - Y_{2H} - {}^3/_7 \cdot Y_{1B} + {}^3/_{16} \cdot E_{1B} - {}^4/_9 \cdot Y_{2B} + {}^3/_8 \cdot E_{2B} + {}^{92}/_{100} \cdot E_F$$

$$dE_{2H}/dt = Y_{2H} - E_{2H} - K_{2H}$$

$$dY_{1B}/dt = -{}^3/_{10} \cdot Y_{1H} + {}^1/_3 \cdot E_{1H} - {}^3/_8 \cdot Y_{2H} + {}^1/_5 \cdot E_{2H} - Y_{1B} - {}^4/_9 \cdot Y_{2B} + {}^5/_{16} \cdot E_{2B} + {}^{93}/_{100} \cdot E_F \qquad (7.21)$$

$$dE_{1B}/dt = Y_{1B} - E_{1B} - K_{1B}$$

$$dY_{2B}/dt = -{}^3/_{10} \cdot Y_{1H} + {}^1/_4 \cdot E_{1H} - {}^3/_8 \cdot Y_{2H} + {}^4/_{15} \cdot E_{2H} - {}^3/_7 \cdot Y_{1B} + {}^5/_{16} \cdot E_{1B} - Y_{2B} + {}^{94}/_{100} \cdot E_F$$

$$dE_{2B}/dt = Y_{2B} - E_{2B} - K_{2B}$$

$$dY_F/dt = {}^1/_3 \cdot E_{1H} + {}^1/_5 \cdot E_{2H} + {}^1/_4 \cdot E_{1B} + {}^3/_{16} \cdot E_{2B} - Y_F$$

$$dE_F/dt = Y_F - E_F + K_F$$

Dynamic model (7.21) assumes general form $\mathbf{dX/dt = \Delta \cdot X - K_F}$. Its steady-state solution $^*\mathbf{X} = \mathbf{\Delta^{-1}} \cdot \mathbf{K_F}$ provides revenues and outlays – of Country One ($^*Y_{1H}$, $^*E_{1H}$, $^*Y_{1B}$, $^*E_{1B}$) and Country Two ($^*Y_{2H}$, $^*E_{2H}$, $^*Y_{2B}$, $^*E_{2B}$) households and businesses as well as of the joint banking sector ($^*Y_F$, $^*E_F$) – as linear functions of the monetary increment's magnitude K_F and allocation (K_{1H}, K_{2H}, K_{1B}, K_{2B}) among accepting sectors of both areas.

7.4. Banking system balance sheet

Annual expansion K_F of the international banking balance sheet may be expressed (in *IAU*s or local currencies) in terms of national monetary increments k_{xf} and exchange rates ξ_x as well as in terms of country φ_X and sector κ_{xfxi} shares. A particular distribution (μ_{xh}, μ_{xb}, etc.) of universal liquidity growth K_F may be achieved through innumerable combinations of national monetary increments k_{xf} and exchange rates ξ_x as well as country φ_X and sector κ_{xfxi} shares. Based on Delta's case, Table (7.22) proves that world-wide loan-deposit expansion K_F may be allocated in a vast number of equivalent ways:

(7.22) Equivalent ways of allocating annual loans / deposits

$$\mathbf{K_F} = \left|\begin{array}{c} 0 \\ \xi_1 \cdot k_{1f1h} + \xi_2 \cdot k_{2f1h} \\ 0 \\ \xi_1 \cdot k_{1f2h} + \xi_2 \cdot k_{2f2h} \\ 0 \\ \xi_1 \cdot k_{1f1b} + \xi_2 \cdot k_{2f1b} \\ 0 \\ \xi_1 \cdot k_{1f2b} + \xi_2 \cdot k_{2f2b} \\ 0 \\ -K_F \end{array}\right| = \left|\begin{array}{c} 0 \\ (\varphi_1 \cdot \kappa_{1f1h} + \varphi_2 \cdot \kappa_{2f1h}) \cdot K_F \\ 0 \\ (\varphi_1 \cdot \kappa_{1f2h} + \varphi_2 \cdot \kappa_{2f2h}) \cdot K_F \\ 0 \\ (\varphi_1 \cdot \kappa_{1f1b} + \varphi_2 \cdot \kappa_{2f1b}) \cdot K_F \\ 0 \\ (\varphi_1 \cdot \kappa_{1f2b} + \varphi_2 \cdot \kappa_{2f2b}) \cdot K_F \\ 0 \\ -K_F \end{array}\right| = \left|\begin{array}{c} 0 \\ K_{1H} \\ 0 \\ K_{2H} \\ 0 \\ K_{1B} \\ 0 \\ K_{2B} \\ 0 \\ -K_F \end{array}\right| = \left|\begin{array}{c} 0 \\ \mu_{1h} \\ 0 \\ \mu_{2h} \\ 0 \\ \mu_{1b} \\ 0 \\ \mu_{2b} \\ 0 \\ -1 \end{array}\right| \cdot K_F$$

When there is stability, transparency and convertibility, receipts and payments in all currencies are welcome. Agents and, therefore, sectors everywhere demand and supply the same amount of capital in exchange for one unit of international liquidity per year. However, the global economic system remains dirigible only as long as:

- its steady-state consists of unique, stable and positive flows;
- banking institutions coordinate credit allocation world-wide.

Within this framework, national monetary authorities have to restrict the annual expansion of their liquid obligations (measured in *IAUs*) near the amount ($\xi_x \cdot k_{xf} = K_{xF} = \varphi_x \cdot K_F$) agreed with the IFO. Each country may modify distinct factors (ξ_x; k_{xf1h}, k_{xf2h}, k_{xf1b}, k_{xf2b}) under control, provided their product $\xi_x \cdot (k_{xf1h} + k_{xf1b} + k_{xf2h} + k_{xf2b})$ approximates the acceptable limit. At the same time, national banking systems are not free to allocate at will the annual increase of their deposits as loans to Country One and Country Two consumers and producers. Governments, regulators and other stakeholders expect them to conduct their affairs so as to minimise shortages or surpluses and thus inflation or deflation world-wide. Ideally, annual increase and apportionment of universal liquidity among households and businesses as well as countries or zones should be supervised by a global financial authority. Euclidean approach considers self-evident that national banking sectors require outside prodding in order to improve international economic equilibrium. Monetary institutions around the world should be motivated to issue deposits and grant loans in the most advantageous way, i.e. by taking into account potential for employment and production in the various countries. A global coordinating body, such as the IFO, should ensure that national networks receive credit in line with their free resources.

In today's interdependent world, financial and economic issues cannot be solved by any one country in isolation. For example, gaps between demanded and available liquid securities are bridgeable by multilateral annual discount, monetary growth and exchange rate adjustments. During the 2007 - 2015 Eurozone crises, full employment and other goals were overlooked in favour of financial stability in the European Union. Given that in the EU the clearing combination of the aforementioned tools is mostly decided from a system perspective, debt-ridden Portugal, Italy, Spain and Greece have had leeway only for fiscal and structural measures. This limitation may have worsened welfare indicators and amplified social tensions in Southern Europe.

7.5. Dynamic properties of economy Delta

Comparisons of alternative positions $^*\mathbf{X}$ are meaningful provided dynamic system $\mathbf{dX/dt = \Delta \cdot X - K_F}$ that generates them is globally stable. Exchange network (7.21) is universally stable if, and only if, the ten Routh-Hurwitz determinants rh_n, $n = 1, 2, \ldots, 10$, consisting of the coefficients of characteristic polynomial $|\mathbf{\Delta - \lambda \cdot I}|$, are greater than zero. Delta's characteristic polynomial $|\mathbf{\Delta - \lambda \cdot I}|$ is depicted by (7.23).

(7.23) Delta's characteristic polynomial

$$\begin{vmatrix} -1-\lambda & 0 & -^3/_8 & ^1/_3 & -^3/_7 & ^1/_4 & -^4/_9 & ^1/_8 & 0 & ^{91}/_{100} \\ 1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -^3/_{10} & ^1/_{12} & -1-\lambda & 0 & -^3/_7 & ^3/_{16} & -^4/_9 & ^3/_8 & 0 & ^{92}/_{100} \\ 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ -^3/_{10} & ^1/_3 & -^3/_8 & ^1/_5 & -1-\lambda & 0 & -^4/_9 & ^5/_{16} & 0 & ^{93}/_{100} \\ 0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 \\ -^3/_{10} & ^1/_4 & -^3/_8 & ^4/_{15} & -^3/_7 & ^5/_{16} & -1-\lambda & 0 & 0 & ^{94}/_{100} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 \\ 0 & ^1/_3 & 0 & ^1/_5 & 0 & ^1/_4 & 0 & ^3/_{16} & -1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda \end{vmatrix} = |\mathbf{\Delta - \lambda \cdot I}|$$

$$|\mathbf{\Delta - \lambda \cdot I}| = \delta_0 \cdot \lambda^{10} + \delta_1 \cdot \lambda^9 + \delta_2 \cdot \lambda^8 + \delta_3 \cdot \lambda^7 + \delta_4 \cdot \lambda^6 + \delta_5 \cdot \lambda^5 + \delta_6 \cdot \lambda^4 + \delta_7 \cdot \lambda^3 + \delta_8 \cdot \lambda^2 + \delta_9 \cdot \lambda + \delta_{10} \qquad (7.23)$$

Computing (with *Mathematica*) the coefficients of the above polynomial, we obtain:

$$\delta_0 = 1 \ ; \ \delta_1 = 10 \ ; \ \delta_2 = {}^{74101}/_{1680} \ ; \ \delta_3 = {}^{13850063}/_{120960} \ ; \ \delta_4 = {}^{41958451}/_{216000}$$

$$\delta_5 = {}^{7163182173761}/_{725760000} \ ; \ \delta_6 = {}^{64970624767}/_{362880000} \ ; \ \delta_7 = {}^{278492568223}/_{2903040000} \qquad (7.24)$$

$$\delta_8 = {}^{188181161339}/_{5806080000} \ ; \ \delta_9 = {}^{134113541}/_{22680000} \ ; \ \delta_{10} = {}^{99478399}/_{276480000} = |\Delta|$$

(7.25) Minors of Delta’s characteristic polynomial

$$
\begin{vmatrix}
\delta_1 & \delta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_3 & \delta_2 & \delta_1 & \delta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_5 & \delta_4 & \delta_3 & \delta_2 & \delta_1 & \delta_0 & 0 & 0 & 0 & 0 \\
\delta_7 & \delta_6 & \delta_5 & \delta_4 & \delta_3 & \delta_2 & \delta_1 & \delta_0 & 0 & 0 \\
\delta_9 & \delta_8 & \delta_7 & \delta_6 & \delta_5 & \delta_4 & \delta_3 & \delta_2 & \delta_1 & \delta_0 \\
0 & \delta_{10} & \delta_9 & \delta_8 & \delta_7 & \delta_6 & \delta_5 & \delta_4 & \delta_3 & \delta_2 \\
0 & 0 & 0 & \delta_{10} & \delta_9 & \delta_8 & \delta_7 & \delta_6 & \delta_5 & \delta_4 \\
0 & 0 & 0 & 0 & 0 & \delta_{10} & \delta_9 & \delta_8 & \delta_7 & \delta_6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_{10} & \delta_9 & \delta_8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_{10}
\end{vmatrix}
$$

(7.26) Static system associated with Delta’s dynamic model

$$
\begin{vmatrix}
-1 & 0 & -\frac{3}{8} & \frac{1}{3} & -\frac{3}{7} & \frac{1}{4} & -\frac{4}{9} & \frac{1}{8} & 0 & \frac{91}{100} \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{3}{10} & \frac{1}{12} & -1 & 0 & -\frac{3}{7} & \frac{3}{16} & -\frac{4}{9} & \frac{3}{8} & 0 & \frac{92}{100} \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{3}{10} & \frac{1}{3} & -\frac{3}{8} & \frac{1}{5} & -1 & 0 & -\frac{4}{9} & \frac{5}{16} & 0 & \frac{93}{100} \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
-\frac{3}{10} & \frac{1}{4} & -\frac{3}{8} & \frac{4}{15} & -\frac{3}{7} & \frac{5}{16} & -1 & 0 & 0 & \frac{94}{100} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & \frac{1}{3} & 0 & \frac{1}{5} & 0 & \frac{1}{4} & 0 & \frac{3}{16} & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{vmatrix}
\cdot
\begin{vmatrix}
{}^{*}Y_{1H} \\ {}^{*}E_{1H} \\ {}^{*}Y_{2H} \\ {}^{*}E_{2H} \\ {}^{*}Y_{1B} \\ {}^{*}E_{1B} \\ {}^{*}Y_{2B} \\ {}^{*}E_{2B} \\ {}^{*}Y_{1F} \\ {}^{*}E_{1F}
\end{vmatrix}
= \mathbf{\Delta} \cdot \mathbf{X} =
$$

$$
\mathbf{\Delta} \cdot \mathbf{X} =
\begin{vmatrix}
0 \\ \xi_1 \cdot k_{1f1h} + \xi_2 \cdot k_{2f1h} \\ 0 \\ \xi_1 \cdot k_{1f2h} + \xi_2 \cdot k_{2f2h} \\ 0 \\ \xi_1 \cdot k_{1f1b} + \xi_2 \cdot k_{2f1b} \\ 0 \\ \xi_1 \cdot k_{1f2b} + \xi_2 \cdot k_{2f2b} \\ 0 \\ -K_F
\end{vmatrix}
=
\begin{vmatrix}
0 \\ (\varphi_1 \cdot \kappa_{1f1h} + \varphi_2 \cdot \kappa_{2f1h}) \cdot K_F \\ 0 \\ (\varphi_1 \cdot \kappa_{1f2h} + \varphi_2 \cdot \kappa_{2f2h}) \cdot K_F \\ 0 \\ (\varphi_1 \cdot \kappa_{1f1b} + \varphi_2 \cdot \kappa_{2f1b}) \cdot K_F \\ 0 \\ (\varphi_1 \cdot \kappa_{1f2b} + \varphi_2 \cdot \kappa_{2f2b}) \cdot K_F \\ 0 \\ -K_F
\end{vmatrix}
=
\begin{vmatrix}
0 \\ K_{1H} \\ 0 \\ K_{2H} \\ 0 \\ K_{1B} \\ 0 \\ K_{2B} \\ 0 \\ -K_F
\end{vmatrix}
=
\begin{vmatrix}
0 \\ \mu_{1h} \\ 0 \\ \mu_{2h} \\ 0 \\ \mu_{1b} \\ 0 \\ \mu_{2b} \\ 0 \\ -1
\end{vmatrix}
\cdot K_F = \mathbf{K_F}
\qquad (7.26)
$$

The Routh-Hurwitz conditions, which apply to dynamic exchange network (7.21), are obtained by placing the values of coefficients δ_n in the ten minor determinants of Table (7.25). With mathematical software it can be proven that minor determinants rh_n, $n = 1, \dots, 10$ are greater than zero. It is noted that every globally stable linear differential equation system is associated with a unique steady state $^*\mathbf{X}$ because its structural matrix $\mathbf{\Delta}$ yields a positive determinant $|\mathbf{\Delta}|$. Consequently, the international economy duplicated by mathematical model (7.21), possesses a universally stable and thus unique equilibrium $^*\mathbf{X}$. Global stability implies that Delta returns to steady state $^*\mathbf{X}$ after each modification of policy parameters $\mathbf{\Pi}(\xi_1, \xi_2; k_{1f}, k_{2f}; k_{1f1h}, k_{1f2h}, k_{1f1b}, k_{1f2b}; k_{2f1h}, k_{2f2h}, k_{2f1b}, k_{2f2b}) = \mathbf{\Pi}(K_F, \mu_{1H}, \mu_{2H}, \mu_{1B}, \mu_{2b})$ and / or shakeup of initial flows $^0\mathbf{X}(^0Y_{1H}, {}^0E_{1H}; {}^0Y_{2H}, {}^0E_{2H}; {}^0Y_{1B}, {}^0E_{1B}; {}^0Y_{2B}, {}^0E_{2B}; {}^0Y_F, {}^0E_F)$. However, fulfilment of stability and uniqueness does not imply that consumer, producer and bank income and expenditure $(^*Y_{1H}, {}^*E_{1H}; {}^*Y_{2H}, {}^*E_{2H}; {}^*Y_{1B}, {}^*E_{1B}; {}^*Y_{2B}, {}^*E_{2B}; {}^*Y_F, {}^*E_F)$ stay positive and, hence, uniformly dirigible, irrespective of credit allocation among accepting sectors world-wide.

7.6. Equilibrium and other properties of Delta

Linear differential equation model (7.21) is associated with simultaneous linear equation system $\mathbf{\Delta} \cdot \mathbf{X} = \mathbf{K_F}$ depicted by (7.26). Time invariant model $\mathbf{\Delta} \cdot \mathbf{X} = \mathbf{K_F}$ yields the equilibrium receipts and payments $^*\mathbf{X}$ of Delta's households, businesses and financiers, because determinant $|\mathbf{\Delta}| \neq 0$. Solution $^*\mathbf{X}$ **(**consisting of the steady-state flows of consumers, producers and bankers around the world**)** is the linear transformation of universal credit policy vector $\mathbf{K_F}$ **(**representing expansion and distribution of the international banking balance sheet**)** by structural matrix $\mathbf{\Delta}$ **(**portraying the interaction of sector receipts and payments with respect to traded goods**)**. Thanks to our premises (e.g. autonomous nature of quotations, absence of money illusion, convertibility of flows into capital, etc.) national monetary financial sectors are totally united to the extent that their combined balance sheet is an integral whole measured in *IAU*s. Delta's latest survey has revealed that the aggregate banking sector purchases perpetual bonds from households and businesses world-wide in the following proportions:

$\mu_{1h} = {}^6/_{12} = 50\%$; $\mu_{2h} = {}^3/_{12} = 25\%$; $\mu_{1b} = {}^1/_{12} \approx 8.33\%$; $\mu_{2b} = {}^2/_{12} \approx 16.67\%$

Given behaviour coefficients and credit shares of Countries One and Two, the revenues and outlays of Delta's sectors are computed by solving $\mathbf{\Delta} \cdot \mathbf{X} = \mathbf{K_F}$ using Cramer's rule. Table (7.27) presents the aforementioned flows as functions of annual world credit K_F.

(7.27) Sector flows of economy Delta when $\mu_{1h} = {}^{6}/_{12}$, $\mu_{1b} = {}^{1}/_{12}$, $\mu_{2b} = {}^{3}/_{12}$, $\mu_{2b} = {}^{2}/_{12}$

Economy Delta	**Income**	**Expenditure**
Country One Households	${}^{*}Y_{1H} = 0.797451 \cdot K_F$	${}^{*}E_{1H} = 0.297451 \cdot K_F$
Country Two Households	${}^{*}Y_{2H} = 0.885107 \cdot K_F$	${}^{*}E_{2H} = 0.635107 \cdot K_F$
Country One Businesses	${}^{*}Y_{1B} = 0.963365 \cdot K_F$	${}^{*}E_{1B} = 0.880031 \cdot K_F$
Country Two Businesses	${}^{*}Y_{2B} = 1.04977 \cdot K_F$	${}^{*}E_{2B} = 0.883102 \cdot K_F$
Worldwide Financiers	${}^{*}Y_{F} = 0.611761 \cdot K_F$	${}^{*}E_{F} = 1.61176 \cdot K_F$

By adding consumer, producer and bank income and expenditure vertically, as shown in Table (7.27), we obtain international revenue and outlay as functions of the universal liquidity increment K_F measured in *IAU*s:

$$\begin{aligned} {}^{*}Y &= ({}^{*}Y_{1H} + {}^{*}Y_{2H} + {}^{*}Y_{1B} + {}^{*}Y_{2B} + {}^{*}Y_{F}) = \\ &= (0.797451 + 0.885107 + 0.963365 + 1.04977 + 0.611761) \cdot K_F = c \cdot K_F = 4.30745 \cdot K_F \\ {}^{*}E &= ({}^{*}E_{1H} + {}^{*}E_{2H} + {}^{*}E_{1B} + {}^{*}E_{2B} + {}^{*}E_{F}) = \\ &= (0.297451 + 0.635107 + 0.880031 + 0.883102 + 1.61176) \cdot K_F = c \cdot KF = 4.30745 \cdot K_F \end{aligned} \tag{7.28}$$

It appears, therefore, that c = 4.30745 is the exchange network's Euclidean multiplier or scale factor. Equations (7.28) prove that receipts and payments are distributed among the households and businesses of Countries One and Two as well as the financiers of economy Delta in the following proportions:

$$\begin{aligned} (y_{1h} + y_{2h} + y_{1b} + y_{2b}) + (y_{fs}) &= (18.5\% + 20.5\% + 22.4\% + 24.4\%) + (14.2\%) = 100\% \\ (e_{1h} + e_{2h} + e_{1b} + e_{2b}) + (e_{fs}) &= (6.9\% + 14.7\% + 20.4\% + 20.5\%) + (37.5\%) = 100\% \end{aligned} \tag{7.29}$$

From the rows of Table (7.20), which displays the expenditure coefficients of consumers, entrepreneurs and bankers, we derive aggregate disbursements on services (${}^{*}R_{1W}$, ${}^{*}R_{2W}$), products (${}^{*}R_{1U}$ and ${}^{*}R_{2U}$) and securities (${}^{*}R_{FS}$) traded world-wide:

$$\begin{aligned} {}^{*}R_{1W} &= {}^{1}/_{3} \cdot {}^{*}E_{2H} + {}^{1}/_{4} \cdot {}^{*}E_{1B} + {}^{1}/_{8} \cdot {}^{*}E_{2B} + {}^{1}/_{100} \cdot {}^{*}E_{F} \\ {}^{*}R_{2W} &= {}^{1}/_{12} \cdot {}^{*}E_{1H} + {}^{3}/_{16} \cdot {}^{*}E_{1B} + {}^{3}/_{8} \cdot {}^{*}E_{2B} + {}^{1}/_{50} \cdot {}^{*}E_{F} \\ {}^{*}R_{1U} &= {}^{1}/_{3} \cdot {}^{*}E_{1H} + {}^{1}/_{5} \cdot {}^{*}E_{2H} + {}^{5}/_{16} \cdot {}^{*}E_{2B} + {}^{3}/_{100} \cdot {}^{*}E_{F} \\ {}^{*}R_{2U} &= {}^{1}/_{4} \cdot {}^{*}E_{1H} + {}^{4}/_{15} \cdot {}^{*}E_{2H} + {}^{5}/_{16} \cdot {}^{*}E_{1B} + {}^{1}/_{25} \cdot {}^{*}E_{F} \\ {}^{*}R_{FS} &= {}^{1}/_{3} \cdot {}^{*}E_{1H} + {}^{1}/_{5} \cdot {}^{*}E_{2H} + {}^{1}/_{4} \cdot {}^{*}E_{1B} + {}^{3}/_{16} \cdot {}^{*}E_{2B} + {}^{9}/_{10} \cdot {}^{*}E_{F} \end{aligned} \tag{7.30}$$

In Delta's case, where $\mu_{1h} = {}^{6}/_{12}$, $\mu_{1b} = {}^{1}/_{12}$, $\mu_{2h} = {}^{3}/_{12}$, $\mu_{2b} = {}^{2}/_{12}$, annual sales ${}^{*}R = ({}^{*}R_{1W} + {}^{*}R_{2W} + {}^{*}R_{1U} + {}^{*}R_{2U} + {}^{*}R_{FS})$ are distributed among the five markets as follows:

$$(\rho_{1w} + \rho_{2w} + \rho_{1u} + \rho_{2u}) + (\rho_{fs}) = (13.0\% + 12.8\% + 12.8\% + 13.5\%) + (47.9\%) = 100\% \qquad (7.31)$$

With *Mathematica*, it can be proven that in the event universal liquidity increment K_F was allocated $\mu_{1h} = \mu_{2h} = \mu_{1b} = \mu_{2b} = ¼$ – instead of $\mu_{1h} = {}^{6}/_{12}$, $\mu_{1b} = {}^{1}/_{12}$, $\mu_{2h} = {}^{3}/_{12}$, $\mu_{2b} = {}^{2}/_{12}$ – the division of Delta's monetary flows among countries, sectors and traded goods would be:

$$(y_{1h} + y_{2h} + y_{1b} + y_{2b}) + (y_{fs}) = (15.7\% + 17.4\% + 22.9\% + 31.7\%) + (12.3\%) = 100\%$$

$$(e_{1h} + e_{2h} + e_{1b} + e_{2b}) + (e_{fs}) = (11.0\% + 10.8\% + 19.1\% + 8.9\%) + (50.2\%) = 100\% \qquad (7.32)$$

$$(\rho_{1w} + \rho_{2w} + \rho_{1u} + \rho_{2u}) + (\rho_{fs}) = (9.9\% + 8.9\% + 10.1\% + 13.6\%) + (57.5\%) = 100\%$$

– instead of (7.29) and (7.31). The above analysis and synthesis have affirmed that bank lending policies apportion receipts and payments (measured in *IAU*s) among the sovereign countries $X = 1, 2$, institutional sectors ${}^{*}\mathbf{X}({}^{*}Y_{XH}, {}^{*}E_{XH}, {}^{*}Y_{XB}, {}^{*}E_{XB}, {}^{*}Y_{FS}, {}^{*}E_{FS})$ $I = H, B, F$, and traded goods ${}^{*}\mathbf{R}({}^{*}R_{XW}, {}^{*}R_{XU}, {}^{*}R_{FS})$ $J = W, U, S$, which comprise the international economy.

7.7. Quantity or quotation gaps of market goods

Table (7.33) records the average salaries, prices and values as well as the available quantities of services, products and securities brought to light by Delta's statistical survey.

(7.33) Average quotations and available quantities of market goods

$P_{1W} =$ 1 070.00 *IAUs*	${}^{\#}q_{1w} = 1 \cdot 10^{6}$ years of Country One labour
$P_{2W} =$ 980.00 *IAUs*	${}^{\#}q_{2w} = 1 \cdot 10^{6}$ years of Country Two labour
$P_{1U} =$ 0.98 *IAUs*	${}^{\#}q_{1u} = 1 \cdot 10^{9}$ pieces of Country One output
$P_{2U} =$ 1.09 *IAUs*	${}^{\#}q_{2u} = 1 \cdot 10^{9}$ pieces of Country Two output
$P_{FS} =$ 40.00 *IAUs*	${}^{\#}q_{fs} = 1 \cdot 10^{8}$ units of international consols.

The said survey has also revealed the annual credit increment to be $K_F = 2x10^{9}$ *IAU*s.

(7.34) Gross turnover of market goods

Country One services	${}^{*}R_{1W} = 1.11643 \cdot 10^{9}$ *IAUs* $= p_{1w} \cdot q_{1w}$
Country Two services	${}^{*}R_{2W} = 1.10638 \cdot 10^{9}$ *IAU*s $= p_{2w} \cdot q_{2w}$
Country One products	${}^{*}R_{1U} = 1.10099 \cdot 10^{9}$ *IAU*s $= p_{1u} \cdot q_{1u}$
Country Two products	${}^{*}R_{2U} = 1.16641 \cdot 10^{9}$ *IAU*s $= p_{2u} \cdot q_{2u}$
International securities	${}^{*}R_{FS} = 4.12469 \cdot 10^{9}$ *IAUs* $= p_{fs} \cdot q_{fs}$

Substituting $K_F = 2x10^9$*IAU*s in Table (7.27) and in equations (7.30), we derive annual payments on labour and output sold by Country One and Country Two as well as on capital borrowed by all institutional sectors. The figures, measured in *IAU*s, are shown in Table (7.34). By definition, aggregate demand $^*R_{XJ}$ in respect of traded good *J* is equal to average quotation P_{XJ} times quantity traded q_{xj}. Dividing the gross turnover by the average quotation, found in Tables (7.34) and (7.33) respectively, we obtain the quantity exchanged per calendar year. Division results are listed in Table (7.35).

(7.35) Quantities traded per annum at market equilibrium

$*q_{1w} = 1.11643 \cdot 10^9$ IAUs / 1 070 IAUs $\approx 1.04 \cdot 10^6$ years of Country One labour
$*q_{2w} = 1.10638 \cdot 10^9$ IAUs / 980 IAUs $\approx 1.13 \cdot 10^6$ years of Country Two labour
$*q_{1u} = 1.10099 \cdot 10^9$ IAUs / 0.98 IAUs $\approx 1.12 \cdot 10^9$ pieces of Country One output
$*q_{2u} = 1.16641 \cdot 10^9$ IAUs / 1.09 IAUs $\approx 1.07 \cdot 10^9$ pieces of Country Two output
$*q_{fs} = 4.12469 \cdot 10^9$ IAUs / 40.00 IAUs $\approx 1.03 \cdot 10^8$ units of international consols

Subtracting demanded $^*q_{xj}$ from available $^\#q_{xj}$ quantities, we derive the shortage (–) or surplus (+) of man hours, output pieces and consol units displayed in Table (7.36).

(7.36) Steady-state shortage (–) or surplus (+) of traded goods

$^\#q_{1w} - {}^*q_{1w} \approx -4.34 \cdot 10^4$ years of Country One labour
$^\#q_{2w} - {}^*q_{2w} \approx -1.29 \cdot 10^5$ years of Country Two labour
$^\#q_{1u} - {}^*q_{1u} \approx -1.23 \cdot 10^8$ pieces of Country One output
$^\#q_{2u} - {}^*q_{2u} \approx -7.01 \cdot 10^7$ pieces of Country Two output
$^\#q_{fs} - {}^*q_{fs} \approx -3.12 \cdot 10^6$ units of international consols

Tables (7.34) to (7.36) allow calculation of the average quotations which would equate demanded $^*q_{xj}$ with available amounts $^\#q_{xj}$. Table (7.37) lists the balancing quotations.

(7.37) Notional quotations which would equalise demanded with available amounts

$^\#P_{1W} = 1.11643 \cdot 10^9$ *IAUs* / $10^6 \approx 1\,116$ *IAUs* per year of Country One labour
$^\#P_{2W} = 1.10638 \cdot 10^9$ *IAU*s / $10^6 \approx 1\,106$ *IAUs* per year of Country Two labour
$^\#P_{1U} = 1.10099 \cdot 10^9$ *IAU*s / $10^9 \approx 1.101$ *IAUs* per piece of Country One output
$^\#P_{2U} = 1.16641 \cdot 10^9$ *IAU*s / $10^9 \approx 1.166$ *IAUs* per piece of Country Two output
$^\#P_{FS} = 4.12469 \cdot 10^9$ *IAUs* / $10^8 \approx 41.247$ *IAUs* per international consol unit

Within each annual period, salaries, prices and values are decided by autonomous factors which usually ignore prevailing shortages or surpluses of services, products and securities. As a result, overhang inflation or deflation is not eliminated. The potential rise (or fall) of quotations is equal to the weighted difference between equilibrium and prevailing salaries, prices and values. Table (7.38) records the sources of Delta's inflation.

(7.38). International quotation trends

$^{\#}P_{1W} - P_{1W}$ => (1 116 – 1 070) / salary per year of Country One labour = + *4. 34%*

$^{\#}P_{2W} - P_{2W}$ => (1 106 – 980) / salary per year of Country Two labour = + *12. 90%*

$^{\#}P_{1U} - P_{1U}$ => (1.10 – 0.98) / price per piece of Country One output = + *12. 35%*

$^{\#}P_{2U} - P_{2U}$ => (1.17 – 1.09) / price per piece of Country Two output = + *7. 01%*

$^{\#}P_{FS} - P_{FS}$ => (41.25 – 40.00) / value per international consol unit = + *3. 12%*

Equation (7.31), reflecting the distribution of gross turnover among the five traded goods, gives one of the eligible sets of weights for estimating Delta's average inflation. From this all-inclusive perspective, potential annual rise of market quotations is estimated below:

$$\pi = (13.0 \text{ x } 4.34) + (12.8 \text{ x } 12.9) + (12.8 \text{ x } 12.35) + (13.5 \text{ x } 7.01) + (47.9 \text{ x } 1.79) = 6.24\% \qquad (7.39)$$

World economy is composed of sovereign countries, each of which maintains a national government, a central bank, a currency unit, a statistics bureau, etc. The officially reported inflation (or deflation) rate depends on the weight accorded to each market good by the appropriate authority. Most cost-of-living and cost-of-operating indices exclude financial securities for well-justified reasons already discussed in Section 5.4. Within a calendar year, i.e. $t_o \leq t_1 \leq t_o+1$, available man hours ($^{\#}q_{1w}$, $^{\#}q_{2w}$), output pieces ($^{\#}q_{1u}$, $^{\#}q_{2u}$) and liquidity units ($^{\#}q_{fs}$) tend to be more or less fixed. As shown by (7.39), without preventive action, salaries prices and values in Countries One and Two would rise by *6.24%* on average so as to eliminate shortages during the following period. This would not necessarily be a happy development especially for consumers and producers in their capacity as buyers. Fortunately, the IFO and national monetary authorities – by regulating country liquidity supplies, interest rates and exchange parities as well as accepting sector credit shares worldwide – are in position to control the demanded years of services $^{*}q_{xw}$, pieces of products $^{*}q_{xu}$, $X = 1, 2$, and units of consols $^{*}q_{fs}$ (and thus the exchange network's potential inflation or deflation rate) during the ensuing period, i.e. $t_1 \leq t_2 \leq t_1+2$.

Bank authorities, by guiding the issue and allocation of loans among the four accepting sectors on the basis of a mathematical economic model such as (7.21), may minimise deficits of services, products and securities and thus the expected rise of quotations. The number of markets that can be influenced is equal to the number of control variables. Bearing in mind that μ_{1h}, μ_{2h}, μ_{1b} and μ_{2b} are linearly dependent because $(\mu_{1h} + \mu_{2h} + \mu_{1b} + \mu_{2b}) = 1$, system (7.26) may be solved to yield the annual flows ($^*Y_{1H}$, $^*E_{1H}$; $^*Y_{2H}$, $^*E_{2H}$; $^*Y_{1B}$, $^*E_{1B}$; $^*Y_{2B}$, $^*E_{2B}$; $^*Y_F$, $^*E_F$) of Delta's sectors as functions of universal liquidity increment (K_F) and credit shares (μ_{1h}, μ_{2h}, μ_{1b} or μ_{2b}) of households and businesses worldwide. Other things being equal, consumer, producer and bank revenue and outlay respond to changes of the international monetary financial balance sheet as shown below:

(7.40) Delta's sector flows as functions of K_F and μ_{1H}, μ_{2H}, μ_{1B} or μ_{2B}

Income of Households One

$$^*Y_{1H} = -10\cdot(-55529517+9100812\cdot\mu_{1b}-7126240\cdot\mu_{1h}+11215305\cdot\mu_{2h})\cdot K_F/696348793$$

Expenditure of Households One

$$^*E_{1H} = -3\cdot(-185098390+30336040\cdot\mu_{1b}+208362131\cdot\mu_{1h}+37384350\cdot\mu_{2h})\cdot K_F/696348793$$

Income of Households Two

$$^*Y_{2H} = 8\cdot(61552065+7390425\cdot\mu_{1b}+13063290\cdot\mu_{1h}+33373421\cdot\mu_{2h})\cdot K_F/696348793$$

Expenditure of Households Two

$$^*E_{2H} = 15\cdot(32827768+3941560\cdot\mu_{1b}+6967088\cdot\mu_{1h}-28624095\cdot\mu_{2h})\cdot K_F/696348793$$

Income of Businesses One

$$^*Y_{1B} = 7\cdot(14516840+3544345\cdot\mu_{1b}-2568444\cdot\mu_{1h}+650360\cdot\mu_{2h})\cdot K_F/99478399\cdot$$

Expenditure of Businesses One

$$^*E_{1B} = -4\cdot(-\,25404470+18666996\cdot\mu_{1b}+4494777\cdot\mu_{1h}-1138130\cdot\mu_{2h})\cdot K_F/994783990$$

Income of Businesses Two

$$^*Y_{2B} = -27\cdot(-36054187+11560507\cdot\mu_{1b}+11574043\cdot\mu_{1h}+8918059\cdot\mu_{2h})\cdot K_F/696348793$$

Expenditure of Businesses Two

$$^*E_{2B} = 16\cdot(17319641+24013444\cdot\mu_{1b}+23990602\cdot\mu_{1h}+28472575\cdot\mu_{2h})\cdot K_F/696348793$$

Income of International Financiers

$$^*Y_F = (513371907-77140000\cdot\mu_{1b}-146952500\cdot\mu_{1h}-29872000\cdot\mu_{2h})\cdot K_F/696348793$$

Expenditure of International Financiers

$$^*E_F = -100\cdot(-12097207+771400\cdot\mu_{1b}+1469525\cdot\mu_{1h}+298720\cdot\mu_{2h})\cdot K_F/696348793$$

Substituting sector flows (7.40) in market flows (7.30), we derive annual payments on labour and output sold by Countries One and Two, as well as on capital borrowed by consumers, producers and bankers worldwide, as functions of the universal liquidity increment and sector credit shares. The said functions are reproduced by Table (7.41).

(7.41) Delta's Market Flows as functions of K_F and μ_{1h}, μ_{2h}, μ_{1b} or μ_{2b}

$$^{*}R_{1W} = P_{1W} \cdot q_{1w} = (55529517 - 9100812\cdot\mu_{1b} + 7126240\cdot\mu_{1h} - 11215305\cdot\mu_{2h})\cdot K_F / 99478399$$

$$^{*}R_{2W} = P_{2W} \cdot q_{2w} = 5\cdot(61552065 + 7390425\cdot\mu_{1b} + 13063290\cdot\mu_{1h} + 33373421\cdot\mu_{2h})\cdot K_F / 696348793$$

$$^{*}R_{1U} = P_{1U} \cdot q_{1u} = 4\cdot(14516840 + 3544345\cdot\mu_{1b} - 2568444\cdot\mu_{1h} + 650360\cdot\mu_{2h})\cdot K_F / 99478399$$

$$^{*}R_{2U} = P_{2U} \cdot q_{2u} = 15\cdot(-36054187 + 11560507\cdot\mu_{1b} + 11574043\cdot\mu_{1h} + 8918059\cdot\mu_{2h})\cdot K_F / 696348793$$

$$^{*}R_{FS} = P_{FS} \cdot q_{fs} = (1602120537 - 146566000\cdot\mu_{1b} - 279209750\cdot\mu_{1h} - 56756800\cdot\mu_{2h})\cdot K_F / 696348793$$

Within the time unit, average quotations (P_{1W}, P_{2W}; P_{1U}, P_{2U}; P_{FS}) as well as available quantities ($^{\#}q_{1w}$, $^{\#}q_{2w}$; $^{\#}q_{1u}$, $^{\#}q_{2u}$; $^{\#}q_{fs}$) of services, products and securities are autonomous. Consequently, system (7.41) – reproduced in the form $\mathbf{\Psi} \times \mathbf{\Pi} = \mathbf{R}$ by (7.42) – involves five linear equations, five target variables (R_{1W}, R_{2W}, R_{1U}, R_{2U}, R_{FS}) and five control parameters, i.e. K_F, μ_{1h}, μ_{2h}, μ_{1b} and $P_{FS} = (1 IAU/r)$.

(7.42) Delta's market equilibrium as a system of linear equations

$$\begin{vmatrix} \frac{55529517}{99478399} & \frac{-9100812}{99478399} & \frac{+7126240}{99478399} & \frac{11215305}{99478399} & 0 \\ \frac{307760325}{696348793} & \frac{5278875}{696348793} & \frac{65316450}{696348793} & \frac{166867105}{696348793} & 0 \\ \frac{58067360}{99478399} & \frac{14177380}{99478399} & \frac{-10273776}{99478399} & \frac{2601440}{99478399} & 0 \\ \frac{540812805}{696348793} & \frac{-24772515}{696348793} & \frac{-173610645}{696348793} & \frac{-133770885}{696348793} & 0 \\ \frac{1602120537}{696348793} & \frac{-2093800}{99478399} & \frac{-279209750}{696348793} & \frac{-56756800}{696348793} & -10^{8} \end{vmatrix} \cdot \begin{vmatrix} K_F \\ K_F\cdot\mu_{1B} \\ K_F\cdot\mu_{1H} \\ K_F\cdot\mu_{2H} \\ P_{FS} \end{vmatrix} = \begin{vmatrix} 1.07\cdot10^{9} \\ 0.98\cdot10^{9} \\ 0.98\cdot10^{9} \\ 1.09\cdot10^{9} \\ 0 \end{vmatrix}$$

System (7.42) can be solved for the credit policy set $\mathbf{\Pi}(K_F;\ \mu_{1h}, \mu_{2h}, \mu_{1b};\ P_{FS} = 1 IAU/r)$ that will eliminate all quantity and quotation gaps during the next period because determinant $|\mathbf{\Psi}| = (1\,539\text{x}10^{11} / 99\,478\,399) \neq 0$. The solution is $\mathbf{\Pi}(K_F = 1.8480\text{x}10^{9}$, $\mu_{1h} = 0.591701$, $\mu_{2h} = 0.136049$, $\mu_{1b} = 0.043574$, $\mu_{2b} = 0.228676$, $P_{FS} = 37.6118467$ *IAU* and thus r = 2.6587%). The IFO and national central banks may clear Delta's markets (i.e. $^{\#}q_{1w} = {}^{*}q_{1w}$, $^{\#}q_{2w} = {}^{*}q_{2w}$; $^{\#}q_{1u} = {}^{*}q_{1u}$, $^{\#}q_{2u} = {}^{*}q_{2u}$; $^{\#}q_{fs} = {}^{*}q_{fs}$) by guiding monetary financial institutions to:

- reduce the annual liquidity supply from $K_F = 2.00 \times 10^{9}$ *IAU*s to $K_F \approx 1.85 \times 10^{9}$ *IAU*s;
- increase the credit share of Country One consumers to $\mu_{1h} \approx 59.27\%$ from $\mu_{1h} \approx 50.0\%$;
- reduce the credit share of Country One producers to $\mu_{1b} \approx 4.36\%$ from $\mu_{1b} \approx 8.33\%$;
- reduce the credit share of Country Two consumers to $\mu_{2h} \approx 13.6\%$ from $\mu_{2h} \approx 25.0\%$;
- increase the credit share of Country Two producers to $\mu_{2b} \approx 22.87\%$ from $\mu_{2b} \approx 16.67\%$;

- raise the annual discount rate to r ≈ 2. 66 per cent from r ≈ 2. 50 per cent.

(7.43) Delta's equilibrium when $\mu_{1h} = {}^6/_{12}$, $\mu_{1b} = {}^1/_{12}$, $\mu_{2h} = {}^3/_{12}$, $\mu_{2b} = {}^2/_{12}$

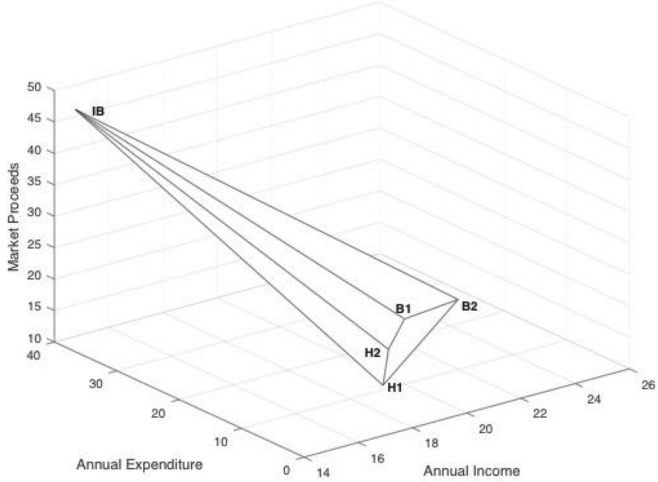


Figure (7.43) depicts the distribution of Delta's income, expenditure and sales among the households **H1**(18.5%, 6.9%, 13.0%), **H2**(20.5%, 14.7%, 12.8%), the businesses **B1**(22.4%, 20.4%, 12.8%), **B2**(24.4%, 20.5%, 13.5%) as well as the combined banking system **F**(14.2%, 37.5%, 47.9%) of Countries One and Two. Beyond the pentahedron's vertices there are exist forces acting in both directions of the three axes, which compel the aforesaid variables to return to their steady-state combinations.

(7.44) Delta's equilibrium when $\mu_{1h} \approx 0.5917$, $\mu_{2h} \approx 0.1360$, $\mu_{1b} \approx 0.0436$, $\mu_{2b} \approx 0.2287$

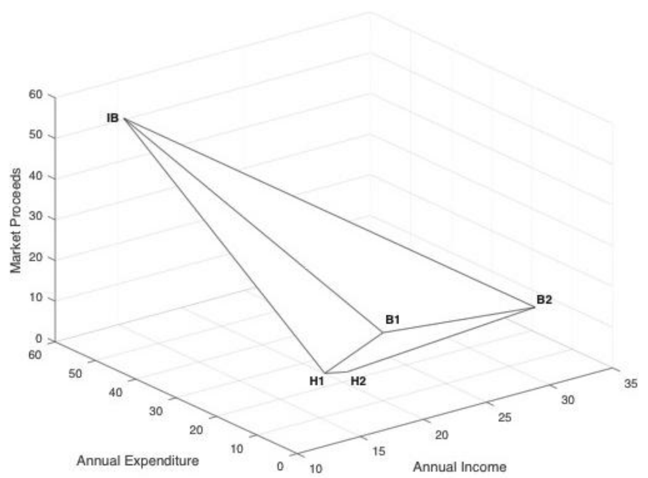


Figure (7.44) reflects the reallocation of Delta's annual flows among consumers **H1**(15.7%, 11.0%, 9.9%), **H2**(17.4%, 10.8%, 8.9%), producers **B1**(22.9%, 19.1%, 10.1%), **B2**(31.7%, 8.9%, 13.6%) and bankers **F**(12.3%, 50.2%, 57.5%) caused by reapportionment of the universal liquidity increment. The new credit policy has completely altered the global economy's structure and thus the corresponding pentahedron's shape.

Remember that the percentage of loans destined for a country or sector may be raised if the percentage of loans destined for another country or sector may be reduced. Euclidean methodology takes for granted that the IFO and national central banks are able to persuade other monetary financial institutions to expand, allocate and value their assets and liabilities according to worldwide statistical data and mathematical economic model (7.21). Within this framework, functions (7.40) enable us to compute the changes of annual flows brought about by redistribution of credit among sovereign countries and accepting sectors. For example, partial derivatives listed in Table (7.45) reflect adjustments of incomes and expenditures brought about by simultaneous rise of μ_{2h} (i.e. worldwide loans to Country Two households) and fall of μ_{2b} (i.e. worldwide loans to Country Two businesses).

(7.45) Responses of equilibrium flows to sector credit redistribution

$\partial^* Y_{1H} / \partial\mu_{2h} = -\,0.161059 \cdot K_F$ $\qquad$ $\partial^* E_{1H} / \partial\mu_{2h} = -\,0.161059 \cdot K_F$

$\partial^* Y_{2H} / \partial\mu_{2h} = +\,0.38341 \cdot K_F$ $\qquad$ $\partial^* E_{2H} / \partial\mu_{2h} = -\,0.61659 \cdot K_F$

$\partial^* Y_{1B} / \partial\mu_{2h} = +\,0.045764 \cdot K_F$ $\qquad$ $\partial^* E_{1B} / \partial\mu_{2h} = +\,0.045764 \cdot K_F$

$\partial^* Y_{2B} / \partial\mu_{2h} = -\,0.345786 \cdot K_F$ $\qquad$ $\partial^* E_{2B} / \partial\mu_{2h} = +\,0.65421 \cdot K_F$

$\partial^* Y_{F} / \partial\mu_{2h} = -\,0.042898 \cdot K_F$ $\qquad$ $\partial^* E_{F} / \partial\mu_{2h} = -\,0.042898 \cdot K_F$

Certain allocations of credit between Country One and Country Two, as well as among consumers and producers internationally, are totally objectionable because they cause some country and / or sector flows to become lower than zero. In order to explore this serious matter, we regard Table (7.40) as a system of ten linear inequalities in terms of μ_{xh} and μ_{xb}. Starting from $\mu_{1h} = {}^{6}/_{12}$, $\mu_{1b} = {}^{1}/_{12}$, $\mu_{2h} = {}^{3}/_{12}$, $\mu_{2b} = {}^{2}/_{12}$, *Mathematica* enables us to compute the credit share ranges within which all countries and sectors enjoy positive revenues and outlays, i.e. $^*Y_{XH} > 0$, $^*E_{XH} > 0$, $^*Y_{XB} > 0$, $^*E_{XB} > 0$, $^*Y_F > 0$, $^*E_F > 0$.

(7.46) Country and sector credit shares within which Delta's flows stay positive

COUNTRY ONE ($\mu_{1h} + \mu_{1b}$)				**COUNTRY TWO ($\mu_{2h} + \mu_{2b}$)**			
Min		**Max**		**Min**		**Max**	
0.00		1.00		0.00		1.00	
μ_{1h}		**μ_{1b}**		**μ_{2h}**		**μ_{2b}**	
min	max	min	max	min	max	min	Max
0.00	0.563337	0.00	1.00	0.00	1.00	0.00	1.00

According to Table (7.46), the incomes and expenditures of Delta's households, businesses and financiers will be greater than zero as long as Country One consumers receive less than 56.3337% of the yearly monetary increment. Since the 1992 Maastricht Treaty, the EU has paid special attention to fiscal rules which are conducive to financial stability. From the 2007 - 2015 crises, everyone has realized that this strategy is inadequate. Many observers have called for further political integration and other structural measures in order to deal with deteriorating conditions, especially in Portugal, Italy, Spain and Greece. We conclude by pointing out the need to reassess the magnitude, division and cost of annual credit in the Eurozone and around the world on the basis of a Euclidean economic model.

7.8. The loci of world-wide portfolio balance

It is reiterated that: i) agents, sectors, countries, etc., do not suffer from money illusion because they compare nominal flows and capital stocks in *IAU*s; ii) policy makers achieve their targets only when the global economic system's steady state consists of stable and positive flows; iii) annual liquidity allotment among countries and zones is accomplished by means of international agreements and strong-arm tactics. As shown by (7.22) and (7.26), parameters K_{XI} and k_{xf}, ξ_x, φ_x, are linearly as well as otherwise connected. Subject to their mutual dependence, they determine:

- expansion and allocation of the world banking balance sheet;
- sector $^*\mathbf{X}$ and market $^*\mathbf{R}$ flows via world structural matrix $\mathbf{\Delta}$.

Certain well-defined relationships must prevail among the money supplies, discount rates and exchange parities of interacting countries (or zones) for national currency denominated consols (deposits) to circulate in parallel. The aforementioned relationships are reviewed from three different perspectives below:

(i) Iron laws, such as (7.1) or (7.47), are always valid.

$$K_F = \varphi_1 \cdot K_F + \varphi_2 \cdot K_F = \xi_1 \cdot k_{1f} + \xi_2 \cdot k_{2f} \quad \Rightarrow \quad K_{1F} = \xi_1 \cdot k_{1f} \quad \Rightarrow \quad K_{2F} = \xi_2 \cdot k_{2f} \qquad (7.47)$$

The above definition confirms that a country's annual liquidity increment (measured in *IAU*s) is equal to the product of ξ_x and k_{xf}. The consequent loci between ξ_x and k_{xf} signify that from one year to the next, a national banking system acting alone may:

- accelerate the issue of local money while devaluing the exchange parity;

- augment the exchange parity while decelerating the issue of local money.

Economic interaction is characterised by relentless efforts by sectors, countries, etc., to improve their welfare against unyielding mathematical constraints, such as yearly budgets, limited resources, etc. Certain people find it hard to admit that average quotations and cleared quantities move in opposite directions and that in the short run there are no internal forces to correct the situation. For example, at any moment, some financial officials may self-servingly tamper with monetary growth, annual discount and exchange parity rates under the impression that the resulting shortages or surpluses (of labour services, business products, financial securities, public takings, international reserves, etc.) will vanish automatically. Frequently, the consequences emanating from violation of economic laws are as harsh as the consequences emanating from violation of natural laws.

(ii) The next equation describes the locus where i) consumers, producers and financiers of both areas demand and supply the same amount of capital in exchange for one *IAU* per year and ii) Country One and Country Two (rest of the world) perpetual bonds (bank deposits) circulate in parallel as perfect substitutes:

$$1\ IAU/(\xi_1 \cdot r_1) = 1\ IAU/(\xi_2 \cdot r_2) \quad => (\xi_1 \cdot r_1) = (\xi_2 \cdot r_2) \qquad (7.48)$$

Relationship (7.48) confirms that unilateral attempts by a central bank to offer higher (or lower) interest rate r_x to those investing (or borrowing) via financial securities denominated in its national currency are short lived because of the unavoidable devaluation (revaluation) ξ_x of its exchange parity. This Iron Law – also known as the Uncovered Interest Parity Principle – defines a locus between a monetary system's interest and exchange rates. The said rule may not apply to the country or zone whose currency constitutes the international numeraire. In Section 8.7, it will be demonstrated that choices (by other countries or zones) regarding lower (higher) discount rates and higher (lower) exchange rates depend on various factors, including the IFO's plans on bridging quantity gaps worldwide.

(iii) Conclusions reached by means of (7.47) and (7.48) acquire further validity when portfolio balance is approached from the allocation of global money supply point of view. The present values of national liquidity increments (extrapolated to infinity) are as follows:

$$\begin{aligned} M_1 &= \xi_1 \cdot k_1 / r_1 \quad => \quad (dM_1/dt)/M_1 = (d\xi_1/dt)/\xi_1 + (dk_1/dt)/k_1 - (dr_1/dt)/r_1 \\ M_2 &= \xi_2 \cdot k_2 / r_2 \quad => \quad (dM_2/dt)/M_2 = (d\xi_2/dt)/\xi_2 + (dk_2/dt)/k_2 - (dr_2/dt)/r_2 \end{aligned} \qquad (7.49)$$

Abstracting from differences in productive potentials, countries and zones should expand their money supplies (measured in *IAU*s) at the same pace when they do not wish to antagonize each other. Stable shares of the world loan and deposit supply imply:

$$(dM_1/dt)/M_1 = (dM_2/dt)/M_2 \;=>$$

$$(d\xi_1/dt)/\xi_1 + (dk_1/dt)/k_1 - (dr_1/dt)/r_1 = (d\xi_2/dt)/\xi_2 + (dk_2/dt)/k_2 - (dr_2/dt)/r_2 \qquad (7.50)$$

Perspective **(iii)** incorporates deductions derived from **(i)** and **(ii)**. Within a single time unit, relations (7.47) to (7.50) may not be automatically true because those in charge may modify liquidity increments k_{xf}, discount rates r_x and exchange parities ξ_x independently in accordance with various expediencies. In the long run, however, k_x, r_x and ξ_x should conform with loci (7.47) to (7.50) because all other combinations give rise to speculative flows, economic warfare and eventual rebalancing of relationships among credit growth, annual discount and exchange parity rates of the countries or zones involved.

In case universal K_F and local k_{xf} money supplies increased at the same pace, any alteration of a country's ξ_x number of *IAUs* per currency unit would be reflected in the liquidity shares φ_x of other countries via relationship:

$$\Sigma\xi_x \cdot k_{xf} = \Sigma\varphi_x \cdot K_F \qquad (7.51)$$

Arbitrary changes lead to cycles of action and reaction until the countries and zones involved recognise the need for financial coordination through a Euclidean economic model. Exchange rates are useful tools when the IFO and the banking systems concerned agree to modify country shares of international liquidity φ_x without affecting loan supply k_{xf} and nominal wages p_{xw} in local currency terms. Although Eurozone consists of sovereign states, ξ_x are not available to them as instruments because $d\xi_x/dt = 0$. Consequently, the optimum annual credit shares of Euro area members should be estimated in accordance with Euclidean methodology and implemented by the European Central Bank in cooperation with national monetary authorities. In Chapter **8**, among other things, we will measure the costs and benefits which accompany amalgamation of sovereign countries and currency zones into a global trade and payments network.

Chapter 8. Economic Union and Policy Coordination

Initially, it is assumed that the world consists of two monetary economies or currency zones which operate separately. Subsequently, both governments recognise the possibility of additional earnings and thus allow households, businesses and financiers to sell and purchase services, products and securities in accordance with agreed rules on trade and payments. For the sake of convenience, the two countries or areas are called Alpha and Gamma, while their combined network is called Zeta. Euclidean propositions and their corollaries are applicable under all circumstances. For example:

- Each economy is differentiated by a distinct apportionment of sector flows among feasible sources and pertinent destinations;
- Foreign demand for at least one market good supplied by a country is necessary for joining the global network;
- Sectors do not suffer from money illusion and thus consider national currency denominated consols as perfect substitutes.

This exercise pertains to the theory of international trade and finance. We aim to demonstrate the wide applicability of Euclidean methodology by comparing monetary flows of national sectors as well as annual turnovers of market goods before and after commencement of cooperation and competition between Alpha and Gamma. Our task and subsequent comparisons are simplified by assuming that both economies start from:

- exchange parity *vis-à-vis* the International Accounting Unit;
- identical annual credit expansion and credit allocation rates;
- equal reserves of labour years, output pieces and liquidity units.

As soon as they become familiar with each other's services and products, Alpha and Gamma shape their substitution elasticities between local and foreign real goods (i.e. labour and output). During the year under review, the said relative preference is $\alpha/(1-\alpha)$, $0<\alpha<1$ in Alpha and $\gamma/(1-\gamma)$, $0<\gamma<1$ in Gamma. It will be shown that the two national economies are mathematically joined by ratio z, $0<z<1$, $\alpha/\gamma = z/(1-z)$ that reflects the combined network's relative preference for Alpha *vis-à-vis* Gamma services and products. Given behaviour coefficients and liquidity allocations, we are ready to study the impact of substitution elasticities on Zeta's equilibrium, stability and controllability properties.

Quantity gaps are functions of the magnitudes and distributions of aggregate demand and credit supply. Financial tools – such as the rates of monetary expansion, annual discount and exchange parity – are generally thought to have parallel effects on countries in the same currency zone. In this connection, Euro area regulations require that the business cycles of applicant countries must agree with those of member states as far as possible. However, during most of the 2007 - 2015 period, the turnover of the German economy was expanding while the turnovers of some member states were contracting. Some analysts commented that this aberration was unrelated to delay in improving financial management in the Eurozone. Consistent with this interpretation, the twelve premises direct us to conclude that countries are like markets which face dissimilar demand relative to optimum turnover almost every year. Regarding concerted action– in the Eurozone and elsewhere – harmonisation of business cycles is less important than simultaneous availability of i) a model of the closed economic system, ii) a complete set of the relevant statistics and iii) a selection of tools that match the targets. Euclidean approach confirms that advantageous policies – which lead to minimisation of quantity deficits and surpluses – involve reallocation of loans and deposits among countries as well as taxes and subsidies among sectors. More than 22 centuries ago, Archimedes said "*Give me a place to stand, a long enough lever and a fulcrum and I can move the Earth*".

8.1. Statistical survey of country Alpha

As a result of biological, psychological and other temporarily inflexible factors, households allocate their (annual) receipts and payments among the feasible sources and the applicable destinations, respectively, as follows:

Income ratio from:	Services 8/9	Securities 1/9
Expenditure ratio for:	Products 7/8	Securities 1/8

As a result of managerial, technological and other temporarily inflexible factors, businesses allocate their (annual) takings and outgoings among the possible sources and pertinent destinations, respectively, as shown below:

Income ratio from:	Products 6/7	Securities 1/7
Expenditure ratio for:	Services 5/6	Securities 1/6

Due to legal restrictions, monetary financial institutions derive 100% of their (annual) revenue from acceptance of guaranteed securities issued by households and businesses.

Simultaneously, they apportion their (annual) outlay among market goods according to operational requirements and various expediencies, as follows:

Expenditure ratio for: Securities $^{439}/_{3168}$ Services 1/2 Products $^{1145}/_{3168}$

The central bank and other monetary financial institutions buy and sell guaranteed securities. They divide the permissible expansion of their balance sheet equally among Alpha's consumers and producers:

Credit shares: Households 1/2 Enterprises 1/2

Within any time unit, i.e. $t_0 \leq t \leq t_0 + 1$, sector behaviour coefficients are assumed to remain constant. Consumers, producers and bankers trade services, products and consols as reproduced by system (8.1). Alpha's mathematical model, which consists of six first-order non-homogeneous linear differential equations, has the general form $\mathbf{dX/dt = A \cdot X - K_A}$. Solution of simultaneous linear equation system $\mathbf{X^A = A^{-1} \cdot K_A}$ yields the steady-state revenue and outlay of households (y_h^A, e_h^A), businesses (y_b^A, e_b^A) and financiers (y_f^A, e_f^A), provided matrix **A** is not empty.

$$\begin{vmatrix} dy_h/dt \\ de_h/dt \\ dy_b/dt \\ de_b/dt \\ dy_f/dt \\ de_f/dt \end{vmatrix} = \begin{vmatrix} -1 & 0 & -^1/_7 & ^5/_6 & 0 & ^{2023}/_{3168} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -^1/_9 & ^7/_8 & -1 & 0 & 0 & ^{1584}/_{3168} \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & ^1/_8 & 0 & ^1/_6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h \\ e_h \\ y_b \\ e_b \\ y_f \\ e_f \end{vmatrix} - \begin{vmatrix} 0 \\ ½ \\ 0 \\ ½ \\ 0 \\ -1 \end{vmatrix} \cdot \mathrm{K_A} \qquad (8.1)$$

In this case $|\mathbf{A}| = {}^{253109}/_{1368576}$ and hence $\mathbf{X^A} = \xi_A \cdot k_\alpha \cdot (0.59342, 0.09342, 0.52361, 0.02362, 0.01561, 1.01561)$ where $\xi_A = 1 IAU/\$$. Equilibrium positions generated by different parameters may be compared meaningfully provided the dynamic system is globally stable.

$$\left| \mathbf{A - \lambda \cdot I} \right| = \begin{vmatrix} -1-\lambda & 0 & -^1/_7 & ^5/_6 & 0 & ^{2023}/_{3168} \\ 1 & -1-\lambda & 0 & 0 & 0 & 0 \\ -^1/_9 & ^7/_8 & -1-\lambda & 0 & 0 & ^{1584}/_{3168} \\ 0 & 0 & 1 & -1-\lambda & 0 & 0 \\ 0 & ^1/_8 & 0 & ^1/_6 & -1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 1 & -1-\lambda \end{vmatrix} = \qquad (8.2)$$

$$= \lambda^6 + 6\cdot\lambda^5 + ({}^{944}/{}_{63})\cdot\lambda^4 + ({}^{30473}/{}_{1512})\cdot\lambda^3 + ({}^{2601727}/{}_{177408})\cdot\lambda^2 + ({}^{11556863}/{}_{2395008})\cdot\lambda + ({}^{253109}/{}_{1368576})$$
$$= a_0\cdot\lambda^6 + a_1\cdot\lambda^5 + a_2\cdot\lambda^4 + a_3\cdot\lambda^3 + a_4\cdot\lambda^2 + a_5\cdot\lambda + a_6 = 0 \tag{8.2}$$

Exchange network (8.1) is globally stable if, and only if, the Routh-Hurwitz determinants, made up of the coefficients of Alpha's characteristic polynomial $|\mathbf{A} - \lambda\cdot\mathbf{I}|$, are greater than zero. The said polynomial is equal to determinant (8.2). With *Mathematica*'s assistance, the above coefficients a_i, i = 0, ... , 6 are substituted in the six Routh-Hurwitz conditions of system (8.1). In this way we obtain $rh_1 = 6$, $rh_2 = 69.751$, $rh_3 = 906.766$, $rh_4 = 8733.36$, $rh_5 = 42141.9$ and $rh_6 = 0.184943$. As long as the stability conditions are fulfilled, monetary flows of consumers, producers and bankers converge towards $\mathbf{X^A}$ regardless of initial values. By substituting sector flows $\mathbf{X^A}$ in equations (4.16), we obtain annual sales $\mathbf{R^A} = (\rho_w{}^A, \rho_u{}^A, \rho_s{}^A)$ of labour, output and liquidity in economy Alpha. Due to the exchange network's global stability, any turnover combination $\mathbf{R^0}$ tends towards the equilibrium vector $\mathbf{R^A}$. Measured in *IAU*s, Alpha's aggregate receipts Y^A, total payments E^A and market sales R^A are given by (8.3):

$$\begin{aligned} Y^A &= (0.59342 + 0.52361 + 0.01561)\cdot\xi_A\cdot k_\alpha \approx 1.13264\cdot\xi_A\cdot k_\alpha = 1.13264\cdot K_A \\ E^A &= (0.09342 + 0.02362 + 1.01561)\cdot\xi_A\cdot k_\alpha \approx 1.13264\cdot\xi_A\cdot k_\alpha = 1.13264\cdot K_A \\ R^A &= (0.52748 + 0.44880 + 0.15635)\cdot\xi_A\cdot k_\alpha \approx 1.13264\cdot\xi_A\cdot k_\alpha = 1.13264\cdot K_A \end{aligned} \tag{8.3}$$

8.2. Statistical survey of country Gamma

As a result of biological, psychological and other temporarily inflexible factors, households allocate their (annual) receipts and payments among the feasible sources and the applicable destinations, respectively, as follows:

Income ratio from:	Services 1/3	Securities 2/3
Expenditure ratio for:	Products 1/4	Securities 3/4

As a result of managerial, technological and other temporarily inflexible factors, businesses allocate their (annual) takings and outgoings among the possible sources and pertinent destinations, respectively, as follows:

Income ratio from:	Products 1/5	Securities 4/5
Expenditure ratio for:	Services 1/6	Securities 5/6

Due to legal restrictions, monetary financial institutions derive 100% of their (annual) revenue from acceptance of guaranteed securities issued by households and businesses. Simultaneously, they apportion their (annual) outlay among all market goods according to operational requirements and various expediencies, as follows:

Expenditure ratio for: Securities 18/24 Services 4/24 Products 2/24

The central bank and other monetary financial institutions buy and sell guaranteed securities. They divide the permissible expansion of their balance sheet equally among Gamma's consumers and producers:

Credit shares: Households 1/2 Enterprises 1/2

Within the time unit, i.e. $t_0 \leq t \leq t_0 + 1$, sector behaviour coefficients remain constant. As a result, dynamic system (8.4), consisting of six first-order non-homogeneous linear differential equations, replicates interaction of monetary flows earned from and spent on market goods by Gamma's institutional sectors.

$$\begin{vmatrix} dy_h/dt \\ de_h/dt \\ dy_b/dt \\ de_b/dt \\ dy_f/dt \\ de_f/dt \end{vmatrix} = \begin{vmatrix} -1 & 0 & -^4/_5 & ^1/_6 & 0 & ^{11}/_{12} \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -^2/_3 & ¼ & -1 & 0 & 0 & ^5/_6 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & ¾ & 0 & ^5/_6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h \\ e_h \\ y_b \\ e_b \\ y_f \\ e_f \end{vmatrix} - \begin{vmatrix} 0 \\ ½ \\ 0 \\ ½ \\ 0 \\ -1 \end{vmatrix} \cdot \mathrm{K}_\Gamma \tag{8.4}$$

Gamma's mathematical model possesses the general form $\mathbf{dX/dt = \Gamma \cdot X - K_\Gamma}$. Provided matrix $\mathbf{\Gamma}$ is not empty, equilibrium receipts and payments of households (y_h^Γ, e_h^Γ), businesses (y_b^Γ, e_b^Γ) and financiers (y_f^Γ, e_f^Γ) are obtained by solving static equation system $\mathbf{X^\Gamma = \Gamma^{-1} \cdot K_\Gamma}$ using Cramer's rule. In this case $|\mathbf{\Gamma}| = {}^{59}/_{864}$ and, hence, $\mathbf{X^\Gamma} = \xi_\Gamma \cdot k_\gamma \cdot$(0.574576, 0.0745763, 0.550847, 0.0508475, 0.0983051, 1.09831), where $\xi_\Gamma = 1 IAU/€$.

$$|\mathbf{\Gamma - \lambda \cdot I}| = \begin{vmatrix} -1-\lambda & 0 & -^4/_5 & ^1/_6 & 0 & ^{11}/_{12} \\ 1 & -1-\lambda & 0 & 0 & 0 & 0 \\ -^2/_3 & ¼ & -1-\lambda & 0 & 0 & ^5/_6 \\ 0 & 0 & 1 & -1-\lambda & 0 & 0 \\ 0 & ¾ & 0 & ^5/_6 & -1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 1 & -1-\lambda \end{vmatrix} = \tag{8.5}$$

$= \lambda^6 + 6\cdot\lambda^5 + (^{217}/_{15})\cdot\lambda^4 + (^{812}/_{45})\cdot\lambda^3 + (^{8143}/_{720})\cdot\lambda^2 + (^{3199}/_{1080})\cdot\lambda + (^{59}/_{864}) =$

$$= g_0\cdot\lambda^6 + g_1\cdot\lambda^5 + g_2\cdot\lambda^4 + g_3\cdot\lambda^3 + g_4\cdot\lambda^2 + g_5\cdot\lambda + g_6 = 0 \qquad (8.5)$$

Exchange network (8.4) is globally stable if, and only if, the Routh-Hurwitz conditions, composed of the coefficients of Gamma's characteristic polynomial $|\mathbf{\Gamma} - \lambda\cdot\mathbf{I}|$, are greater than zero. The said polynomial is equal to determinant (8.5). With *Mathematica*'s assistance, coefficients g_i, $i = 0, \dots, 6$, are substituted in the Routh-Hurwitz determinants of (8.4) to obtain $rh_1 = 6$, $rh_2 = 68.622$, $rh_3 = 858.022$, $rh_4 = 6983.81$, $rh_5 = 20686.3$ and $rh_6 = 0.68287$. As long as the stability conditions are fulfilled, consumer, producer and bank monetary flows converge toward $\mathbf{X}^{\Gamma}$ regardless of initial values. After any economic jolt, income from and expenditure on services, products and securities, also tend towards equilibrium position $\mathbf{R}^{\Gamma} = (\rho_w^{\Gamma}, \rho_u^{\Gamma}, \rho_s^{\Gamma}) = \xi_{\Gamma}\cdot k_{\gamma}\cdot(0.191525, 0.110169, 0.922034)$ determined by substituting $\mathbf{X}^{\Gamma}$ in market equations (4.16). Measured in *IAU*s, Gamma's aggregate revenue Y^{Γ}, total outlay E^{Γ} and market turnover R^{Γ} are given by (8.6):

$$\begin{aligned}
Y^{\Gamma} &= (0.574576 + 0.550847 + 0.0983051)\cdot\xi_{\Gamma}\cdot k_{\gamma} \approx 1.22373\cdot\xi_{\Gamma}\cdot k_{\gamma} = 1.22373\cdot K_{\Gamma} \\
E^{\Gamma} &= (0.0745763 + 0.0508475 + 1.09831)\cdot\xi_{\Gamma}\cdot k_{\gamma} \approx 1.22373\cdot\xi_{\Gamma}\cdot k_{\gamma} = 1.22373\cdot K_{\Gamma} \qquad (8.6) \\
R^{\Gamma} &= (0.191525 + 0.110169 + 0.922034)\cdot\xi_{\Gamma}\cdot k_{\gamma} \approx 1.22373\cdot\xi_{\Gamma}\cdot k_{\gamma} = 1.22373\cdot K_{\Gamma}
\end{aligned}$$

Up to the point the two economies were operating separately, the Euclidean multiplier of Alpha was 1.1326 while that of Gamma was 1.2237. By definition, the combined liquidity increment was equal to $K_F = \xi_A\cdot k_{\alpha} + \xi_{\Gamma}\cdot k_{\gamma} = K_A + K_{\Gamma}$. Their aggregate receipts, payments and sales were equal to $Y^A + Y^{\Gamma} = E^A + E^{\Gamma} = R^A + R^{\Gamma} = (1.1326\cdot\xi_A\cdot k_{\alpha} + 1.2237\cdot\xi_{\Gamma}\cdot k_{\gamma})$. With a view to expanding the turnover of traded goods, both governments have decided to allow their consumers, producers and bankers to engage in international transactions at the agreed exchange rate 1 *IAU* = 1\$ = 1€.

8.3. World income demand and expenditure supply

Since the start of competition and cooperation, each area's demand for labour and output has been modified according to the relative preference of its households, businesses and financiers for local *vis-à-vis* foreign real goods; the substitution elasticity is (α, 1-α) in Alpha and (γ, 1-γ) in Gamma. From then on, both national currency denominated consols (deposits) have been used as perfect substitutes. Table (8.7) displays how the five sectors divide their takings and outgoings among the services, products and securities of Zeta.

(8.7) Allocation of income and expenditure among Alpha and Gamma

SECTOR / FLOWS	HOUSEHOLDS **Alpha**		ENTERPRISES **Alpha**		HOUSEHOLDS **Gamma**		ENTERPRISES **Gamma**		**BANKING SYSTEM**	
	Y_{AH}	E_{AH}	Y_{AB}	E_{AB}	$Y_{\Gamma H}$	$E_{\Gamma H}$	$Y_{\Gamma B}$	$E_{\Gamma B}$	Y_F	E_F
Labour										
Y_{AW}	$^8/_9$	-	-	-	-	-	-	-	-	-
E_{AW}	-	-	-	$^{5\alpha}/_6$	-	-	-	$^{(1-\gamma)}/_6$	-	$^1/_4$
$Y_{\Gamma W}$	-	-	-	-	$^1/_3$	-		-	-	-
$E_{\Gamma W}$	-	-	-	$^{5(1-\alpha)}/_6$	-	-	-	$^{\gamma}/_6$	-	$^1/_{12}$
Production										
Y_{AU}	-	-	$^6/_7$	-	-	-		-	-	-
E_{AU}	-	$^{7\alpha}/_8$	-	-	-	$^{(1-\gamma)}/_4$	-	-	-	$^{1145}/_{6336}$
$Y_{\Gamma U}$	-	-	-	-	-	-	$^1/_5$	-	-	-
$E_{\Gamma U}$	-	$^{7(1-\alpha)}/_8$	-	-	-	$^{\gamma}/_4$	-	-	-	$^1/_{24}$
Liquidity										
Y_{FS}	$^1/_9$	-	$^1/_7$	-	$^2/_3$	-	$^4/_5$	-	1	-
E_{FS}	-	$^1/_8$	-	$^1/_6$	-	$^3/_4$	-	$^5/_6$	-	$^{2815}/_{6336}$

System (8.8) represents Zeta's mathematical model derived by substituting the behaviour coefficient data of Table (8.7) for the algebraic symbols of general system (7.17).

(8.8) Dynamic interaction among the sectors of Alpha and Gamma

$$dY_{AH}/dt = -Y_{AH} - {}^2/_3 \cdot Y_{2H} - {}^1/_7 \cdot Y_{AB} + {}^{5\alpha}/_6 \cdot E_{AB} - {}^4/_5 \cdot Y_{\Gamma B} + {}^{(1-\gamma)}/_6 \cdot E_{\Gamma B} + {}^{4399}/_{6336} \cdot E_F$$

$$dE_{AH}/dt = Y_{AH} - E_{AH} - \mathrm{K_{AH}}$$

$$dY_{\Gamma H}/dt = -{}^1/_9 \cdot Y_{AH} - Y_{\Gamma H} - {}^1/_7 \cdot Y_{AB} + {}^{5(1-\alpha)}/_6 \cdot E_{AB} - {}^4/_5 \cdot Y_{\Gamma B} + {}^{\gamma}/_6 \cdot E_{\Gamma B} + {}^{3343}/_{6336} \cdot E_F$$

$$dE_{\Gamma H}/dt = Y_{\Gamma H} - E_{\Gamma H} - \mathrm{K_{\Gamma H}}$$

$$dY_{AB}/dt = -{}^1/_9 \cdot Y_{AH} + {}^{7\alpha}/_8 \cdot E_{AH} - {}^2/_3 \cdot Y_{\Gamma H} + {}^{(1-\gamma)}/_4 \cdot E_{\Gamma H} - Y_{AB} - {}^4/_5 \cdot Y_{\Gamma B} + {}^5/_8 \cdot E_F$$

$$dE_{AB}/dt = Y_{AB} - E_{AB} - \mathrm{K_{AB}}$$

$$dY_{\Gamma B}/dt = -{}^1/_9 \cdot Y_{AH} + {}^{7(1-\alpha)}/_8 \cdot E_{AH} - {}^2/_3 \cdot Y_{\Gamma H} + {}^{\gamma}/_4 \cdot E_{\Gamma H} - {}^1/_7 \cdot Y_{AB} - Y_{\Gamma B} + {}^{3079}/_{6336} \cdot E_F$$

$$dE_{\Gamma B}/dt = Y_{\Gamma B} - E_{\Gamma B} - \mathrm{K_{\Gamma B}}$$

$$dY_F/dt = {}^1/_8 \cdot E_{AH} + {}^3/_4 \cdot E_{\Gamma H} + {}^1/_6 \cdot E_{AB} + {}^5/_6 \cdot E_{\Gamma B} - Y_F$$

$$dE_F/dt = Y_F - E_F + \mathrm{K_F}$$

Table (8.9) and system (8.10) are obtained by replacing α with z and γ with (1–z) in (8.7) and (8.8). Sector flows interact according to general model $\mathbf{dX/dt = Z{\cdot}X - K_F}$ displayed by system (8.10). As a result, we can derive Zeta's equilibrium position, dynamic stability and dirigibility properties as functions of preference ratio z.

(8.9) Allocation of income demand and expenditure supply in Zeta

SECTOR FLOWS	HOUSEHOLDS **Alpha**		ENTERPRISES **Alpha**		HOUSEHOLDS **Gamma**		ENTERPRISES **Gamma**		**BANKING SYSTEM**	
	Y_{AH}	E_{AH}	Y_{AB}	E_{AB}	$Y_{\Gamma H}$	$E_{\Gamma H}$	$Y_{\Gamma B}$	$E_{\Gamma B}$	Y_F	E_F
Labour										
Y_{AW}	$^8/_9$	-	-	-	-	-	-	-	-	-
E_{AW}	-	-	-	$^{5z}/_6$	-	-	-	$^z/_6$	-	$^1/_4$
$Y_{\Gamma W}$	-	-	-	-	$^1/_3$	-		-	-	-
$E_{\Gamma W}$	-	-	-	$^{5(1-z)}/_6$	-	-	-	$^{1-z}/_6$	-	$^1/_{12}$
Production										
Y_{AU}	-	-	$^6/_7$	-	-	-		-	-	-
E_{AU}	-	$^{7z}/_8$	-	-	-	$^z/_4$	-	-	-	$^{1145}/_{6336}$
$Y_{\Gamma U}$	-	-	-	-	-	-	$^1/_5$	-	-	-
$E_{\Gamma U}$	-	$^{7(1-z)}/_8$	-	-	-	$^{1-z}/_4$	-	-	-	$^1/_{24}$
Liquidity										
Y_{FS}	$^1/_9$	-	$^1/_7$	-	$^2/_3$	-	$^4/_5$	-	1	-
E_{FS}	-	$^1/_8$	-	$^1/_6$	-	$^3/_4$	-	$^5/_6$	-	$^{2815}/_{6336}$

(8.10) Dynamic interaction among Zeta's sectors in the form $\mathbf{dX/dt = Z{\cdot}X - K_F}$

$$dY_{AH}/dt = -Y_{AH} - {}^2/_3{\cdot}Y_{\Gamma H} - {}^1/_7{\cdot}Y_{AB} + {}^{5z}/_6{\cdot}E_{AB} - {}^4/_5{\cdot}Y_{\Gamma B} + {}^z/_6{\cdot}E_{\Gamma B} + {}^{4399}/_{6336}{\cdot}E_F$$

$$dE_{AH}/dt = Y_{AH} - E_{AH} - K_{AH}$$

$$dY_{\Gamma H}/dt = -{}^1/_9{\cdot}Y_{AH} - Y_{\Gamma H} - {}^1/_7{\cdot}Y_{AB} + {}^{5(1-z)}/_6{\cdot}E_{AB} - {}^4/_5{\cdot}Y_{\Gamma B} + {}^{1-z}/_6{\cdot}E_{\Gamma B} + {}^{3343}/_{6336}{\cdot}E_F$$

$$dE_{\Gamma H}/dt = Y_{\Gamma H} - E_{\Gamma H} - K_{\Gamma H}$$

$$dY_{AB}/dt = -{}^1/_9{\cdot}Y_{AH} + {}^{7z}/_8{\cdot}E_{AH} - {}^2/_3{\cdot}Y_{\Gamma H} + {}^z/_4{\cdot}E_{\Gamma H} - Y_{AB} - {}^4/_5{\cdot}Y_{\Gamma B} + {}^5/_8{\cdot}E_F$$

$$dE_{AB}/dt = Y_{AB} - E_{AB} - K_{AB}$$

$$dY_{\Gamma B}/dt = -{}^1/_9{\cdot}Y_{AH} + {}^{7(1-z)}/_8{\cdot}E_{AH} - {}^2/_3{\cdot}Y_{\Gamma H} + {}^{1-z}/_4{\cdot}E_{\Gamma H} - {}^1/_7{\cdot}Y_{1B} - Y_{\Gamma B} + {}^{3079}/_{6336}{\cdot}E_F$$

$$dE_{\Gamma B}/dt = Y_{\Gamma B} - E_{\Gamma B} - K_{\Gamma B}$$

$$dY_F/dt = {}^1/_8{\cdot}E_{1H} + {}^3/_4{\cdot}E_{\Gamma H} + {}^1/_6{\cdot}E_{1B} + {}^5/_6{\cdot}E_{\Gamma B} - Y_F$$

$$dE_F/dt = Y_F - E_F + K_F$$

(8.11) Zeta's characteristic polynomial

$$\begin{vmatrix} -1-\lambda & 0 & -^{2}/_{3} & 0 & -^{1}/_{7} & ^{5z}/_{6} & -^{4}/_{5} & ^{z}/_{6} & 0 & ^{4399}/_{6336} \\ 1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -^{1}/_{9} & 0 & -1-\lambda & 0 & -^{1}/_{7} & ^{5(1-z)}/_{6} & -^{4}/_{5} & ^{1-z}/_{6} & 0 & ^{3743}/_{6336} \\ 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ -^{1}/_{9} & ^{7z}/_{8} & -^{2}/_{3} & ^{z}/_{4} & -1-\lambda & 0 & -^{4}/_{5} & 0 & 0 & ^{5}/_{8} \\ 0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 \\ -^{1}/_{9} & ^{7(1-z)}/_{8} & -^{2}/_{3} & ^{1-z}/_{4} & -^{1}/_{7} & 0 & -1-\lambda & 0 & 0 & ^{3079}/_{6336} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 \\ 0 & ^{1}/_{8} & 0 & ^{3}/_{4} & 0 & ^{1}/_{6} & 0 & ^{5}/_{6} & -1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda \end{vmatrix} =$$

$$= \left| \mathbf{Z} - \boldsymbol{\lambda}\cdot\mathbf{I} \right| = \zeta_0\cdot\lambda^{10} + \zeta_1\cdot\lambda^9 + \zeta_2\cdot\lambda^8 + \zeta_3\cdot\lambda^7 + \zeta_4\cdot\lambda^6 + \zeta_5\cdot\lambda^5 + \zeta_6\cdot\lambda^4 + \zeta_7\cdot\lambda^3 + \zeta_8\cdot\lambda^2 + \zeta_9\cdot\lambda^1 + \zeta_{10}\cdot\lambda^0 \qquad (8.11)$$

where: $\zeta_0 = 1$; $\zeta_1 = 10$; $\zeta_2 = 41654/945$; $\zeta_3 = (865700 - 9789\cdot z)/7560$;

$\zeta_4 = (1038746571 - 45502688\cdot z - 2217600\cdot z^2)/5322240$;

$\zeta_5 = (1821496759 - 188034528\cdot z - 26122800\cdot z^2)/7983360$;

$\zeta_6 = (10590079853 - 1992093669\cdot z - 519583680\cdot z^2)/57480192$

$\zeta_7 = (14385023543 - 4211702364\cdot z - 1691236800\cdot z^2)/143700480$;

$\zeta_8 = (9828919691 - 4226441616\cdot z - 1888501500\cdot z^2)/287400960$;

$\zeta_9 = (455158841 - 295729923\cdot z - 73750950\cdot z^2)/71850240$;

$\zeta_{10} = (40614704 - 34550313\cdot z - 3909300\cdot z^2)/95800320$

(8.12) Minors of Zeta's characteristic polynomial

$$\begin{vmatrix} \zeta_1 & \zeta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \zeta_3 & \zeta_2 & \zeta_1 & \zeta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \zeta_5 & \zeta_4 & \zeta_3 & \zeta_2 & \zeta_1 & \zeta_0 & 0 & 0 & 0 & 0 \\ \zeta_7 & \zeta_6 & \zeta_5 & \zeta_4 & \zeta_3 & \zeta_2 & \zeta_1 & \zeta_0 & 0 & 0 \\ \zeta_9 & \zeta_8 & \zeta_7 & \zeta_6 & \zeta_5 & \zeta_4 & \zeta_3 & \zeta_2 & \zeta_1 & \zeta_0 \\ 0 & \zeta_{10} & \zeta_9 & \zeta_8 & \zeta_7 & \zeta_6 & \zeta_5 & \zeta_4 & \zeta_3 & \zeta_2 \\ 0 & 0 & 0 & \zeta_{10} & \zeta_9 & \zeta_8 & \zeta_7 & \zeta_6 & \zeta_5 & \zeta_4 \\ 0 & 0 & 0 & 0 & 0 & \zeta_{10} & \zeta_9 & \zeta_8 & \zeta_7 & \zeta_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_{10} & \zeta_9 & \zeta_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_{10} \end{vmatrix}$$

International economic system (8.10) is globally stable if, and only if, the Routh-Hurwitz determinants, made up of the coefficients of Zeta's characteristic polynomial $|\mathbf{Z} - \lambda \cdot \mathbf{I}|$, are greater than zero. The characteristic polynomial of (8.10) is provided by (8.11). Coefficients ζ_i, $i = 0, \ldots, 10$, of characteristic equation (8.11) are functions of z up to the second degree. By substituting the values of ζ_i in the ten minors of (8.12), we obtain Routh-Hurwitz determinants rh_n, $n = 1, \ldots, 10$, as polynomial functions of z up to the fourteenth degree. With *Mathematica*, it can be proven that the system of ten polynomial inequalities $rh_n > 0$ is solved by all $0 < z < 1$, and thus by all $0 < \alpha < 1$ and $0 < \gamma < 1$. Given that all rh_n are greater than zero, exchange network (8.10) is universally stable and, consequently, possesses a unique solution ${}^{\mathbf{Z}}\mathbf{X}({}^{Z}Y_{XH}, {}^{Z}E_{XH}, {}^{Z}Y_{XB}, {}^{Z}E_{XB}, {}^{Z}Y_{F}, {}^{Z}E_{F})$, $X = A, \Gamma$. Zeta returns to steady state ${}^{\mathbf{Z}}\mathbf{X}$ after any shock to i) sector flows $\mathbf{X}$, ii) behaviour coefficients $\mathbf{Z}$ and iii) credit shares $\mathbf{K_F}$. The fact that households and businesses are free to apportion their receipts and payments among Zeta's market goods without destabilising the international economy does not imply that all ratios $0 < z < 1$, $0 < \alpha < 1$ and $0 < \gamma < 1$ also yield sector flows which are greater than zero. It should be remembered that equilibrium vectors which incorporate negative revenue or outlay are unacceptable to those concerned. In such perverse situations, the exchange network constitutes a 'left-handed screw' system that cannot be regulated by means of uniform expansionary or contractionary policies.

8.4. Gains or losses from global trade and payments?

Next, we aim to compare the monetary flows of Alpha and Gamma, first as separate, and then as united, economies. When they were operating as two distinct networks, their Euclidean multipliers were 1.13264 and 1.22373, as shown by (8.3) and (8.6), respectively. With an eye on benefits from free trade and payments, Alpha and Gamma have agreed to fix their exchange rates at 1*IAU*/\$1 = 1*IAU*/€1. They have also agreed that their monetary financial institutions will expand national currency liabilities – and thus the assets of both countries' consumers and producers – by the same amount of 10^3 *IAUs* per year. On this basis, just before globalisation, their combined Euclidean multiplier was:

$$\begin{aligned} &Y^A + Y^\Gamma = E^A + E^\Gamma = R^A + R^\Gamma = 1.13264 \cdot \xi_A \cdot k_\alpha + 1.22373 \cdot \xi_\Gamma \cdot k_\gamma = c_0 \cdot K_F = \\ &= 1.13264 \cdot K_{AF} + 1.22374 \cdot K_{\Gamma F} = [(1.13264 + 1.22374)/2] \cdot 2 \cdot K_{XF} = (1.17818) \cdot K_F \end{aligned} \tag{8.13}$$

Permissible expansion $K_F = 2 \cdot 10^3$ *IAU* of Zeta's banking balance sheet is equally divided among consumers and producers of Alpha and Gamma as shown by (8.14):

$K_F = (\mu_{\alpha h} + \mu_{\gamma h} + \mu_{\alpha b} + \mu_{\gamma b}) \cdot K_F = (¼ + ¼ + ¼ + ¼)\cdot K_F = (½ + ½)\cdot K_F$ (8.14)

Preference ratio $0 < z < 1$ ($\alpha / \gamma = z / 1\text{-}z$) has taken effect from the moment the two countries or zones started operating as a single trade and payments area. Their joint exchange network (8.10) assumes the general form $\mathbf{dX/dt = Z \cdot X - K_F}$, which contains static equation system $\mathbf{Z} \cdot {}^{\mathbf{Z}}\mathbf{X} = \mathbf{K_F}$. With help from *Mathematica*, $\mathbf{Z} \cdot {}^{\mathbf{Z}}\mathbf{X} = \mathbf{K_F}$ can be solved using Cramer's rule because $|\mathbf{Z}| \neq 0$ for all $0 < z < 1$. Table (8.15) displays sector incomes and expenditures (${}^{Z}Y_{AH}$, ${}^{Z}E_{AH}$, ${}^{Z}Y_{AB}$, ${}^{Z}E_{AB}$, ${}^{Z}Y_{\Gamma H}$, ${}^{Z}E_{\Gamma H}$, ${}^{Z}Y_{\Gamma B}$, ${}^{Z}E_{\Gamma B}$, ${}^{Z}Y_F$, ${}^{Z}E_F$) as functions of preference ratio z and universal liquidity increment K_F. All sector flows have $\Omega = (-40614704 + 34550313 \cdot z + 3909300 \cdot z^2)$ as their denominator.

(8.15) Zeta's sector flows as functions of K_F and preference ratio z

Income of Alpha's Households =

${}^{Z}Y_{AH} = (27/4)\cdot(-1729728 + 1571741\cdot z + 8868\cdot z^2)\cdot K_F / \Omega$

Expenditure of Alpha's Households =

${}^{Z}E_{AH} = -(1/2)\cdot(3043976 - 3943347\cdot z + 1834932\cdot z^2)\cdot K_F / \Omega$

Income of Gamma's Households =

${}^{Z}Y_{\Gamma H} = 18\cdot(-460135 + 196357\cdot z + 232692\cdot z^2)\cdot K_F / \Omega$

Expenditure of Gamma's Households =

${}^{Z}E_{\Gamma H} = (7484984 - 20412609\cdot z + 12844524\cdot z^2)\cdot K_F / \Omega$

Income of Alpha's Businesses =

${}^{Z}Y_{AB} = (21/4)\cdot(-1667120 + 1210641\cdot z + 274485\cdot z^2)\cdot K_F / \Omega$

Expenditure of Alpha's Businesses =

${}^{Z}E_{AB} = (1/4)\cdot(5605184 - 9126852\cdot z + 1854885\cdot z^2)\cdot K_F / \Omega$

Income of Gamma's Businesses =

${}^{Z}Y_{\Gamma B} = 45\cdot(1152736 - 1230513\cdot z + 119225\cdot z^2)\cdot K_F / 4\cdot\Omega$

Expenditure of Gamma's Businesses =

${}^{Z}E_{\Gamma B} = -(1/4)\cdot(11258416 - 20822772\cdot z + 9274425\cdot z^2)\cdot K_F / \Omega$

Income of International Financiers =

${}^{Z}Y_F = -(1/4)\cdot(-898768 + 376887\cdot z + 438780\cdot z^2)\cdot K_F / \Omega$

Expenditure of International Financiers =

${}^{Z}E_F = 14256\cdot(-2912 + 2450\cdot z + 305\cdot z^2)\cdot K_F / \Omega$

Using *Mathematica*, it can be proven that revenues and outlays of Zeta's five sectors are greater than zero (i.e. ${}^{Z}Y_{XH} > 0$, ${}^{Z}E_{XH} > 0$, ${}^{Z}Y_{XB} > 0$, ${}^{Z}E_{XB} > 0$; ${}^{Z}Y_F > 0$, ${}^{Z}E_F > 0$) within the range $0.719291 < z < 0.90744$. We deduce that households, businesses and financiers of Alpha and Gamma enjoy positive monetary flows only for α and γ within the boundaries $0.719291 < \alpha < 0.90744$ and $0.09256 < \gamma < 0.280709$. As z rises and 1-z falls, the joint economy's consumers and producers increasingly prefer the labour and output of Alpha; monetary circulation is redistributed against the sectors and markets of Gamma. Consequently, z determines whether the monetary flows of Zeta's households, businesses and financiers become smaller, equal to, or larger than, those prevailing when Alpha and Gamma were operating as separate economies. Takings and outgoings of Zeta's five sectors, shown by (8.15), are summed up in (8.16). The net gain or loss depends on the combined Euclidean multiplier of the two economies before and after globalisation, i.e. c_0 and c_z provided by (8.13) and (8.16), respectively.

$$
\begin{aligned}
{}^{Z}\mathrm{Y} &= {}^{Z}Y_{AH} + {}^{Z}Y_{\Gamma H} + {}^{Z}Y_{AB} + {}^{Z}Y_{\Gamma B} + {}^{Z}Y_{F} = \\
&= (13\cdot(-13100776 + 10682985\cdot z + 1472880\cdot z^{2}) / 4\cdot\Omega)\cdot \mathrm{K_F} = c_z \cdot \mathrm{K_F} \qquad (8.16)\\
&= {}^{Z}E_{AH} + {}^{Z}E_{\Gamma H} + {}^{Z}E_{AB} + {}^{Z}E_{\Gamma B} + {}^{Z}E_{F} = {}^{Z}\mathrm{E}
\end{aligned}
$$

Relationship (8.16) illustrates that as elasticity rate z goes from $z \approx 0$ ($\alpha \approx 0$ and $\gamma \approx 1$) to $z \approx 1$ ($\alpha \approx 1$ and $\gamma \approx 0$), the value of Zeta's Euclidean multiplier c_z varies from $c_z \min = 1.04833$ to $c_z \max = 1.42498$. By relating the initial value c_0 – estimated by means of (8.13) – to the lowest and highest limit of c_z – estimated by means of (8.16) – we conclude that merger of two or more economies is not necessarily accompanied by greater turnover for all sectors and countries, if the universal liquidity increment K_F is kept constant. Shifting preferences from low-end goods (e.g. those produced by Portugal, Italy, Spain and Greece) to high-end goods (e.g. those produced by Germany) may be one of the causes of diverging growth rates among EU economies during the 2007 - 2015 crises. Net cost or benefit emanating from globalisation is ultimately distributed among sovereign countries and currency zones in accordance with the choices of households, businesses, governments and financiers regarding their worldwide sources of income and destinations of expenditure. For example, assuming $z = {}^{72}/_{100}$, we can compare the monetary flows of Alpha and Gamma before and after the formation of an international network.

$$
\begin{aligned}
{}^{Z}\mathrm{Y} &= ({}^{Z}Y_{AH} + {}^{Z}Y_{\Gamma H} + {}^{Z}Y_{AB} + {}^{Z}Y_{\Gamma B} + {}^{Z}Y_{F}) = \\
&= (0.292153 + 0.260091 + 0.250083 + 0.269579 + 0.0291678)\cdot \mathrm{K_F} = (1.10108)\cdot \mathrm{K_F} \qquad (8.17)\\
{}^{Z}\mathrm{E} &= ({}^{Z}E_{AH} + {}^{Z}E_{\Gamma H} + {}^{Z}E_{\Gamma B} + {}^{Z}E_{\Gamma B} + {}^{Z}E_{F}) = \\
&= (0.042153 + 0.010091 + 0.00008345 + 0.0195794 + 1.02917)\cdot \mathrm{K_F} = (1.10108)\cdot \mathrm{K_F}
\end{aligned}
$$

Replacing z with $^{72}/_{100}$ in (8.16), we derive equations (8.17) which show Zeta's receipts and payments as functions of combined credit increment K_F. Evidently, the Euclidean multiplier 1.10108 of Zeta is smaller than the multiplier 1.13264 of Alpha and the multiplier 1.22374 of Gamma when they were operating separately. With the universal liquidity increment K_F unchanged, merger of two or more economies is not inevitably accompanied by greater turnover for all sectors and markets. It remains an open question whether this conclusion implies that, under similar circumstances, some EU and Eurozone member countries would be better off by returning to their previous status.

8.5. Distributions of steady-state flows

As soon as commercial and banking agreements link Alpha and Gamma, their constituent sectors begin supplying to and demanding from each other real goods and financial paper. Under the assumption $z = {}^{72}/_{100}$, international network (8.10) is globally stable. Zeta's gross takings ${}^{Z}Y$ and outgoings ${}^{Z}E$ – displayed by (8.17) – are apportioned among consumers h, producers b and the combined banking sector f of Alpha α and Gamma γ, as shown below:

(8.18) Division of international receipts and payments among sectors and countries

$$(y_{\alpha h}+y_{\gamma h}) + (y_{\alpha b}+y_{\gamma b}) + y_f = y_h + y_b + y_f = (26.5\%+23.6\%) + (22.8\%+24.5\%) + 2.6\% = 100\%$$

$$(y_{\alpha h}+y_{\alpha b}) + (y_{\gamma h}+y_{\gamma b}) + y_f = y_\alpha + y_\gamma + y_f = (26.5\%+22.8\%) + (23.6\%+24.5\%) + 2.6\% = 100\%$$

$$(e_{\alpha h}+e_{\gamma h}) + (e_{\alpha b}+e_{\gamma b}) + e_f = e_h + e_b + e_f = (3.8\%+0.9\%) + (0.1\%+1.8\%) + 93.4\% = 100\%$$

$$(e_{\alpha h}+e_{\alpha b}) + (e_{\gamma h}+e_{\gamma b}) + e_f = e_\alpha + e_\gamma + e_f = (3.8\%+0.1\%) + (0.9\%+1.8\%) + 93.4\% = 100\%$$

It is noted that households and businesses, respectively, earn 50.1% and 47.3% of global income but realize only 4.7% and 1.9% of global expenditure; they are the savers of the economic system. Looking at the same figures from another angle, we see that Alpha and Gamma, respectively, bring in 49.3% and 48.1% of international receipts but run through only 3.8% and 2.7% of international disbursements. Conversely, bankers get only 2.6% of revenue but spend 93.4% of outlay due to their right to acquire services, products and securities by paying with their own liabilities world-wide. By being profligate, they 'balance' the universal exchange network. Among other things, the international banking system deals with agents, sectors, countries and zones by i) handling gaps between their receipts and payments and by ii) keeping record of their loans and deposits. Table (8.19) displays the monetary flows of Alpha and Gamma (when $K_{AF} = K_{\Gamma F} = 10^3$ *IAU* and thus $K_F = 2{\cdot}10^3$ *IAU*) before ($c_0 = 1.17818$) and after ($c_z = 1.10108$) integration.

(8.19) Comparison of sector flows before and after globalisation

IAUs	SEPARATE ECONOMIES			UNITED ECONOMIES		
SECTORS	**Alpha**	**Gamma**	**TOTAL**	**Alpha**	**Gamma**	**TOTAL**
Households	593.42	574.58	1 168.00	584.31	520.18	1 104.49
Businesses	523.62	550.85	1 074.47	500.17	539.16	1 039.33
Sub-total	*1 117.04*	*1 125.43*	2 242.47	*1 084.48*	*1059.34*	2 143.82
Financiers	15.61	98.31	113.92	-	-	58.34
Income	1 132.65	1 223.74	**2 356.39**			**2 202.16**
Households	93.42	74.58	168.00	84.31	20.18	104.49
Businesses	23.62	50.85	74.47	0.17	39.16	39.33
Sub-total	*117.04*	*125.43*	242.47	*84.48*	*59.34*	143.82
Financiers	1 015.61	1 098.31	2 113.92	-	-	2 058.34
Expenditure	1 132.65	1 223.74	**2 356.39**			**2 202.16**

From the relevant lines of Tables (8.7) and (8.9), we derive annual expenditures on services and products supplied by Alpha and Gamma as well as on securities issued by all sectors:

$$
\begin{aligned}
{}^{Z}R_{AW} &= {}^{5\alpha}/_{6}\cdot{}^{Z}E_{AB} + {}^{(1-\gamma)}/_{6}\cdot{}^{Z}E_{\Gamma B} + {}^{1}/_{4}\cdot{}^{Z}E_{F} = {}^{5z}/_{6}\cdot{}^{Z}E_{AB} + {}^{z}/_{6}\cdot{}^{Z}E_{\Gamma B} + {}^{1}/_{4}\cdot{}^{Z}E_{F} \\
{}^{Z}R_{\Gamma W} &= {}^{5(1-\alpha)}/_{6}\cdot{}^{Z}E_{AB} + {}^{\gamma}/_{6}\cdot{}^{Z}E_{\Gamma B} + {}^{1}/_{12}\cdot{}^{Z}E_{F} = {}^{5(1-z)}/_{6}\cdot{}^{Z}E_{AB} + {}^{(1-z)}/_{6}\cdot{}^{Z}E_{\Gamma B} + {}^{1}/_{12}\cdot{}^{Z}E_{F} \\
{}^{Z}R_{AU} &= {}^{7\alpha}/_{8}\cdot{}^{Z}E_{AH} + {}^{(1-\gamma)}/_{4}\cdot{}^{Z}E_{\Gamma H} + {}^{1145}/_{6336}\cdot{}^{Z}E_{F} = {}^{7z}/_{8}\cdot{}^{Z}E_{AH} + {}^{z}/_{4}\cdot{}^{Z}E_{\Gamma H} + {}^{1145}/_{6336}\cdot{}^{Z}E_{F} \qquad (8.20) \\
{}^{Z}R_{\Gamma U} &= {}^{7(1-\alpha)}/_{8}\cdot{}^{Z}E_{AH} + {}^{\gamma}/_{4}\cdot{}^{Z}E_{\Gamma H} + {}^{1}/_{24}\cdot{}^{Z}E_{F} = {}^{7(1-z)}/_{8}\cdot{}^{Z}E_{AH} + {}^{(1-z)}/_{4}\cdot{}^{Z}E_{\Gamma H} + {}^{1}/_{24}\cdot{}^{Z}E_{F} \\
{}^{Z}R_{FS} &= {}^{1}/_{8}\cdot{}^{Z}E_{AH} + {}^{3}/_{4}\cdot{}^{Z}E_{\Gamma H} + {}^{1}/_{6}\cdot{}^{Z}E_{AB} + {}^{5}/_{6}\cdot{}^{Z}E_{\Gamma B} + {}^{2815}/_{6336}\cdot{}^{Z}E_{F}
\end{aligned}
$$

Setting $z = {}^{72}/_{100}$ in system (8.20), we get Zeta's gross turnover ${}^{Z}R$ as linear function of universal liquidity increment K_F:

$$
\begin{aligned}
&({}^{Z}R_{AW} + {}^{Z}R_{\Gamma W} + {}^{Z}R_{AU} + {}^{Z}R_{\Gamma U} + {}^{Z}R_{FS}) \approx \\
&\approx (0.2597 + 0.0867 + 0.2144 + 0.0539 + 0.4864)\cdot K_F \approx 1.10108\cdot K_F \qquad (8.21)
\end{aligned}
$$

As reflected by (8.17) and (8.21), the Euclidean multiplier of the combined network is 1.10108. Equations (8.22) show the apportionments of Zeta's annual sales among services and products of Alpha and Gamma as well as securities issued by all institutional sectors:

(8.22) Division of international turnover among markets and countries

$$(\rho_{\alpha w}+\rho_{\gamma w}) + (\rho_{\alpha u}+\rho_{\gamma u}) + \rho_{fs} = \rho_{w} + \rho_{u} + \rho_{fs} = (23.6\%+7.9\%) + (19.4\%+4.9\%) + 44.2\% = 100\%$$

$$(\rho_{\alpha w}+\rho_{\alpha u}) + (\rho_{\gamma w}+\rho_{\gamma u}) + \rho_{fs} = \rho_{\alpha} + \rho_{\gamma} + \rho_{fs} = (23.6\%+19.4\%) + (7.9\%+4.9\%) + 44.2\% = 100\%$$

It may be observed that services and products, respectively, make up 31.5% and 24.3% of the world-wide economy's turnover. Although equally endowed, Alpha and Gamma originate 43.0% and 12.8%, respectively, of real goods sold internationally. Financial securities take up the remaining 44.2% of the trading total. Table (8.23) illustrates that – in case $K_F = K_{AF} + K_{\Gamma F} = K_A + K_\Gamma = 2 \cdot 10^3$ *IAU* – integration reduces the annual flows of both sovereign countries while redistributing sales of labour and output in favour of Alpha and against Gamma.

(8.23) Comparison of market flows before and after globalisation

IAUs	**CLOSED ECONOMIES**			**OPEN ECONOMIES**		
TURNOVER	**Alpha**	**Gamma**	**Total**	**Alpha**	**Gamma**	**Total**
Labour A	527.48	-	527.48	519.38	-	519.38
Output A	448.81	-	448.81	428.72	-	428.72
Sub-total A	***976.29***			***948.10***		
Labour Γ	-	191.53	191.53	-	173.40	173.40
Output Γ	-	110.17	110.17	-	107.83	107.83
Sub-total Γ		***301.70***			***281.23***	
Liquidity R_{AS}	156.35	-	-	-	-	-
Liquidity $R_{\Gamma S}$	-	922.03	-	-	-	-
Sub-total F	-	-	1 078.38	-	-	972.83
Annual Sales	1 132.65	1 223.74	***2 356.39***			***2 202.16***

In case $z = {}^{72}/_{100}$ and $K_{AF} = K_{\Gamma F} = 1000$ *IAU*, Table (8.24) shows that Alpha would import (0.04 + 20.66) *IAU* = 20.70 *IAU* worth of labour and output from Gamma, while Gamma would import only (4.70 + 3.64) *IAU* = 8.34 *IAU* worth of labour and output from Alpha. Deficits are always equal to surpluses as far as trade is concerned. Owing to the twelve major propositions, transactions among sectors and markets as well as countries and zones are recorded by the global banking system according to double-entry accounting principles.

(8.24) Demand and supply by sector, market and country after globalisation

OUTLAY/ /SALES	**Consumers**		**Producers**		**International**	***IAUs***
	ALPHA	**GAMMA**	**ALPHA**	**GAMMA**	**Bankers**	**TOTAL**
Labour A	-	-	0.10	4.70	514.58	519.38
Labour Γ	-	-	0.04	1.83	171.53	173.40
Sub-total W	-	-				
Output A	53.11	3.64	-	-	371.97	428.72
Output Γ	20.66	1.41	-	-	85.76	107.83
Sub-total U						
Liquidity S_F	10.54	15.14	00.03	32.63	914.49	972.83
TOTAL R						**2 202.16**

Just after the union of Alpha and Gamma, the applicable scale factor (Euclidean multiplier) diminishes from 1.17818 to 1.10108; the resulting monetary flow changes are displayed by Tables (8.19), (8.23) and (8.24). Within the time unit, services, products and securities are quoted autonomously, irrespective of quantity gaps. Annual sales of any traded good (${}^{z}R_{AW}$, ${}^{z}R_{AU}$, ${}^{z}R_{\Gamma W}$, ${}^{z}R_{\Gamma U}$, ${}^{z}R_{FS}$) (measured in *IAU*s) divided by its average quotation (P_{AW}, $P_{\Gamma W}$, P_{AU}, $P_{\Gamma U}$, P_{FS}) (measured in *IAU*s) yields the quantity demanded (${}^{z}q_{\alpha w}$, ${}^{z}q_{\gamma w}$, ${}^{z}q_{\alpha u}$, ${}^{z}q_{\gamma u}$, ${}^{z}q_{fs}$) per year. Deficits or surpluses of labour, output and liquidity as well as likely rises or falls of salaries, prices and values are estimated by comparing demanded with obtainable amounts (${}^{\#}q_{\alpha w}$, ${}^{\#}q_{\gamma w}$, ${}^{\#}q_{\alpha u}$, ${}^{\#}q_{\gamma u}$, ${}^{\#}q_{fs}$). In this way, mathematical economists are able to:

- relate the nominal with the real magnitudes of an exchange network;
- render advice about the marginal financing of sectors and countries.

The distributions of sector and country payments among market goods combined with the allocations of bank loans among sectors and countries, decide the net effects which accompany integration of two or more economies into a free trade and payments area. Without a Euclidean mathematical model, it would be impossible to predict how much two or more sovereign countries would gain or lose by becoming members of a common market with joint currency rules. Modern software provides the indispensable tools for exploring the effects of behaviour coefficients and economic policies on sector, market, country and zone flows in relation to the prevailing quotations and available quantities.

8.6. Aspects of international economic coordination

Ideally, every banking system should determine the rates of growth, discount and exchange applicable to its assets and liabilities, without reference to another country's monetary financial institutions. As discussed in Section 7.8, lenders and borrowers realise that they cannot escape from interdependence in general and the Uncovered Interest Parity Principle in particular. Under freedom of current payments and capital movements, one unit of international liquidity per annum should have the same present value irrespective of the currency in which it is received or paid. In the long-run, quotations become as flexible as necessary to restore equilibrium. Any protracted effort to attract (repel) capital by offering a higher (lower) interest rate relative to other countries is doomed to failure because of the unavoidable devaluation (revaluation) of the currency. Monetary authorities understand that liquidity growth, annual discount and foreign exchange rates must be conducive to portfolio balance and economic stability world-wide. Banking systems have no choice but to jointly manage issues of additional deposits and acceptances of new loans. In view of this inescapable reality, the monetary financial institutions of Alpha and Gamma have concurred to expand, allocate and evaluate their assets and liabilities in accordance with the recommendations of IFO and Euclidean modelling methodology. Their ultimate goal is to:

- direct sectors, markets and countries toward stable and unique equilibrium vectors consisting of positive receipts and payments;
- minimise quantity deficits and surpluses, as well as potential inflation or deflation in all markets and countries, simultaneously.

Like all Euclidean models, worldwide exchange network (8.10) assumes the general form $\mathbf{dX/dt = Z \cdot X - K_F}$. Alternative equilibrium vectors $^{\mathbf{Z}}\mathbf{X}$ may be meaningfully compared provided the dynamic system that generates them is globally stable. International economic system $\mathbf{dX/dt = Z \cdot X - K_F}$ is universally stable if, and only if, the Routh-Hurwitz determinants rh_n, n = 1, ... , 10, consisting of the coefficients of characteristic polynomial $|\mathbf{Z} - \lambda \cdot \mathbf{I}|$, are greater than zero. When z = $^{72}/_{100}$, the characteristic polynomial of worldwide economy Zeta is given by (8.25).

(8.25) Zeta's characteristic polynomial

$$\begin{vmatrix} -1-\lambda & 0 & -^2/_3 & 0 & -^1/_7 & ^3/_5 & -^4/_5 & ^3/_{25} & 0 & ^{4399}/_{6336} \\ 1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -^1/_9 & 0 & -1-\lambda & 0 & -^1/_7 & ^7/_{30} & -^4/_5 & ^7/_{150} & 0 & ^{3343}/_{6336} \\ 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ -^1/_9 & ^{63}/_{100} & -^2/_3 & ^9/_{50} & -1-\lambda & 0 & -^4/_5 & 0 & 0 & ^5/_8 \\ 0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 \\ -^1/_9 & ^{49}/_{200} & -^2/_3 & ^7/_{100} & -^1/_7 & 0 & -1-\lambda & 0 & 0 & ^{3079}/_{6336} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 \\ 0 & ^1/_8 & 0 & ^3/_4 & 0 & ^1/_6 & 0 & ^5/_6 & -1-\lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda \end{vmatrix} = |\mathbf{Z} - \lambda \cdot \mathbf{I}|$$

$$= \zeta_0 \cdot \lambda^{10} + \zeta_1 \cdot \lambda^9 + \zeta_2 \cdot \lambda^8 + \zeta_3 \cdot \lambda^7 + \zeta_4 \cdot \lambda^6 + \zeta_5 \cdot \lambda^5 + \zeta_6 \cdot \lambda^4 + \zeta_7 \cdot \lambda^3 + \zeta_8 \cdot \lambda^2 + \zeta_9 \cdot \lambda + \zeta_{10} \qquad (8.25)$$

With *Mathematica*'s assistance, computation of coefficients ζ_i, i = 0,…, 10, yields:

$\zeta_0 = 1$; $\zeta_1 = 10$; $\zeta_2 = {}^{41654}/_{945}$; $\zeta_3 = {}^{1533307}/_{13500}$; $\zeta_4 = {}^{1674725053}/_{8870400}$;

$\zeta_5 = {}^{418142459 83}/_{19958400}$; $\zeta_6 = {}^{1110802528951}/_{7185024000}$; $\zeta_7 = {}^{52379303419}/_{718502400}$;

$\zeta_8 = {}^{145172063747}/_{7185024000}$; $\zeta_9 = {}^{5100020099}/_{1796256000}$; $\zeta_{10} = {}^{171398719}/_{1197504000}$.

By placing the values of coefficients ζ_i in the minors of (8.12), we discover that $rh_1 = 10 > 0$; $rh_2 = 327.205 > 0$; $rh_3 = 20378.5 > 0$; $rh_4 = 1.65946 \text{x} 10^6 > 0$; $rh_5 = 1.30546 \text{x} 10^8 > 0$; $rh_6 = 7.52682 \text{x} 10^9 > 0$; $rh_7 = 2.35717 \text{x} 10^{11} > 0$; $rh_8 = 2.6132 \text{x} 10^{12} > 0$; $rh_9 = 5.45225 \text{x} 10^{12} > 0$; $rh_{10} = 7.80381 \text{x} 10^{11} > 0$. One should bear in mind that a globally-stable linear differential equation system is associated with a single equilibrium vector because the determinant of its structural matrix is always positive and, therefore, different from zero. This implies that the exchange network will return to a unique steady state $^{\mathbf{Z}}\mathbf{X}$ after any rearrangement of initial flows $^{\mathbf{0}}\mathbf{X}(^0Y_{AH}, {}^0E_{AH}; {}^0Y_{\Gamma H}, {}^0E_{\Gamma H}; {}^0Y_{AB}, {}^0E_{AB}; {}^0Y_{\Gamma B}, {}^0E_{\Gamma B}; {}^0Y_F, {}^0E_F)$ and / or any modification of credit policies $\mathbf{K_F}(K_F; \mu_{\alpha h}, \mu_{\gamma h}, \mu_{\alpha b}, \mu_{\gamma b})$. Fulfilment of stability $rh_n > 0$, $n = 1, \ldots, 10$, and uniqueness $|\mathbf{Z}| \neq 0$ conditions does not entail that equilibrium revenues and outlays – of Zeta's households, businesses and financiers – are greater than zero and, hence, controllable in the same direction.

(8.26) Zeta's simultaneous linear equation system

$$\begin{vmatrix} -1 & 0 & -^2/_3 & 0 & -^1/_7 & ^3/_5 & -^4/_5 & ^3/_{25} & 0 & ^{4399}/_{6336} \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -^1/_9 & 0 & -1 & 0 & -^1/_7 & ^7/_{30} & -^4/_5 & ^7/_{150} & 0 & ^{3343}/_{6336} \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -^1/_9 & ^{63}/_{100} & -^2/_3 & ^9/_{50} & -1 & 0 & -^4/_5 & 0 & 0 & ^5/_8 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -^1/_9 & ^{49}/_{200} & -^2/_3 & ^7/_{100} & -^1/_7 & 0 & -1 & 0 & 0 & ^{3079}/_{6336} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & ^1/_8 & 0 & ^3/_4 & 0 & ^1/_6 & 0 & ^5/_6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} ^ZY_{1H} \\ ^ZE_{1H} \\ ^ZY_{2H} \\ ^ZE_{2H} \\ ^ZY_{1B} \\ ^ZE_{1B} \\ ^ZY_{2B} \\ ^ZE_{2B} \\ ^ZY_{1F} \\ ^ZE_{1F} \end{vmatrix} = \mathbf{Z} \cdot \mathbf{X} =$$

(8.26)

$$= \begin{vmatrix} 0 \\ \xi_A \cdot k_{\alpha f\alpha h} + \xi_\Gamma \cdot k_{\gamma f\alpha h} \\ 0 \\ \xi_A \cdot k_{\alpha f\gamma h} + \xi_\Gamma \cdot k_{\gamma f\gamma h} \\ 0 \\ \xi_A \cdot k_{\alpha f\alpha b} + \xi_\Gamma \cdot k_{\gamma f\alpha b} \\ 0 \\ \xi_A \cdot k_{\alpha f\gamma b} + \xi_\Gamma \cdot k_{\gamma f\gamma b} \\ 0 \\ -K_F \end{vmatrix} = \begin{vmatrix} 0 \\ (\varphi_\alpha \cdot \kappa_{\alpha f\alpha h} + \varphi_\gamma \cdot \kappa_{\gamma f\alpha h}) \cdot K_F \\ 0 \\ (\varphi_\alpha \cdot \kappa_{\alpha f\gamma h} + \varphi_\gamma \cdot \kappa_{\gamma f\gamma h}) \cdot K_F \\ 0 \\ (\varphi_\alpha \cdot \kappa_{\alpha f\alpha b} + \varphi_\gamma \cdot \kappa_{\gamma f\alpha b}) \cdot K_F \\ 0 \\ (\varphi_\alpha \cdot \kappa_{\alpha f\gamma b} + \varphi_\gamma \cdot \kappa_{\gamma f\gamma b}) \cdot K_F \\ 0 \\ -K_F \end{vmatrix} = \begin{vmatrix} 0 \\ K_{AH} \\ 0 \\ K_{\Gamma H} \\ 0 \\ K_{AB} \\ 0 \\ K_{\Gamma B} \\ 0 \\ -K_F \end{vmatrix} = \begin{vmatrix} 0 \\ \mu_{\alpha h} \\ 0 \\ \mu_{\gamma h} \\ 0 \\ \mu_{\alpha b} \\ 0 \\ \mu_{\gamma b} \\ 0 \\ -1 \end{vmatrix} \cdot K_F = \mathbf{K_F}$$

Intending to explore further the consequences of competition and cooperation between Alpha and Gamma, we have solved linear equation system (8.26). Table (8.27) displays the takings and outgoings of Zeta's sectors as functions of universal liquidity increment K_F and allocation of new loans μ_{ix} among consumers and producers world-wide.

(8.27) Zeta's sector flows as functions of K_F and $\mu_{\alpha h}$, $\mu_{\gamma h}$ and $\mu_{\alpha b}$

Income of Alpha's Households =

$${}^{z}Y_{AH} = 81\cdot(2191208 + 8764832\cdot\mu_{\alpha b} + 35665807\cdot\mu_{\alpha h} + 8625082\cdot\mu_{\gamma h})\cdot K_F / 4284967975$$

Expenditure of Alpha's Households =

$${}^{z}E_{AH} = 2\cdot(88743924 + 354975696\cdot\mu_{\alpha b} - 698018804\cdot\mu_{\alpha h} + 349315821\cdot\mu_{\gamma h})\cdot K_F / 4284967975$$

Income of Gamma's Households =

$${}^{z}Y_{\Gamma H} = 12\cdot(10614836 + 42459344\cdot\mu_{\alpha b} + 3228789469\cdot\mu_{\alpha b} + 57786394\cdot\mu_{\gamma b})\cdot K_F / 4284967975$$

Expenditure of Gamma's Households =

$${}^{z}E_{\Gamma H} = (127378032 + 509512128\cdot\mu_{\alpha b} + 2745473628\cdot\mu_{\alpha h} - 3591531247\cdot\mu_{\gamma h})\cdot K_F / 4284967975$$

Income of Alpha's Businesses =

$${}^{z}Y_{AB} = 7\cdot(12686278 + 50745112\cdot\mu_{\alpha b} + 23364387\cdot\mu_{\alpha b} - 2386088\cdot\mu_{\gamma b})\cdot K_F / 856993595$$

Expenditure of Alpha's Businesses =

$${}^{z}E_{AB} = (88803946 - 501777811\cdot\mu_{\alpha B} + 163550709\cdot\mu_{\alpha H} - 16702612\cdot\mu_{\gamma H})\cdot K_F / 856993595$$

Income of Gamma's Businesses =

$${}^{z}Y_{\Gamma B} = 5\cdot(4363377 + 17453508\cdot\mu_{\alpha b} + 4024324\cdot\mu_{\alpha h} - 1966891\cdot\mu_{\gamma h})\cdot K_F / 171398719$$

Expenditure of Gamma's Businesses =

$${}^{z}E_{\Gamma B} = (-149581834 + 258666259\cdot\mu_{\alpha b} + 191520339\cdot\mu_{\alpha h} + 161564264\cdot\mu_{\gamma h})\cdot K_F / 171398719$$

Income of International Financiers =

$${}^{z}Y_{F} = (-584913083 + 1088322048\cdot\mu_{\alpha b} + 1202179968\cdot\mu_{\alpha h} + 149136768\cdot\mu_{\gamma h})\cdot K_F / 856993595$$

Expenditure of International Financiers =

$${}^{z}E_{F} = 38016\cdot(7157 + 28628\cdot\mu_{\alpha b} + 31623\cdot\mu_{\alpha h} + 3923\cdot\mu_{\gamma h})\cdot K_F / 856993595$$

Given that $K_F > 0$, equations (8.27) may be considered as a system of ten inequalities in terms of $\mu_{\alpha h}$, $\mu_{\gamma h}$, $\mu_{\alpha b}$ or $\mu_{\gamma b}$. Table (8.28) presents the ranges of country and sector credit shares – in the neighbourhood of $\mu_{\alpha h} = \mu_{\gamma h} = \mu_{\alpha b} = \mu_{\gamma b} = ¼$ – which yield ${}^{z}Y_{XH} > 0$, ${}^{z}E_{XH} > 0$, ${}^{z}Y_{XB} > 0$, ${}^{z}E_{XB} > 0$, ${}^{z}Y_{F} > 0$, ${}^{z}E_{F} > 0$.

(8.28) Credit share limits within which monetary flows remain positive

ALPHA ($\mu_{\alpha h} + \mu_{\alpha b}$)				**GAMMA ($\mu_{\gamma h} + \mu_{\gamma b}$)**			
Min 48.66%		**Max** 62.95%		**Min** 11.62%		**Max** 28.42%	
$\mu_{\alpha h}$		**$\mu_{\alpha b}$**		**$\mu_{\gamma h}$**		**$\mu_{\gamma b}$**	
min	max	min	max	min	max	min	Max
24.96%	37.94%	23.70%	25.01%	22.92%	25.43%	11.62%	28.42%

Other things being equal, households, businesses and financiers enjoy positive revenues and outlays as long as the accepting sectors of Alpha receive more than 48.66% but less than 62.95%, while those of Gamma received more than 11.62% but less than 28.42% of the annual liquidity increment K_F. Normally, banking institutions apportion credit among borrowers taking into account a number of factors, which include collateral security and productive potential. World monetary authorities should give top priority to national capacities for additional employment and production when considering the annual expansion and distribution of the banking system balance sheet. With *Mathematica*, we substitute sector flows (8.27) in market system (8.20) to derive equations (8.29); they show that the yearly sales of services, products and securities are functions of the universal liquidity increment and credit allocation in terms of *IAUs*:

$$\begin{aligned}
{}^zR_{AW} &= 72\cdot(2191208 + 8764832\cdot\mu_{\alpha b} + 35665807\cdot\mu_{\alpha h} + 8625082\cdot\mu_{\gamma h})\cdot K_F / 4284967975 \\
{}^zR_{\Gamma W} &= 4\cdot(10614836 + 42459344\cdot\mu_{\alpha b} + 228789469\cdot\mu_{\alpha h} + 57786394\cdot\mu_{\gamma h})\cdot K_F / 4284967975 \\
{}^zR_{AU} &= 6\cdot(12686278 + 50745112\cdot\mu_{\alpha b} + 23364387\cdot\mu_{\alpha h} - 2386088\cdot\mu_{\gamma h})\cdot K_F / 856993595 \\
{}^zR_{\Gamma U} &= (4363377 + 17453508\cdot\mu_{\alpha b} + 4024324\cdot\mu_{\alpha h} - 1966891\cdot\mu_{\gamma h})\cdot K_F / 171398719 \\
{}^zR_{FS} &= (-464031353 + 1571848968\cdot\mu_{\alpha b} + 1736292438\cdot\mu_{\alpha h} + 215396238\cdot\mu_{\gamma h})\cdot K_F / 856993595
\end{aligned} \tag{8.29}$$

Annual demand ${}^zR_{XJ}$ for market good XJ is by definition equal to the average quotation P_{XJ} (in *IAUs*) times the quantity traded q_{xj}. Replacing $K_F = 2x10^9$ *IAUs* and $\mu_{\alpha h} = \mu_{\gamma h} = \mu_{\alpha b} = \mu_{\gamma b} = ¼$ in (8.29), we derive equations (8.30) which show steady-state payments on labour and output sold by Alpha and Gamma as well as on capital borrowed by all sectors in *IAU*:

$$\begin{aligned}
{}^zR_{AW} &= 519.383 \cdot 10^6\, IAUs = P_{AW} \cdot {}^zq_{\alpha w} \\
{}^zR_{\Gamma W} &= 173.394 \cdot 10^6\, IAUs = P_{\Gamma W} \cdot {}^zq_{\gamma w} \\
{}^zR_{AU} &= 428.714 \cdot 10^6\, IAUs = P_{AU} \cdot {}^zq_{1u} \\
{}^zR_{\Gamma U} &= 107.832 \cdot 10^6\, IAUs = P_{\Gamma U} \cdot {}^zq_{\gamma u} \\
{}^zR_{FS} &= 972.827 \cdot 10^6\, IAUs = P_{FS} \cdot {}^zq_{fs}
\end{aligned} \tag{8.30}$$

(8.31) Average quotations and available quantities

P_{AW} = 500.00 *IAUs*	${}^{\#}q_{\alpha w} = 1\,000\cdot10^3$ years of Alpha labour
$P_{\Gamma W}$ = 200.00 *IAUs*	${}^{\#}q_{\gamma w} = 1\,000\cdot10^3$ years of Gamma labour
P_{AU} = 4.00 *IAUs*	${}^{\#}q_{\alpha u} = 100\cdot10^6$ pieces of Alpha output
$P_{\Gamma U}$ = 1.00 *IAUs*	${}^{\#}q_{\gamma u} = 100\cdot10^6$ pieces of Gamma output
P_{FS} = 10.0 *IAUs*	${}^{\#}q_{\alpha s} = 50\cdot10^6$ Alpha liquidity consols
P_{FS} = 10.0 *IAUs*	${}^{\#}q_{\gamma s} = 50\cdot10^6$ Gamma liquidity consols

Table (8.31) displays average quotations and available quantities in Zeta. Set (8.32) shows the amounts traded ${}^{z}q_{xj}$ obtained by dividing annual sales ${}^{z}R_{XJ}$ by average quotations P_{XJ}.

$$
\begin{aligned}
{}^{z}q_{\alpha w} &= 519.38 \cdot 10^{6}\, IAUs / 500\, IAUs = 1\,038.770 \cdot 10^{3} \text{ years of Alpha labour;}\\
{}^{z}q_{\gamma w} &= 173.39 \cdot 10^{6}\, IAUs / 200\, IAUs = 866.97 \cdot 10^{3} \text{ years of Gamma labour;}\\
{}^{z}q_{\alpha u} &= 428.71 \cdot 10^{6}\, IAUs / 4.00\, IAUs = 107.18 \cdot 10^{6} \text{ pieces of Alpha output;}\\
{}^{z}q_{\gamma u} &= 107.83 \cdot 10^{6}\, IAUs / 1.00\, IAUs = 107.83 \cdot 10^{6} \text{ pieces of Gamma output;}\\
{}^{z}q_{fs} &= 97.28 \cdot 10^{7}\, IAUs / 10.00\, IAUs = 97.28 \cdot 10^{6} \text{ international liquidity consols.}
\end{aligned}
\tag{8.32}
$$

Set (8.33) presents the prevailing shortages (–) and surpluses (+) derived by subtracting demanded (8.32) from available (8.31) quantities of labour, output and liquidity.

$$
\begin{aligned}
{}^{\#}q_{\alpha w} - {}^{z}q_{\alpha w} &= -\,38.77 \cdot 10^{3} \text{ years of Alpha labour;}\\
{}^{\#}q_{\gamma w} - {}^{z}q_{\gamma w} &= +\,133.30 \cdot 10^{3} \text{ years of Gamma labour;}\\
{}^{\#}q_{\alpha u} - {}^{z}q_{\alpha u} &= -\,7.18 \cdot 10^{6} \text{ pieces of Alpha output;}\\
{}^{\#}q_{\gamma u} - {}^{z}q_{\gamma u} &= -\,7.83 \cdot 10^{6} \text{ pieces of Gamma output;}\\
{}^{\#}q_{fs} - {}^{z}q_{fs} &= +\,2.72 \cdot 10^{6} \text{ number of international consols}
\end{aligned}
\tag{8.33}
$$

8.7. The nexus between international and local quotations

Given accessible quantities ${}^{\#}q_{xj}$, relations (8.30) yield the *IAU* salaries, prices and values which would balance all markets. The clearing *IAU* quotations are displayed by (8.34).

$$
\begin{aligned}
{}^{\#}P_{AW} &= 519.383 \cdot 10^{6}\, IAUs / 1\,000 \cdot 10^{3} = 519.38\, IAUs \text{ per year of Alpha labour;}\\
{}^{\#}P_{\Gamma W} &= 173.394 \cdot 10^{6}\, IAUs / 1\,000 \cdot 10^{3} = 173.39\, IAUs \text{ per year of Gamma labour;}\\
{}^{\#}P_{AU} &= 428.714 \cdot 10^{6}\, IAUs / 100 \cdot 10^{6} = 4.29\, IAUs \text{ per piece of Alpha output;}\\
{}^{\#}P_{\Gamma U} &= 107.832 \cdot 10^{6}\, IAUs / 100 \cdot 10^{6} = 1.08\, IAUs \text{ per piece of Gamma output;}\\
{}^{\#}P_{FS} &= 972.8 \cdot 10^{6}\, IAUs / 100 \cdot 10^{6} = 9.728\, IAUs \text{ per international liquidity consol.}
\end{aligned}
\tag{8.34}
$$

Differences between prevailing (8.31) and clearing (8.34) quotations are indicative of worldwide inflationary and deflationary trends shown below:

$$
\begin{aligned}
{}^{\#}P_{AW} - {}^{z}P_{AW} &= (519.38 - 500.00) \text{ / salary per year of Alpha labour} = +\,\mathit{3.88\%}\\
{}^{\#}P_{\Gamma W} - {}^{z}P_{\Gamma W} &= (173.39 - 200.00) \text{ / salary per year of Gamma labour} = -\,\mathit{13.30\%}\\
{}^{\#}P_{AU} - {}^{z}P_{AU} &= (4.29 - 4.00) \text{ / price per piece of Alpha output} = +\,\mathit{7.25\%}\\
{}^{\#}P_{\Gamma U} - {}^{z}P_{\Gamma U} &= (1.08 - 1.00) \text{ / price per piece of Gamma output} = +\,\mathit{8.00\%}\\
{}^{\#}P_{FS} - {}^{z}P_{FS} &= (9.728 - 9.00) \text{ / value per international liquidity consol} = +\,\mathit{8.10\%}
\end{aligned}
\tag{8.35}
$$

The distribution of global turnover among all traded goods, given by (8.22), is one of the eligible weighing sets. From this broad perspective, the underlying inflation rate is:

$$\pi = 23.6\% \text{ x } \mathit{3.88} + 7.9\% \text{ x } (\mathit{-13.30}) + 19.4\% \text{ x } \mathit{7.25} + 4.9\% \text{ x } \mathit{8.0} + 44.2\% \text{ x } \mathit{8.10} = \mathit{5.24\%} \qquad (8.36)$$

Without intervention, salaries, prices and values measured in *IAU*s will rise by *5.24%* on average during the next period. Zeta's hypothetical world is composed of two sovereign countries, each of which maintains a national government, a central bank, a currency unit, etc. The authorities of Alpha and Gamma may influence the gross sales of a number of market goods that is equal to the number of policy instruments under joint control. For political and other reasons, monetary financial institutions may be reluctant to decrease and reallocate annual credit among accepting sectors so as to zero quantity and quotation gaps. International and local salaries, prices and values are irrevocably linked since each *IAU* quotation P_{XJ} is equal to the local salary, price or value p_{xj} times the exchange rate ξ_X. To explore the nexus, we combine (8.31) and (8.32) to obtain the labour years and output pieces of Alpha and Gamma as well as the consol units demanded as functions of i) their respective *IAU* quotations and ii) their local quotations times the applicable exchange rate. System (8.37) generates the *IAU* salaries, prices and values that would balance demanded with available quantities; it thus enables us to connect the international clearing with the corresponding local quotations via the relevant exchange rates.

(8.37) Quantity equilibriums as functions of *IAU* and national quotations

${}^{\#}q_{\alpha w} = {}^{z}q_{\alpha w}$	$10^6 = 519.38 \cdot 10^6 / P_{AW}$	$10^6 = 519.38 \cdot 10^6 / \xi_A \cdot p_{\alpha w}$
${}^{\#}q_{\alpha u} = {}^{z}q_{\alpha u}$	$10^8 = 428.71 \cdot 10^6 / P_{AU}$	$10^8 = 428.71 \cdot 10^6 / \xi_A \cdot p_{\alpha u}$
${}^{\#}q_{\gamma w} = {}^{z}q_{\gamma w}$	$10^6 = 173.39 \cdot 10^6 / P_{\Gamma W}$	$10^6 = 173.39 \cdot 10^6 / \xi_\Gamma \cdot p_{\gamma w}$
${}^{\#}q_{\gamma u} = {}^{z}q_{\gamma u}$	$10^8 = 107.83 \cdot 10^6 / P_{\Gamma U}$	$10^8 = 107.83 \cdot 10^6 / \xi_\Gamma \cdot p_{\gamma u}$
${}^{\#}q_{\alpha s} = {}^{z}q_{\alpha s}$	$50 \cdot 10^6 = 486.415 \cdot 10^6 / P_{AS}$	$50 \cdot 10^6 = 486.415 \cdot 10^6 \cdot r_\alpha / \xi_A$
${}^{\#}q_{\gamma s} = {}^{z}q_{\gamma s}$	$50 \cdot 10^6 = 486.415 \cdot 10^6 / P_{\Gamma S}$	$50 \cdot 10^6 = 486.415 \cdot 10^6 \cdot r_\gamma / \xi_\Gamma$

System (8.37) has a unique solution in terms of *IAUs* but multiple solutions in terms of local salaries, prices and values because P_{XJ} and p_{xj} are connected via two exchange (ξ_A, ξ_Γ) and two interest (r_α, r_γ) rates. The problem is simplified as soon as it is recognised that:

- countries are unequal from an economic, political or any other point of view;
- the *IAU* always coincides with the currency of one country, e.g. $\xi_A = 1\$ = 1\mathit{IAU}$;
- exchange and interest rates are linked by the Uncovered Interest Parity Principle.

As proponents of free market forces, Alpha's authorities have chosen to deal with excess demand by letting quotations rise as necessary. On the other hand, Gamma's authorities are faced with a more complex dilemma; they may choose between an expensive currency unit (and thus low domestic inflation) or an inexpensive currency unit (and thus high domestic inflation). Excess supply of labour calls for 13.3% reduction in salaries to strengthen the purchasing power of producers, while excess demand for output calls for 8.0% increase in prices to weaken the purchasing power of consumers. Given that lowering nominal wages is usually accompanied by practical complications – such as social unrest, political collapse, etc. – the authorities of Gamma have decided to keep the same wages in terms of local currency but to devalue the exchange rate by 13.3% so as to achieve the necessary reduction in terms of *IAU*s. Table (8.38) furnishes the *IAU* and national quotations before and after the devaluation Gamma's currency vis-à-vis the *IAU*.

(8.38) Initial and clearing quotations of traded goods

	ECONOMY ALPHA		**ECONOMY GAMMA**	
TRADED GOOD	**Initial $**	**Clearing $**	**Initial €**	**Clearing €**
Year of Alpha labour	500.00	519.38	500.00	599.09
Year of Gamma labour	200.00	173.39	200.00	200.00
Piece of Alpha output	4.00	4.29	4.00	4.95
Piece of Gamma output	1.00	1.08	1.00	1.24
Perpetual bond	10.00	9.728	10.00	11.221
Interest rate	10.0%	10.279%	10.0%	8.91%
Exchange rate	1 *IAU*/$	1 *IAU*/$	1 *IAU*/€	0.86695

(8.39) Ultimate inflation rates in Alpha and Gamma

TRADED GOOD	**Global turnover weight %**	**Rate of change in $ terms**	**Index in $ terms**	**Rate of change in € terms**	**Index in € terms**
Alpha labour	23.6	*+3.87*	0.9133	*+19.82*	4.6775
Gamma labour	7.9	*–13.3*	–1.0507	*0.0*	0.0
Alpha output	19.4	*+7.25*	1.4065	*+23.75*	4.6075
Gamma output	4.9	*+8.00*	0.3920	*+24.00*	1.1760
Perpetual bond	44.2	*–2.72*	–1.2022	*+12.21*	5.3968
TOTAL	100.0	*0.46%*	+0.4589	*+15.86%*	+15.8578

Table (8.39) reveals that (following Euro devaluation) the basket of internationally traded goods has become dearer by *0.46%* in terms of Dollars and by *15.86%* in terms of Euros. Gamma's vexing 13.3% unemployment rate has been eliminated via redistribution of purchasing power from households to businesses world-wide.

8.8. Impact of exchange rate changes

In order to demonstrate the versatility and therefore usefulness of Euclidean methodology, we will focus on the effects of Gamma's exchange rate ξ_Γ on Zeta's flows. As in previous exercises, i) Alpha's currency is the international accounting unit, i.e. $\xi_A = 1\$ = 1IAU$; ii) the banking sectors of Alpha and Gamma expand their balance sheets by $k_{\alpha f} = \$10^9$ and $k_{\gamma f} = €10^9$ per year; while iii) the national credit increments $k_{\alpha f}$ and $k_{\gamma f}$ are divided equally among consumers and producers of both countries. Structural matrix **Z** of equilibrium system $^{\mathbf{Z}}\mathbf{X} = \mathbf{Z}^{-1}\cdot\mathbf{K_F}$ is provided by Table (8.9). When $z = {}^{72}/_{100}$, the international economy described by (8.10) possesses a stable and, thus, unique equilibrium vector $^{\mathbf{z}}\mathbf{X}$.

(8.40) Zeta's flows as functions of Gamma's exchange rate ξ_Γ

Income of Alpha's Households =

$$^{z}Y_{AH} = (50074647930\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Expenditure of Alpha's Households =

$$^{z}E_{AH} = (7224968180\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Income of Gamma's Households =

$$^{z}Y_{\Gamma H} = (44579346120\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Expenditure of Gamma's Households =

$$^{z}E_{\Gamma H} = (1729666370\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Income of Alpha's Businesses =

$$^{z}Y_{\Gamma B} = (42863983050\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Expenditure of Alpha's Businesses =

$$^{z}E_{\Gamma B} = (14303300\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Income of Gamma's Businesses =

$$^{z}Y_{\Gamma B} = (46205561250\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Expenditure of Gamma's Businesses =

$$^{z}E_{\Gamma B} = (3355881500\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Income of International Financiers =

$$^{z}Y_{F} = (4999322600\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Expenditure of International Financiers =

$$^{z}E_{F} = (176398041600\cdot(1+\xi_\Gamma)/171398719)\cdot 10^6$$

Steady-state solution ${}^{\mathbf{Z}}\mathbf{X}$ – reproduced by set (8.40) – is obtained by substituting credit policy vector $\mathbf{K_F}(0,\ 250+\xi_\Gamma{\cdot}250;\ 0,\ 250+\xi_\Gamma{\cdot}250,\ 0,\ 250+\xi_\Gamma{\cdot}250;\ 0,\ 250+\xi_\Gamma{\cdot}250;\ 0,\ -1000 - \xi_\Gamma{\cdot}1000){\cdot}10^6$ in the appropriate column of ${}^{\mathbf{Z}}\mathbf{X} = \mathbf{Z}^{-1}{\cdot}\mathbf{K_F}$. Exchange rates become part of the mathematical model via the increase and allocation of universal liquidity $\mathbf{K_F}$; they determine the magnitude and apportionment of international bank credit and thus the world-wide circulation of 'paper' money and 'real' goods. Like other kinds of quotations, exchange rates decide the purchasing power of sector / country flows and stocks without affecting either the economic structure or the dynamic stability of the closed system.

Next, we consider relationships (8.40) as a system of ten inequalities ${}^zY_{AH} > 0$, ${}^zE_{AH} > 0$, ${}^zY_{\Gamma H} > 0$, ${}^zE_{\Gamma H} > 0$, ${}^zY_{AB} > 0$, ${}^zE_{AB} > 0$, ${}^zY_{\Gamma B} > 0$, ${}^zE_{\Gamma B} > 0$, ${}^zY_F > 0$, ${}^zE_F > 0$. With *Mathematica*'s help, we conclude that Zeta's consumers, producers and bankers enjoy positive flows for all $\xi_\Gamma > 0$. Set (8.41) gives the variations of sector flows induced by appreciation of Gamma's exchange rate vis-à-vis the *IAU*, while everything else remains the same.

(8.41) Variations of Zeta's flows caused by revaluation of Gamma's currency.

$\partial^Z Y_{AH} / \partial\xi_\Gamma = 292.15{\cdot}10^6$	$\partial^Z E_{AH} / \partial\xi_\Gamma = 42.15{\cdot}10^6$
$\partial^Z Y_{\Gamma H} / \partial\xi_\Gamma = 260.09{\cdot}10^6$	$\partial^Z E_{\Gamma H} / \partial\xi_\Gamma = 10.09{\cdot}10^6$
$\partial^Z Y_{AB} / \partial\xi_\Gamma = 250.08{\cdot}10^6$	$\partial^Z E_{AB} / \partial\xi_\Gamma = 0.08{\cdot}10^6$
$\partial^Z Y_{\Gamma B} / \partial\xi_\Gamma = 269.58{\cdot}10^6$	$\partial^Z E_{\Gamma B} / \partial\xi_\Gamma = 19.58{\cdot}10^6$
$\partial^Z Y_F / \partial\xi_\Gamma = 29.17{\cdot}10^6$	$\partial^Z E_F / \partial\xi_\Gamma = 1\,029.17{\cdot}10^6$

Universal liquidity increment K_F as well as revenue ${}^zY$ and outlay ${}^zE$ are positive functions of Gamma's exchange rate ξ_Γ. Appreciation of Gamma's currency raises the *IAU* income $(292.15+250.08){\cdot}10^6$ and expenditure $(42.15+0.08){\cdot}10^6$ of Alpha's accepting sectors more than the *IAU* revenue $(260.09+269.58){\cdot}10^6$ and outlay $(10.09+19.58){\cdot}10^6$ of Gamma's accepting sectors. This stems from the fact that households and businesses prefer local over foreign real goods at the ratio $\alpha/(1-\alpha) = {}^{72}/_{28} = 2.57$ in Alpha, compared with $\gamma/(1-\gamma) = {}^{28}/_{72} = 0.39$ in Gamma. Summing up separately sector receipts and sector payments given by (8.40), we affirm that Zeta's income ${}^zY$ and expenditure ${}^zE$ are equal as shown below:

$${}^zY = {}^zY_{AH} + {}^zY_{\Gamma H} + {}^zY_{AB} + {}^zY_{\Gamma B} + {}^zY_F =$$

$$= (188722860950{\cdot}(1+\xi_\Gamma) / 171398719){\cdot}10^6 = \qquad (8.42)$$

$${}^zE = {}^zE_{AH} + {}^zE_{\Gamma H} + {}^zE_{AB} + {}^zE_{\Gamma B} + {}^zE_F$$

Replacing the algebraic symbols of (8.20) with the matching flows found in (8.40), we derive equations (8.43) which display annual disbursements on labour (R_{AW}, $R_{\Gamma W}$), output (R_{AU}, $R_{\Gamma U}$) and liquidity (R_{FS}) as functions of Gamma's exchange rate.

$$
\begin{aligned}
{}^{z}R_{AW} &= (44510798160\cdot(1+\xi_\Gamma)\,/\,171398719)\cdot 10^6 \\
{}^{z}R_{\Gamma W} &= (14859782040\cdot(1+\xi_\Gamma)\,/\,171398719)\cdot 10^6 \\
{}^{z}R_{AU} &= (36740556900\cdot(1+\xi_\Gamma)\,/\,171398719)\cdot 10^6 \\
{}^{z}R_{\Gamma U} &= (9241112250\cdot(1+\xi_\Gamma)\,/\,171398719)\cdot 10^6 \\
{}^{z}R_{FS} &= (83370611600\cdot(1+\xi_\Gamma)\,/\,171398719)\cdot 10^6
\end{aligned}
\tag{8.43}
$$

Combining (8.20) and (8.40), we get Alpha's trade deficit and Gamma's trade surplus (on the services and products account) as functions of preference ratio z and exchange rate ξ_Γ:

$$
\begin{aligned}
\text{Alpha's trade deficit} &= |({}^{z}/_{4}\cdot{}^{Z}E_{\Gamma H} + {}^{z}/_{6}\cdot{}^{Z}E_{\Gamma B}) - ({}^{7(1\text{-}z)}/_{8}\cdot{}^{Z}E_{AH} + {}^{5(1\text{-}z)}/_{6}\cdot{}^{Z}E_{AB})| = \\
&= |-6356453485\cdot(1+\xi_\Gamma)\,/\,1028392314\cdot 10^6| = \text{Gamma's trade surplus}
\end{aligned}
\tag{8.44}
$$

Owing to Zeta's structural matrix **Z** and substitution elasticity z – between the labour and output of Alpha *vis-à-vis* the corresponding goods of Gamma – appreciation (depreciation) of ξ_Γ expands (contracts) Alpha's trade deficit and, therefore, Gamma's trade surplus on the services and products account. Closer examination of (8.44) reveals that there is no $\xi_\Gamma > 0$ that would eliminate Alpha's deficit and, thus, Gamma's surplus completely. It is a well-known fact that countries in Central Africa, East Asia, Latin America, Southern Europe, etc., that are negatively favoured by global preferences, cannot leap from relative poverty to relative wealth by devaluing their currencies. In the short run, no change in exchange rates can prompt either significant demand for undesirable goods or supply of desirable goods. Current account gaps, such as those depicted by (8.44), may also be expressed and examined as functions of:

- sector, national and zone behaviour coefficients regarding income sources and expenditure destinations;
- taxation and subsidisation parameters that have been incorporated in the grid of incomings and outgoings;
- the increase and allocation of bank credit among institutional sectors, sovereign countries and currency areas.

Deficits and surpluses, like two sides of the same coin, are integral and inseparable components of the global trade and payments system. Granted that salaries, prices and values are inflexible, banking institutions have no alternative but to keep increasing their balance sheets in response to natural growth of labour and other resources. After connecting two exchange networks and explaining their economic and financial links, we are in a better position to construct a multi-country model with the aim of discussing international policies that minimise deficits and surpluses world-wide.

8.9 A world divided into currency zones

The optimum size of a Euclidean exchange network depends on the desired level of disaggregation. From the policy-making point-of-view, whenever:

- fine details are paramount, the mathematical economic model should encompass sectors and markets;
- aggregate figures are sufficient, the mathematical economic model should focus on countries and zones.

This monograph would be incomplete without an example of dynamic interaction among currency unions and the financial system. When the amount of detail is small, as in the ensuing example, little can be said either about behaviour and finance of particular sectors or about quotations and quantities of specific goods. In such models, everything is analysed and composed in terms of country or zone macroeconomic variables. Under all circumstances, the twelve major propositions of Euclidean methodology remain valid. There is no money illusion and thus no discrimination according to the origin or destination of current and capital payments. One unit of international liquidity per year is bought and sold for the same amount of *IAU*s worldwide. When conditions favour balanced portfolios, agents, sectors, countries, etc., derive steady revenue shares from, and spend steady outlay shares on, guaranteed securities.

Let us suppose that a closed economy called Theta consists of three currency zones which use an International Accounting Unit for facilitating their commercial and investment transactions. Theta's exchange network involves only macroeconomic variables. Tables (8.45), (8.46) and (8.47) portray demand for income and supply of expenditure by the three currency areas and the joint banking system during a particular year. The reasoning that successfully produced model (7.21) has also been applied to the derivation of model (8.50).

(8.45) Income demand and expenditure supply measured in *IAU*s

Demand & Supply	Zone One		Zone Two		Zone Three		Finance		TOTAL
$\mathbf{Y_1}$	Y_{11}		Y_{12}		Y_{13}		Y_{1F}		100.0
$\mathbf{E_1}$		E_{11}		E_{12}		E_{13}		E_{1F}	100.0
$\mathbf{Y_2}$	Y_{21}		Y_{22}		Y_{23}		Y_{2F}		100.0
$\mathbf{E_2}$		E_{21}		E_{22}		E_{23}		E_{2F}	100.0
$\mathbf{Y_3}$	Y_{31}		Y_{32}		Y_{33}		Y_{3F}		100.0
$\mathbf{E_3}$		E_{31}		E_{32}		E_{33}		E_{34}	100.0
$\mathbf{Y_F}$	Y_{F1}		Y_{F2}		Y_{F3}		Y_{FF}		100.0
$\mathbf{E_F}$		E_{F1}		E_{F2}		E_{F3}		E_{FF}	100.0
$\mathbf{Y = E}$	$\mathbf{Y_1 - E_1}$		$\mathbf{Y_2 - E_2}$		$\mathbf{Y_3 - E_3}$		$\mathbf{E_F - Y_F}$		

(8.46) Distributions of income demand and expenditure supply

Demand & Supply	Zone One		Zone Two		Zone Three		Finance		TOTAL
Y_1	α_{11}		α_{12}		α_{13}		α_{1f}		100.0
E_1		β_{11}		β_{12}		β_{13}		β_{1f}	100.0
Y_2	α_{21}		α_{22}		α_{23}		α_{2f}		100.0
E_2		β_{21}		β_{22}		β_{23}		β_{2f}	100.0
Y_3	α_{31}		α_{32}		α_{33}		α_{3f}		100.0
E_3		β_{31}		β_{32}		β_{33}		β_{3f}	100.0
Y_F	α_{f1}		α_{f2}		α_{f3}		α_{ff}		100.0
E_F		β_{f1}		β_{f2}		β_{f3}		β_{ff}	100.0
$E_F - Y_F$	$\mu_1 \cdot K_F$		$\mu_2 \cdot K_F$		$\mu_3 \cdot K_F$		$= K_F$		

(8.47) Hypothetical income demand and expenditure supply shares

Demand & Supply	Zone One		Zone Two		Zone Three		Finance		TOTAL
Y_1	α_{11}		α_{12}		α_{13}		$^{10}/_{100}$		
E_1		$^{60}/_{100}$		$^{20}/_{100}$		$^{15}/_{100}$		$^{5}/_{100}$	100.0
Y_2	α_{21}		α_{22}		α_{23}		$^{7}/_{100}$		
E_2		$^{15}/_{100}$		$^{65}/_{100}$		$^{10}/_{100}$		$^{10}/_{100}$	100.0
Y_3	α_{31}		α_{32}		α_{33}		$^{5}/_{100}$		
E_3		$^{10}/_{100}$		$^{5}/_{100}$		$^{70}/_{100}$		$^{15}/_{100}$	100.0
Y_F	α_{f1}		α_{f2}		α_{f3}		$^{100}/_{100}$		
E_F		$^{25}/_{100}$		$^{15}/_{100}$		$^{10}/_{100}$		$^{50}/_{100}$	100.0
$E_F - Y_F$	$^{1}/_{3} \cdot K_F$		$^{1}/_{3} \cdot K_F$		$^{1}/_{3} \cdot K_F$		$= K_F$		

(8.48), (8.49) and (8.50) Exchange network of economy Theta

$$
\begin{aligned}
Y_1 &= (E_{11} + E_{21} + E_{31} + E_{F1}) + (E_{FS} - Y_{2F} - Y_{3F}) \\
E_1 &= Y_1 - K_{1F} \\
Y_2 &= (E_{12} + E_{22} + E_{32} + E_{F2}) + (E_{FS} - Y_{1F} - Y_{3F}) \\
E_2 &= Y_2 - K_{2F} \\
Y_3 &= (E_{13} + E_{23} + E_{33} + E_{F3}) + (E_{FS} - Y_{1F} - Y_{2F}) \\
E_3 &= Y_3 - K_{3F} \\
Y_F &= (E_{1F} + E_{2F} + E_{3F}) \\
E_F &= Y_F + K_F
\end{aligned}
\tag{8.48}
$$

$$
\begin{aligned}
dY_1/dt &= \beta_{11}\cdot E_1 + \beta_{21}\cdot E_2 + \beta_{31}\cdot E_3 + \beta_{f1}\cdot E_F + E_{FS} - (\alpha_{2f}\cdot Y_2 + \alpha_{3f}\cdot Y_3) - Y_1 \\
dE_1/dt &= Y_1 - E_1 - K_{1F} \\
dY_2/dt &= \beta_{12}\cdot E_1 + \beta_{22}\cdot E_2 + \beta_{32}\cdot E_3 + \beta_{f2}\cdot E_F + E_{FS} - (\alpha_{1f}\cdot Y_1 + \alpha_{3f}\cdot Y_3) - Y_2 \\
dE_2/dt &= Y_2 - E_2 - K_{2F} \\
dY_3/dt &= = \beta_{13}\cdot E_1 + \beta_{23}\cdot E_2 + \beta_{33}\cdot E_3 + \beta_{f3}\cdot E_F + E_{FS} - (\alpha_{1f}\cdot Y_1 + \alpha_{2f}\cdot Y_2) - Y_3 \\
dE_3/dt &= Y_3 - E_3 - K_{3F} \\
dY_F/dt &= \beta_{1f}\cdot E_1 + \beta_{2f}\cdot E_2 + \beta_{3f}\cdot E_3 - Y_F \\
dE_F/dt &= Y_F - E_F + K_F
\end{aligned}
\tag{8.49}
$$

$$
\begin{aligned}
dY_1/dt &= {}^3/_5\cdot E_1 + {}^3/_{20}\cdot E_2 + {}^1/_{10}\cdot E_3 + {}^3/_4\cdot E_F - ({}^7/_{100}\cdot Y_2 + {}^1/_{20}\cdot Y_3) - Y_1 \\
dE_1/dt &= Y_1 - E_1 - K_{1F} \\
dY_2/dt &= {}^1/_5\cdot E_1 + {}^{13}/_{20}\cdot E_2 + {}^1/_{20}\cdot E_3 + {}^{13}/_{20}\cdot E_F - ({}^1/_{10}\cdot Y_1 + {}^1/_{20}\cdot Y_3) - Y_2 \\
dE_2/dt &= Y_2 - E_2 - K_{2F} \\
dY_3/dt &= {}^3/_{20}\cdot E_1 + {}^1/_{10}\cdot E_2 + {}^7/_{10}\cdot E_3 + {}^3/_5\cdot E_F - ({}^1/_{10}\cdot Y_1 + {}^7/_{100}\cdot Y_2) - Y_2 \\
dE_3/dt &= Y_3 - E_3 - K_{3F} \\
dY_F/dt &= {}^1/_{20}\cdot E_1 + {}^1/_{10}\cdot E_2 + {}^3/_{20}\cdot E_3 - Y_F \\
dE_F/dt &= Y_F - E_F + K_F
\end{aligned}
\tag{8.50}
$$

In the aforementioned Tables and in systems (8.48), (8.49) and (8.50), Zones One, Two and Three appear as spending on their 'own goods'. This is due to aggregation of agents and sectors under sovereign countries and currency areas. According to Euclidean methodology, the main factors affecting economic performance relative to productive potential are the magnitudes and allocations of total demand and bank credit. Within this framework, we will investigate uniqueness, stability and direction of Theta's annual flows as well as effects of coordination among Theta's monetary financial institutions. Yearly, the united banking system issues additional liabilities with which it buys guaranteed securities from consumers, producers, etc., in all three zones. Any change with respect to magnitude and distribution of zone finance shifts the exchange network to a new equilibrium. Comparison of alternative positions $^{*}\mathbf{X}$ is meaningful provided the dynamic system $\mathbf{dX/dt = \Theta \cdot X - K_F}$ that generates them is globally stable. International economic model (8.50) is universally stable if, and only if, the eight Routh-Hurwitz conditions rh_n, consisting of the coefficients of characteristic polynomial $|\mathbf{\Theta - \lambda \cdot I}|$, are greater than zero. Theta's characteristic polynomial $|\mathbf{\Theta - \lambda \cdot I}|$ is determinant (8.51).

(8.51) Theta's characteristic polynomial

$$\begin{vmatrix}
-1-\lambda & {}^{3}/_{5} & -{}^{7}/_{100} & {}^{3}/_{20} & -{}^{1}/_{20} & {}^{1}/_{10} & 0 & {}^{3}/_{4} \\
1 & -1-\lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
-{}^{1}/_{10} & {}^{1}/_{5} & -1-\lambda & {}^{13}/_{20} & -{}^{1}/_{20} & {}^{1}/_{20} & 0 & {}^{13}/_{20} \\
0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 0 \\
-{}^{1}/_{10} & {}^{3}/_{20} & -{}^{7}/_{100} & {}^{1}/_{10} & -1-\lambda & {}^{7}/_{10} & 0 & {}^{3}/_{5} \\
0 & 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 \\
0 & {}^{1}/_{204} & 0 & {}^{1}/_{10} & 0 & {}^{3}/_{20} & -1-\lambda & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1-\lambda
\end{vmatrix} = |\mathbf{\Theta - \lambda \cdot I}|$$

$$|\mathbf{\Theta - \lambda \cdot I}| = \theta_0 \cdot \lambda^8 + \theta_1 \cdot \lambda^7 + \theta_2 \cdot \lambda^6 + \theta_3 \cdot \lambda^5 + \theta_4 \cdot \lambda^4 + \theta_5 \cdot \lambda^3 + \theta_6 \cdot \lambda^2 + \theta_7 \cdot \lambda + \theta_8 \qquad (8.51)$$

Computing the above polynomial coefficients with *Mathematica*, we obtain:

$\theta_0 = 1$; $\theta_1 = 8$; $\theta_2 = {}^{52\,069}/_{2000}$; $\theta_3 = {}^{442\,627}/_{10\,000}$; $\theta_4 = {}^{1\,673\,009}/_{40\,000}$; $\theta_5 = {}^{854\,611}/_{40\,000}$;

$\theta_6 = {}^{2\,082\,179}/_{400\,000}$; $\theta_7 = {}^{196\,969}/_{400\,000}$; $\theta_8 = {}^{147}/_{20\,000} = |\mathbf{\Theta}|$

By placing the values of coefficients θ_n in the eight minor determinants of Table (8.52), we prove that all Routh-Hurwitz conditions rh_n, n = 1, ... , 8 are always greater than zero. Globally stable exchange network (8.50) is associated with a unique steady state $^{*}\mathbf{X}$ because determinant $\theta_8 = |\mathbf{\Theta}|$ of Theta's structural matrix $\mathbf{\Theta}$ is positive.

(8.52) Minors of Theta's characteristic polynomial

$$\begin{vmatrix} \theta_1 & \theta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \theta_3 & \theta_2 & \theta_1 & \theta_0 & 0 & 0 & 0 & 0 \\ \theta_5 & \theta_4 & \theta_3 & \theta_2 & \theta_1 & \theta_0 & 0 & 0 \\ \theta_7 & \theta_6 & \theta_5 & \theta_4 & \theta_3 & \theta_2 & \theta_1 & \theta_0 \\ \theta_9 & \theta_8 & \theta_7 & \theta_6 & \theta_5 & \theta_4 & \theta_3 & \theta_2 \\ 0 & \theta_{10} & \theta_9 & \theta_8 & \theta_7 & \theta_6 & \theta_5 & \theta_4 \\ 0 & 0 & 0 & \theta_{10} & \theta_9 & \theta_8 & \theta_7 & \theta_6 \\ 0 & 0 & 0 & 0 & 0 & \theta_{10} & \theta_9 & \theta_8 \end{vmatrix}$$

Due to its structural matrix, closed economy Theta – duplicated by Euclidean mathematical model (8.50) – possesses a universally stable and unique equilibrium $^*\mathbf{X}$. Global stability implies that Theta returns to steady state $^*\mathbf{X}$ after each modification of policy parameters $\mathbf{\Pi}(K_F, \mu_1, \mu_2, \mu_3)$ and / or jolt of initial flows $^0\mathbf{X}(^0Y_1, {}^0E_1\, {}^0Y_2, {}^0E_2; {}^0Y_3, {}^0E_3; {}^0Y_F, {}^0E_F)$. Under all circumstances, linear differential equation model (8.50) ends up at timeless linear equation system $\mathbf{\Theta} \cdot {}^*\mathbf{X} = \mathbf{K_F}$ depicted by (8.53). However, stability does not entail that annual flows ($^*Y_1$, $^*E_1$; $^*Y_2$, $^*E_2$; $^*Y_3$, $^*E_3$; $^*Y_F$, $^*E_F$) remain positive and, hence, uniformly dirigible, irrespective of credit allocation among accepting countries and zones.

(8.53) Static system associated with Theta's dynamic model

$$\begin{vmatrix} -1 & {}^3/_5 & -{}^7/_{100} & {}^3/_{20} & -{}^1/_{20} & {}^1/_{10} & 0 & {}^3/_4 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -{}^1/_{10} & {}^1/_5 & -1 & {}^{13}/_{20} & -{}^1/_{20} & {}^1/_{20} & 0 & {}^{13}/_{20} \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ -{}^1/_{10} & {}^3/_{20} & -{}^7/_{100} & {}^1/_{10} & -1 & {}^7/_{10} & 0 & {}^3/_5 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & {}^1/_{204} & 0 & {}^1/_{10} & 0 & {}^3/_{20} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} ^*Y_1 \\ ^*E_1 \\ ^*Y_2 \\ ^*E_2 \\ ^*Y_3 \\ ^*E_3 \\ ^*Y_F \\ ^*E_F \end{vmatrix} = \mathbf{\Theta} \cdot {}^*\mathbf{X} = \begin{vmatrix} 0 \\ \mu_1 \\ 0 \\ \mu_2 \\ 0 \\ \mu_3 \\ 0 \\ -1 \end{vmatrix} \cdot K_F = \mathbf{K_F}$$

Structural matrix $\mathbf{\Theta}$ **(**depicting interaction of zone and bank revenues and outlays**)** linearly transforms credit policy vector $\mathbf{K_F}$ **(**representing the magnitude and distribution of the monetary financial increment**)** into solution $^*\mathbf{X}$ **(**consisting of the steady-state flows of zones and banks**)**. Bearing in mind that $|\mathbf{\Theta}| > 0$, static system (8.53) generates the annual debits and credits of Zones One, Two, Three and their combined banking sector. Therefore, using Cramer's rule, we derive system (8.54), which portrays equilibrium incomes and expenditures ($^*Y_1$, $^*E_1$), ($^*Y_2$, $^*E_2$), ($^*Y_3$, $^*E_3$), ($^*Y_F$, $^*E_F$) as functions of liquidity increment K_F and credit shares μ_1 and μ_2 (because $\mu_3 = 1 - \mu_1 - \mu_2$).

(8.54) Theta's zone and bank flows as functions of K_F and μ_1 and μ_2

Income of Zone One

$$^*Y_1 = (5565 + 1567\cdot\mu_1 + 1505\cdot\mu_2)\cdot K_F / 980$$

Expenditure of Zone One

$$^*E_1 = (5265 + 587\cdot\mu_1 + 1505\cdot\mu_2)\cdot K_F / 980$$

Income of Zone Two

$$^*Y_2 = 5\cdot(641 + 227\cdot\mu_1 - 7\cdot\mu_2)\cdot K_F / 588$$

Expenditure of Zone Two

$$^*E_2 = (3205 + 1135\cdot\mu_1 - 623\cdot\mu_2)\cdot K_F / 588$$

Income of Zone Three

$$^*Y_3 = (10225 + 12763\cdot\mu_1 + 10465\cdot\mu_2)\cdot K_F / 2940$$

Expenditure of Zone Three

$$^*E_3 = (7285 + 15702\cdot\mu_1 + 13405\cdot\mu_2)\cdot K_F / 2940$$

Income of Banking Institutions

$$^*Y_F = (3485 + 3011\cdot\mu_1 + 1925\cdot\mu_2)\cdot K_F / 2940$$

Expenditure of Banking Institutions

$$^*E_F = (6425 + 3011\cdot\mu_1 + 1925\cdot\mu_2)\cdot K_F / 2940$$

It is paramount to know in advance the ranges of μ_1, μ_2 or μ_3, which convert any one of Theta's takings and outgoings from greater than to less than zero. As already mentioned, circuits that combine positive and negative flows are classified as 'clockwise turning' or 'left-handed screw' systems. In Section 6.4 it was pointed out that when equilibrium vector $^*\mathbf{X}$ includes positive and negative elements, exchange network $\mathbf{dX/dt = \Theta\cdot X - K_F}$ is beyond control because interacting sectors, countries, zones, etc., behave counter-intuitively. Unexplainable but also unanticipated reactions render pursuit of coherent monetary, fiscal and other policies impossible. Granted $K_F > 0$, functions (8.54) form a system of eight linear inequalities that can be easily solved for credit portions μ_1 and μ_2, which yield ($^*Y_1 >0$, $^*E_1 >0$, $^*Y_2 >0$, $^*E_2 >0$, $^*Y_3 >0$, $^*E_3 >0$, $^*Y_F >0$, $^*E_F >0$). It is apparent that zone and bank receipts and payments are positive for all $0 < \mu_1 < 1$, $0 < \mu_2 < 1$. Consequently, Theta comprises a dirigible dynamic system during the relevant time period. Returning to equations (8.54), let us add up revenues and outlays separately to obtain international income *Y and expenditure *E. Relations (8.55) present Theta's takings and outgoings as functions of the worldwide liquidity increment (measured in *IAU*s) and the allotment of new loans among the three zones:

$$^{*}\mathrm{Y} = ({}^{*}Y_1 + {}^{*}Y_2 + {}^{*}Y_3 + {}^{*}Y_F) = (4553 + 2615\cdot\mu_1 + 1673\cdot\mu_2)\cdot \mathrm{K_F} / 294$$
$$^{*}\mathrm{E} = ({}^{*}E_1 + {}^{*}E_2 + {}^{*}E_3 + {}^{*}E_F) = (4553 + 2615\cdot\mu_1 + 1673\cdot\mu_2)\cdot \mathrm{K_F} / 294 \quad (8.55)$$

As predicted by Euclidean methodology and proven by relations (8.55), total income is equal to total expenditure. When annual monetary growth is divided equally ($\mu_1 = \mu_2 = \mu_3 = {}^1/_3$) all zones of Theta enjoy positive monetary flows, which add up to:

$$^{*}\mathrm{Y} = (6.41735 + 6.07426 + 6.11145 + 1.74501)\cdot \mathrm{K_F} = c\cdot \mathrm{KF} = 20.3481\cdot \mathrm{K_F}$$
$$^{*}\mathrm{E} = (6.08401 + 5.74093 + 5.77812 + 2.74501)\cdot \mathrm{K_F} = c\cdot \mathrm{KF} = 20.3481\cdot \mathrm{K_F} \quad (8.56)$$

Euclidean multiplier $c = {}^{*}Y/\mathrm{K_F} = {}^{*}E/\mathrm{K_F} = 20.3481$ is the scale factor applied to the quadrangle that depicts the particular system. Due to Theta's economic structure, 1 *IAU* of additional money generates about 20.35 *IAU*s of additional throughput. Table (8.57) presents the annual revenues and outlays of Zones One, Two, Three and their combined banking sector when international monetary expansion is 10^{12} *IAU*s and $\mu_1 = \mu_2 = \mu_3 = {}^1/_3$.

(8.57) Annual takings and outgoings of zones and banks

${}^{*}Y_1 = 6.41735\cdot \mathrm{K_F} = 6.41735\cdot 10^{12}$ *IAU*s	${}^{*}E_1 = 6.08401\cdot \mathrm{K_F} = 6.08401\cdot 10^{12}$ *IAU*s
${}^{*}Y_2 = 6.07426\cdot \mathrm{K_F} = 6.07426\cdot 10^{12}$ *IAU*s	${}^{*}E_2 = 5.74093\cdot \mathrm{K_F} = 5.74093\cdot 10^{12}$ *IAU*s
${}^{*}Y_3 = 6.11145\cdot \mathrm{K_F} = 6.11145\cdot 10^{12}$ *IAU*s	${}^{*}E_3 = 5.77812\cdot \mathrm{K_F} = 5.77812\cdot 10^{12}$ *IAU*s
${}^{*}Y_F = 1.74501\cdot \mathrm{K_F} = 1.74501\cdot 10^{12}$ *IAU*s	${}^{*}E_F = 2.74501\cdot \mathrm{K_F} = 2.74501\cdot 10^{12}$ *IAU*s
${}^{*}Y = 20.3481\cdot \mathrm{K_F} = 20.3481\cdot 10^{12}$ *IAU*s	${}^{*}E = 20.3481\cdot \mathrm{K_F} = 20.3481\cdot 10^{12}$ *IAU*s

Equations (8.57) give rise to pair (8.58), which displays the distributions of Theta's receipts and payments among the three currency areas and their combined banking sector.

$$(y_1 + y_2 + y_3) + (y_f) = (31.54\% + 29.85\% + 30.03\%) + (8.58\%) = 100\%$$
$$(e_1 + e_2 + e_3) + (e_f) = (29.90\% + 28.21\% + 28.40\%) + (13.49\%) = 100\% \quad (8.58)$$

Equations (8.58) confirm that world income and expenditure are perfectly apportioned among the three currency areas and the monetary financial system. Granted that dynamic model (8.50) is dirigible, policy-makers may influence a number of endogenous variables that is equal to the number of control instruments. It goes without saying that the economic objectives of currency unions rank higher than those of banking institutions because the former are run by elected politicians while the latter are run by appointed technocrats. During a debate on international policy formation, some government representatives referred to the productive capacities of Zones One, Two and Three as well as to the quotation trends of perpetual bonds. Convincingly, they argued in favour of:

- increasing international demand by about 6.7%;
- expanding the sales of Zone One services and products below the world average;
- shrinking demand for Zone Two services and products by about 1.5%;
- boosting demand for Zone Three services and products up to 17.5%;
- keeping the turnover of monetary financial institutions relatively constant.

(8.59) Equilibrium and target disbursements of zones and banks

Initial Turnover	**Target Turnover**	**%Δ**
$^{*}E_1 = 6.08401 \cdot 10^{12}$ *IAU*s	$^{\#}E_1 = 6.36792 \cdot 10^{12}$ *IAU*s	+4.67%
$^{*}E_2 = 5.74093 \cdot 10^{12}$ *IAU*s	$^{\#}E_2 = 5.65442 \cdot 10^{12}$ *IAU*s	–1.50%
$^{*}E_3 = 5.77812 \cdot 10^{12}$ *IAU*s	$^{\#}E_3 = 6.78729 \cdot 10^{12}$ *IAU*s	+17.46%
$^{*}E_F = 2.74501 \cdot 10^{12}$ *IAU*s	$^{\#}E_F = 2.90193 \cdot 10^{12}$ *IAU*s	+5.72%
$^{*}\mathrm{E} = 20.3481 \cdot 10^{12}$ *IAU*s	$^{\#}\mathrm{E} = 21.7116 \cdot 10^{12}$ *IAU*s	+6.70%

In response, some mathematical economists undertook to find magnitude $\mathrm{K_F}$ and percentages μ_1, μ_2 or μ_3, which would apportion international demand as reflected by (8.59). Bearing in mind that $\mathbf{Y}(^{\#}Y_1, ^{\#}Y_2, ^{\#}Y_3, ^{\#}Y_F)$ are linearly dependent on $\mathbf{E}(^{\#}E_1, ^{\#}E_2, ^{\#}E_3, ^{\#}E_F)$ – in order to derive policy vector $\mathbf{\Pi}(\mathrm{K_F}, \mu_1, \mu_2, \mu_3)$ that would generate target values $\mathbf{E}(^{\#}E_1, ^{\#}E_2, ^{\#}E_3, ^{\#}E_F)$ – they rearranged (8.54) in the form $\mathbf{\Psi} \cdot \mathbf{\Pi} = \mathbf{E}$ reflected by (8.60).

(8.60) Credit rise and zone shares as functions of target payments

$$\begin{vmatrix} 14110 & -8258 & -7340 & -8845 \\ 8150 & -3810 & -5568 & -4945 \\ 41990 & -19002 & -21300 & -34705 \\ 15430 & -5994 & -7080 & -9005 \end{vmatrix} \cdot \begin{vmatrix} \mathrm{K_F} \\ \mu_1 \cdot \mathrm{K_F} \\ \mu_2 \cdot \mathrm{K_F} \\ \mu_3 \cdot \mathrm{K_F} \end{vmatrix} = \begin{vmatrix} (980) \cdot 6.36792 \cdot 10^{12} \\ (588) \cdot 5.65442 \cdot 10^{12} \\ (2940) \cdot 6.78729 \cdot 10^{12} \\ (2940) \cdot 2.90193 \cdot 10^{12} \end{vmatrix}$$

This system – consisting of four equations and four unknowns – may be solved for $\mathbf{\Pi}$ because determinant $|\mathbf{\Psi}| \neq 0$. Using *Mathematica*, the technocrats proved that all turnover targets are achievable when the yearly liquidity increment stays constant at $\mathrm{K_F} = 10^{12}$ *IAU*s but is allocated among the three currency areas in proportions $\mu_1 = 38.0\%$, $\mu_2 = 50.0\%$ and $\mu_3 = 12.0\%$, as opposed to $\mu_1 = \mu_2 = \mu_3 = 33.33\%$. The new credit shares do not disturb the positive signs either of Routh-Hurwitz conditions or of currency area flows.

International economic model (8.50) and its derivative (8.60) – being expressed in terms of macroeconomic aggregates – do not link Theta's steady state either with behaviour and finance of particular sectors or with quotations and quantities of specific goods. Therefore, any miscalculated gaps between demanded and available quantities in Zones One, Two, Three and the joint banking sector will keep exerting upward or downward pressures on quotations during the ensuing periods. Owing to the new credit allotment ($\mu_1 = 38.0\%$, $\mu_2 = 50.0\%$ and $\mu_3 = 12.0\%$) world receipts and payments will be redistributed among the three currency areas and their banking institutions as reflected by (8.61).

$$\begin{aligned} &(y_1 + y_2 + y_3 + y_f) = (31.08\% + 28.35\% + 31.81\% + 8.76\%) = 100\% \\ &(e_1 + e_2 + e_3 + e_f) = (29.33\% + 26.04\% + 31.26\% + 13.37\%) = 100\% \end{aligned} \tag{8.61}$$

Comparing (8.58) with (8.61), we observe that Zone Three has surpassed Zone One as the largest currency area (in respect of takings and outgoings) solely due to redistribution of the annual money supply. Other things being equal, every year the nominal sizes and relative shares of bank loans decide the incomes and expenditures of sectors, countries and zones compared to the respective productive capacities.

(8.62) Theta's steady state when $\mu_1 = \mu_2 = \mu_3 = 33.33\%$

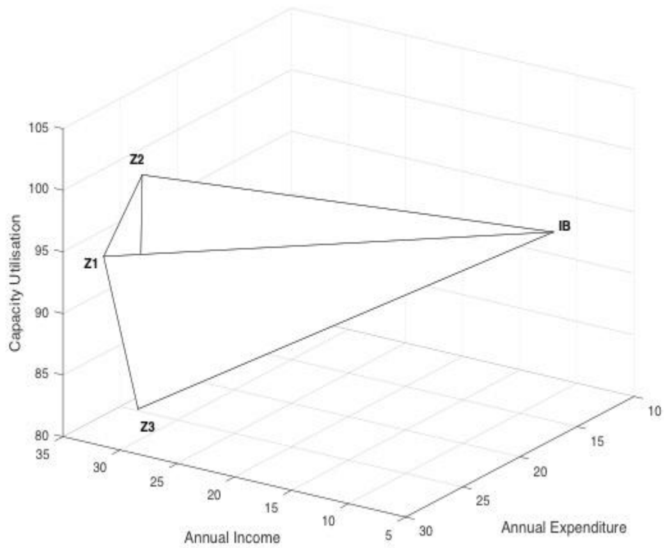


Figure (8.62) captures the equilibrium income and expenditure shares as well as the *capacity utilisation* rates of Zones One **Z1**(31.54%, 29.9%, *95.33*%), Two **Z2**(29.85%, 28.21%, *101.5*%), Three **Z3**(30.03%, 28.4%, *82.54*%) and their combined banking system **IB**(8.58%, 13.49%, *94.28*%) during the initial year. Divergent capacity utilisation rates denote the latent inflationary or deflationary pressures exerted on Theta's constituent parts.

(8.63) Theta's steady state when $\mu_1 = 38.0\%$, $\mu_2 = 50.0\%$ and $\mu_3 = 12.0\%$

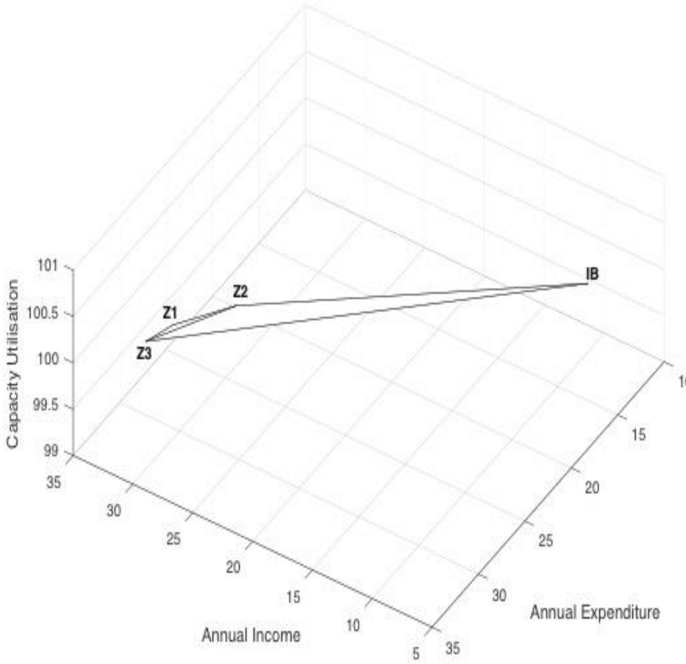


Figure (8.63) depicts the equilibrium income and expenditure shares plus the *capacity utilisation* rates of Zones One **Z1**(31.08%, 29.33%, *100.0*%), Two **Z2**(28.35%, 26.04%, *100.0*%), Three **Z3**(31.81%, 31.26%, *100.0*%) and their combined banking system **IB**(8.76%, 13.37%, *100.0*%) following redistribution of the annual liquidity increment. The new credit policy has rearranged Theta's circulation structure and, in this way, has balanced demand and supply in the said economy's four constituent parts, which are represented by the corresponding tetrahedron's vertices.

In this connection, it is worth mentioning that a series of commercial and central bank decisions before and after 2007 amplified deficits, enlarged demand and accelerated inflation, especially in some Mediterranean members of Eurozone. When credit expansion and apportionment among sectors and countries were brought under control, revenues and outlays in Portugal, Italy, Spain and Greece dropped sharply, leading some households and businesses to bankruptcy. Eventually, those monetary financial institutions which had granted the wrong amounts of credit were burdened with significant losses. Mathematical economists applying a Euclidean exchange model – hopefully to be developed by international financial organisations, such as the IMF, the ECB, the World Bank, etc. – will be well positioned to advise each national banking system on the magnitude and allocation of its annual liquidity increment so as to optimise supply of goods and minimise incidence of inflation or deflation worldwide.

Part III. Didactic and Philosophical Aspects

Chapter 9. Circulation of Money and Goods

The laws of mathematics hold in economic as well as in physical systems. Consumers, businesses and banks trade services, products and securities through networks or circuits. The twelve fundamental premises yield economic systems composed of 2n first-order non-homogeneous linear differential equations that simulate demand and supply of n market goods by n institutional sectors. Households, enterprises, financiers, etc., as well as their respective goods may be classified under sovereign countries, currency zones, etc. Modern mathematical software facilitates exploration of equilibrium, dynamic and other properties of Euclidean models, which possess the general form $\mathbf{dX/dt = A{\cdot}X - K}$. It is noted that the yearly:

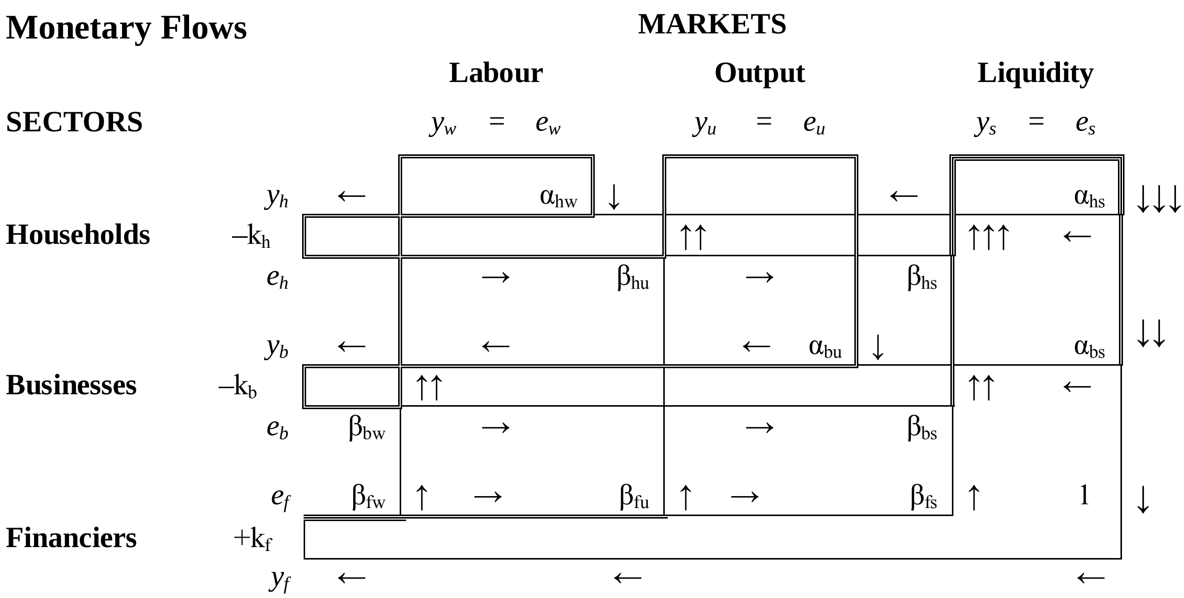
- steady state $\mathbf{X}^* = (y_i^*, e_i^*)$ is the solution of the time-invariant equation system $\mathbf{A{\cdot}X = K}$ associated with the non-homogeneous linear differential equation system $\mathbf{dX/dt = A{\cdot}X - K}$;
- stability conditions are determined by the coefficients of the characteristic equation $|\mathbf{A} - \lambda{\cdot}\mathbf{I}|$ associated with the homogeneous linear differential equation system $\mathbf{dX/dt = A{\cdot}X}$.

The central bank may intervene regarding annual expansion, allocation and cost of credit. Given the scalar nature of quotations, the monetary financial authority affects the exchange network's equilibrium flows $\mathbf{X}^* = (y_h^*, e_h^*, y_b^*, e_b^*, y_f^*, e_f^*)$ and quantities demanded $\mathbf{Q} = (\mathrm{q_w}^*, \mathrm{q_u}^*, \mathrm{q_s}^*)$. Every year the banking system issues liquid deposits (liabilities) in order to trade them for guaranteed bonds (assets) issued by the other sectors. In an elementary economy – consisting of consumers *h*, producers *b* and banks *f* – the monetary financial balance sheet's yearly expansion takes place according to the following equations:

$$\mathrm{dM/dt} = e_f - y_f = \mathrm{k_f} = (y_h - e_h) + (y_b - e_b) = \mathrm{k_h} + \mathrm{k_b} > 0$$
$$\mathrm{k_f} / \mathrm{k_f} = (\mathrm{k_h} + \mathrm{k_b}) / \mathrm{k_f} \Rightarrow 1 = \kappa_h + \kappa_b\,; \tag{9.1}$$

The exchange network is kept going by the double entry operations of the banking system. Euclidean economic models reflect the debits and credits entered on the current and capital accounts of sectors, countries, zones, etc., per time unit. Receipts and payments differ when they concern sectors, countries or zones but coincide when they concern market goods or the entire exchange network. The complexity of each economic system depends on the numbers of sectors and markets as well as countries and zones which engage in transactions. The resulting flows can be illustrated by means of plain graphics or animated pictures. Figure (9.2) presents the circuits of elementary exchange network (4.1).

(9.2) Circuits of elementary economy Alpha



According to combinatorics, the basic closed economy's three sectors and three markets intertwine one complete and six partial sets of circuits involving real goods and financial paper. Two of the partial sets deserve special attention because of their didactic value. The following micro-analysis and micro-synthesis highlight the inseparable mathematical and economic aspects of Euclidean exchange networks as well as the scientific potential of Euclidean modelling methodology. The first circuit concerns interaction of bank income and expenditure in isolation from other sectors (i.e. $\bar{e}_h$ and $\bar{e}_b$ are assumed to be invariant):

$$\begin{aligned} dy_f/dt &= \beta_{hs}\cdot\bar{e}_h + \beta_{bs}\cdot\bar{e}_b - y_f \\ de_f/dt &= y_f + k_f - e_f \end{aligned} \qquad (9.3)$$

9.1. Flows of the money issuing sector

The steady state of differential system (9.3) is provided by synchronous system (9.4), which consists of two linear equations with two unknowns:

$$\begin{vmatrix} -1 & 0 \\ 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_f \\ e_f \end{vmatrix} = \begin{vmatrix} \beta_{hs}\cdot\bar{e}_h + \beta_{bs}\cdot\bar{e}_b \\ -k_f \end{vmatrix} = \mathbf{A}_F\cdot\mathbf{X}_F = \mathbf{K}_F \qquad (9.4)$$

In this case, household and business flows are considered as exogenous data. Due to its linear form and because $|\mathbf{A_F}| = 1 \neq 0$, system (9.4) is solved by applying Cramer's rule:

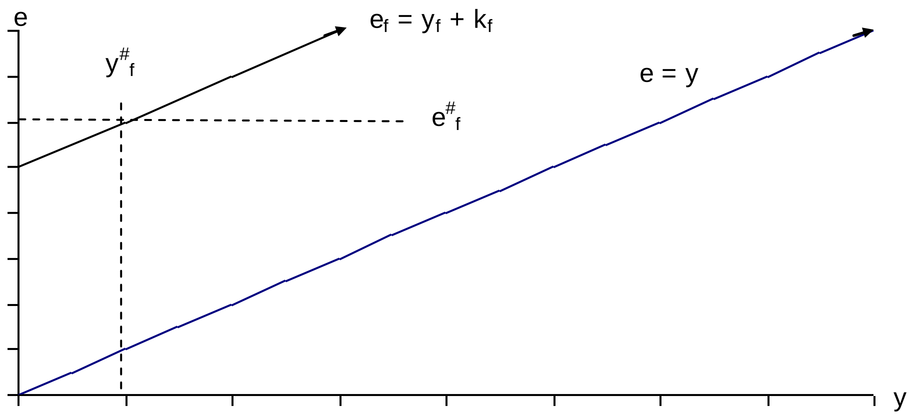
$$\begin{aligned} y^{\#}_f &= \beta_{hs} \cdot \bar{e}_h + \beta_{bs} \cdot \bar{e}_b \\ e^{\#}_f &= y^{\#}_f + k_f = \beta_{hs} \cdot \bar{e}_h + \beta_{bs} \cdot \bar{e}_b + k_f \end{aligned} \tag{9.5}$$

Superscript $^{\#}$ denotes a variable's partial equilibrium. Figure (9.6) shows the lines where:

(a) the revenue $y^{\#}_f$ of financiers is equal to the outlay of consumers $\beta_{hs} \cdot \bar{e}_h$ and producers $\beta_{bs} \cdot \bar{e}_b$ in respect of their debt in the form of perpetual bonds;

(b) the outlay $e^{\#}_f$ of financiers is equal to their revenue $y^{\#}_f$ from consumers and producers plus the authorised increase of their liabilities k_f.

(9.6) Four quadrants created by a subsystem's dead point



Lines (9.5) intersect at point $(y^{\#}_f, e^{\#}_f)$ because:

(a) line $y^{\#}_f$ is vertical on its axis, having a unique value emanating from the money-accepting sectors' expenditure in respect of their borrowing;

(b) line $e^{\#}_f$ is equal to $y^{\#}_f$ plus a constant k_f, representing the money-issuing sector's authorised additional lending (credit expansion) per annum.

Steady state $\mathbf{X}_F^{\#} = \mathbf{A}_F^{-1} \cdot \mathbf{K}_F = (y^{\#}_f, e^{\#}_f) = (\beta_{hs} \cdot \bar{e}_h + \beta_{bs} \cdot \bar{e}_b, \beta_{hs} \cdot \bar{e}_h + \beta_{bs} \cdot \bar{e}_b + k_f)$ reflects the banking system's partial equilibrium. Later, we will examine whether the endogenous variables of (9.3) tend to approach $(y^{\#}_f, e^{\#}_f)$ starting from any initial combination (y^0_f, e^0_f). Stability of equilibrium vector $\mathbf{X}_F^{\#}$ depends on the eigenvalues of the corresponding homogeneous linear differential equation system shown below:

$$\begin{vmatrix} dy_f/dt \\ \\ de_f/dt \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ \\ 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_f \\ \\ e_f \end{vmatrix} = \mathbf{A}_F \cdot \mathbf{X}_F \qquad (9.7)$$

System (9.7) possesses the general form $\mathbf{dX_F/dt} = \mathbf{A_F} \cdot \mathbf{X_F} = \boldsymbol{\lambda} \cdot \mathbf{I} \cdot \mathbf{X_F}$, where $\mathbf{A_F}$ is a square matrix, $\mathbf{X_F}$ is a state vector, and $\boldsymbol{\lambda}$ is the eigenvector of square matrix $\mathbf{A}_F$. A non-homogeneous linear differential equation system is globally stable if, and only if, the eigenvalues $\boldsymbol{\lambda}$ of the corresponding homogeneous system have roots with negative real parts. For this necessary and sufficient condition to hold, the Routh-Hurwitz determinants rh_i, $i = 1, \ldots, n$, must be positive. The Routh-Hurwitz determinants of system (9.3) or (9.7) coincide with coefficients a_1 and a_2 of characteristic equation (9.8).

$$\left| \mathbf{A}_F - \boldsymbol{\lambda}\cdot\mathbf{I} \right| = \begin{vmatrix} \alpha_{11} - \lambda & \alpha_{12} \\ \\ \alpha_{21} & \alpha_{22} - \lambda \end{vmatrix} = \lambda^2 - (\alpha_{11}+\alpha_{22})\cdot\lambda + (\alpha_{11}\cdot\alpha_{22} - \alpha_{12}\cdot\alpha_{21}) = 0 \qquad (9.8)$$

$$\left| \mathbf{A}_F - \boldsymbol{\lambda}\cdot\mathbf{I} \right| = a_0 \cdot \lambda^2 + a_1 \cdot \lambda + a_2 = 0$$

By substituting $\alpha_{11} = -1$, $\alpha_{12} = 0$, $\alpha_{21} = 1$ and $\alpha_{22} = -1$ in a_1 and a_2 above, we conclude that the isolated interaction of bank income and expenditure, as reflected by differential equation system (9.3), is globally stable because:

$$rh_1 = a_1 = 2 > 0 \; ; \quad rh_2 = a_2 = 1 > 0 \qquad (9.9)$$

The steady state of (9.3) is a stable node because trajectories (formed by the two endogenous streams) converge towards ($y^{\#}_f$, $e^{\#}_f$) at ever lower speeds, irrespective of their initial composition (y^0_f, e^0_f). In general:

- for any $y_f < y^{\#}_f$, y_f tends to increase, while for any $y_f > y^{\#}_f$, y_f tends to decrease;
- for any $e_f < e^{\#}_f$, e_f tends to increase, while for any $e_f > e^{\#}_f$, e_f tends to decrease.

According to our premises, the revenue and outlay of any sector as well as the takings and outgoings of any market are sums of (bank account) debits and credits per time unit. Outside equilibrium, every pair of monetary flows (y_i, e_i) is associated with:

i) a vector measuring the acceleration / deceleration of the sector's receipts and payments;
ii) a scalar measuring the differentiation (angle or slope) of the pair's common trajectory.

In a two dimensional model such as (9.3), the aforementioned results concerning velocity and direction can be classified under the four quadrants created by lines $y^{\#}_f$ and $e^{\#}_f$ while passing through $X_F^{\#} = (y^{\#}_f, e^{\#}_f)$. At any point (y_f, e_f) on Euclidean plane (9.6), the vector representing the acceleration and / or deceleration of financier revenue and outlay consists of the ordered pair of differential equations (9.3) seen below:

$$(dy_f/dt,\ de_f/dt) = (\beta_{hs}\cdot\bar{e}_h + \beta_{bs}\cdot\bar{e}_b - y_f,\ \ y_f + \mathrm{k_f} - e_f) \qquad (9.10)$$

At the circuit's dead point $\mathbf{X}_F^{\#} = (y^{\#}_f, e^{\#}_f)$, monetary flows remain constant. As shown in Table (9.11), bank receipts and payments accelerate or decelerate principally depending on the quadrant of the Euclidean plane (i.e. phase of the business cycle) from which each monetary flow begins relatively to its steady state $\mathbf{X}_F^{\#} = (y^{\#}_f, e^{\#}_f)$.

(9.11) Increase (+) or decrease (–) of bank monetary flows

QUADRANT	$y_f < y_f^{\#}$	$y_f^{\#} < y_f$
$e_f < e_f^{\#}$	$(dy_f/dt\ ,\ de_f/dt) = (+, +)$	$(dy_f/dt\ ,\ de_f/dt) = (-, +)$
$e_f^{\#} < e_f$	$(dy_f/dt\ ,\ de_f/dt) = (+, -)$	$(dy_f/dt\ ,\ de_f/dt) = (-, -)$

The mathematical function that provides the angle dy_i/de_i of a trajectory formed by a pair of sector flows on the Euclidean plane equals the ratio of the respective differential equations. The division $(dy_i/dt)/(de_i/dt)$ eliminates the time element t so that the resulting slope becomes independent of the interplay of dynamic forces at a particular point (y_f, e_f). In the case of the banking sector of economy Alpha or model (5.1), system (9.3) yields:

$$dy_f/dt\,/\,de_f/dt = dy_f/de_f = ({}^1/_8\cdot\bar{e}_h + {}^1/_6\cdot\bar{e}_b - y_f)\,/\,(y_f + \mathrm{k_f} - e_f) \qquad (9.12)$$

At any point (y_f, e_f), slope dy_f/de_f gives the ratio of the two flows without disclosing their velocity or direction. Table (9.13) summarises the combinations of dy_f and de_f signs associated with the trajectory of banking system receipts and payments.

(9.13) Sign / direction of slopes

QUADRANT	$y_f < y^{\#}_f$	$y^{\#}_f < y_f$
$e_f < e^{\#}_f$	(+)/(+) = (+) positive	(+)/(–)= (–) negative
$e^{\#}_f < e_f$	(–)/(+) = (–) negative	(–)/(–)= (+) positive

The analysis just applied to the isolated flows of monetary financial institutions may be applied to any small or large sub-system of Euclidean economic models.

9.2. Interaction of households and businesses

In an elementary economy, such as Alpha, consumers and producers make up the 'real' (liquidity accepting) sector in contrast to banks which comprise the 'monetary' (liquidity issuing) sector. Differential equation system (9.14) reflects exchange of services for products and the reverse, in isolation from a continuously operating banking system:

$$
\begin{aligned}
dy_h/dt &= \beta_{bw} \cdot e_b + \beta_{fw} \cdot \bar{e}_f + \beta_{fs} \cdot \bar{e}_f - \alpha_{bs} \cdot y_b - y_h \\
de_h/dt &= y_h - \mathrm{k_h} - e_h \\
dy_b/dt &= \beta_{hu} \cdot e_h + \beta_{fu} \cdot \bar{e}_f + \beta_{fs} \cdot \bar{e}_f - \alpha_{hs} \cdot y_h - y_b \\
de_b/dt &= y_b - \mathrm{k_b} - e_b
\end{aligned}
\tag{9.14}
$$

In the above partial model, bank annual flows and financial stocks are exogenous. By the way, it is mentioned that in old-style exchange networks the necessary liquidity supply depended on lucky discoveries of gold and silver as well as other means of payment and stores of wealth. Therefore, boom and bust largely depended on chance. In modern monetary economies, the banking system balance sheet operates as a driving engine and as a steering wheel. Understandably, it is paramount to be in the best hands. Although out-dated exchange networks incorporate no monetary control tools, it would be interesting to know whether (9.14) i) has a (partial) equilibrium solution $\mathbf{X}_R^{\#}$ and ii) returns to $\mathbf{X}_R^{\#}$ after any shock. The steady state ($dy_h/dt = 0$, $de_h/dt = 0$, $dy_b/dt = 0$, $de_b/dt = 0$) corresponding to (9.14) is reproduced in matrix form here below:

$$
\begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bu} \\ 1 & -1 & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h \\ e_h \\ y_b \\ e_b \end{vmatrix} = \begin{vmatrix} -(\beta_{fw}+\beta_{fs})\cdot\bar{e}_f \\ \mathrm{k_h} \\ -(\beta_{fu}+\beta_{fs})\cdot\bar{e}_f \\ \mathrm{k_b} \end{vmatrix} = \mathbf{A}_R \cdot \mathbf{X}_R = \mathbf{K}_R \tag{9.15}
$$

Non-homogeneous linear equation system (9.15) possesses the static form $\mathbf{A}_R \cdot \mathbf{X}_R = \mathbf{K}_R$. Confirmation of the solution's existence and uniqueness involves a routine mathematical procedure. According to Cramer's rule, the solution $\mathbf{X}_R^{\#}$ is expressed in terms of the elements of $\mathbf{A}_R$ and $\mathbf{K}_R$, provided determinant $|\mathbf{A}_R| \neq 0$. Recognising that $0 < \alpha_{hs} < 1$, $0 < \beta_{hu} < 1$, $0 < \alpha_{bs} < 1$, $0 < \beta_{bu} < 1$, it follows that:

$$
|\mathbf{A}_R| = \alpha_{hu} \cdot \alpha_{bs} + \beta_{hu} \cdot \beta_{bs} + \alpha_{bu} \cdot \beta_{hs} + \beta_{bw} \cdot \alpha_{hs} \neq 0 \tag{9.16}
$$

Households and businesses, which barter labour for output and *vice versa* without a monetary financial system, always end up at a non-trivial (partial) equilibrium vector $\mathbf{X}_R^{\#} = (y^{\#}_h, e^{\#}_h, y^{\#}_b, e^{\#}_b)$. It goes without saying that $(y^{\#}_h, e^{\#}_h, y^{\#}_b, e^{\#}_b)$ is not inevitably better than $(y^{*}_h, e^{*}_h, y^{*}_b, e^{*}_b, y^{*}_f, e^{*}_f)$, which would have been achieved had a banking sector been operating properly. Despite their useful and indispensable role, since medieval times financiers have been viewed as troublemakers or villains because too often they were caught undermining the exchange network's benign operation through questionable purchases of services, products and securities as well as ill-advised allocations of loans and pricing of deposits. Euclidean modelling methodology provides tools for assignment of duties and responsibilities as well as for evaluation of acts and omissions by national and international monetary authorities per time period.

Having established existence and uniqueness of (partial) solution $\mathbf{X}_R^{\#}$, we wish to determine whether (9.14) would return to $\mathbf{X}_R^{\#} = (y^{\#}_h, e^{\#}_h, y^{\#}_b, e^{\#}_b)$ starting from any initial combination $\mathbf{X}_R^{\mathbf{0}} = (y^{0}_h, e^{0}_h, y^{0}_b, e^{0}_b)$ of consumer and producer monetary flows. System (9.14) treats bank flows (y_f, e_f) as exogenous variables. The stability of partial equilibrium $\mathbf{X}_R^{\#}$ – generated by non-homogeneous system (9.14) – depends on the eigenvalues of the associated homogeneous linear differential equation system (9.17).

$$\begin{vmatrix} dy_h/dt \\ de_h/dt \\ dy_b/dt \\ de_b/dt \end{vmatrix} = \begin{vmatrix} -1 & 0 & -\alpha_{bs} & \beta_{bu} \\ 1 & -1 & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1 & 0 \\ 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h \\ e_h \\ y_b \\ e_b \end{vmatrix} = \mathbf{A}_R \cdot \mathbf{X}_R = \mathbf{0} \qquad (9.17)$$

The above dynamic system possesses the general form $\mathbf{dX_R/dt} = \mathbf{A_R} \cdot \mathbf{X_R} = \boldsymbol{\lambda} \cdot \mathbf{I} \cdot \mathbf{X_R}$. The said system reflects interaction of receipts and payments linked to trading of labour and output between consumers and producers, while everything else stays the same. Non-homogeneous system (9.14) tends to return to (partial) equilibrium position $\mathbf{X}_R^{\#}$ if, and only if, the real parts of eigenvalues $\boldsymbol{\lambda}$ of characteristic equation $|\mathbf{A_R} - \boldsymbol{\lambda} \cdot \mathbf{I}| = 0$, associated with homogeneous system (9.17), are negative:

$$|\mathbf{A}_R - \boldsymbol{\lambda} \cdot \mathbf{I}| = \begin{vmatrix} \alpha_{11}-\lambda & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22}-\lambda & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33}-\lambda & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44}-\lambda \end{vmatrix} = \begin{vmatrix} -1-\lambda & 0 & -\alpha_{bs} & \beta_{bu} \\ 1 & -1-\lambda & 0 & 0 \\ -\alpha_{hs} & \beta_{hu} & -1-\lambda & 0 \\ 0 & 0 & 1 & -1-\lambda \end{vmatrix}$$

$$\left| \mathbf{A_R} - \boldsymbol{\lambda} \cdot \mathbf{I} \right| = a_0 \cdot \lambda^4 + a_1 \cdot \lambda^3 + a_2 \cdot \lambda^2 + a_3 \cdot \lambda + a_4 = 0 \tag{9.18}$$

Let a_0, a_1, a_2, a_3 and a_4 be the coefficients of the above fourth degree polynomial in λ. Characteristic equation $\left| \mathbf{A_R} - \boldsymbol{\lambda} \cdot \mathbf{I} \right|$ has four real or imaginary roots (eigenvalues) in terms of λ. One solution x_i, $i = 1,\ldots,4$, of (9.17) corresponds to each one of the roots λ_i.

$$\begin{vmatrix} a_1 & a_0 & 0 & 0 \\ a_3 & a_2 & a_1 & a_0 \\ 0 & a_4 & a_3 & a_2 \\ 0 & 0 & 0 & a_4 \end{vmatrix} \tag{9.19}$$

The necessary and sufficient condition for the real parts of eigenvalues λ_i to have negative signs is that Routh-Hurwitz determinants rh_n, n = 1,…, 4 – i.e. the principle minors of matrix (9.19) formed by the coefficients of characteristic equation (9.18) – be greater than zero. In the case of matrix (9.19), the Routh-Hurwitz conditions are:

$$\begin{aligned} rh_1 &= a_1 \quad ; \quad rh_2 = (a_1 \cdot a_2 - a_0 \cdot a_3) \quad ; \\ rh_3 &= a_3 \cdot (rh_2) - a_4 \cdot a_1^2 \quad ; \quad rh_4 = a_4 \cdot (rh_3) \quad . \end{aligned} \tag{9.20}$$

Granted fundamental assumption $0 < \alpha_{hs} < 1$, $0 < \beta_{hu} < 1$, $0 < \alpha_{bs} < 1$, $0 < \beta_{bu} < 1$, it can be proven that all Routh-Hurwitz determinants are positive, and thus system (9.14) is globally stable. In economy Alpha, where $\alpha_{hw} = {}^8/_9$, $\alpha_{hs} = {}^1/_9$; $\beta_{hu} = {}^7/_8$, $\beta_{hs} = {}^1/_8$; $\alpha_{bu} = {}^6/_7$, $\alpha_{bs} = {}^1/_7$; $\beta_{bw} = {}^5/_6$, $\beta_{bs} = {}^1/_6$, we obtain: $rh_1 = 4$, $rh_2 \approx 19.75$, $rh_3 \approx 75.11$ and $rh_4 \approx 35.48$. In the absence of bank involvement, household and business transactions always tend towards partial equilibrium ($y^{\#}_h$, $e^{\#}_h$, $y^{\#}_b$, $e^{\#}_b$). Equilibrium vector $\mathbf{X}_R^{\#}$ of (9.14) comprises a globally stable node because the trajectories formed by endogenous variables (y_h, e_h, y_b, e_b) converge towards $\mathbf{X}_R^{\#}$ at ever lower speeds, regardless of the latter's initial combination $\mathbf{X}_R^{\mathbf{0}} = (y^0_h, e^0_h, y^0_b, e^0_b)$. In other words:

- for any $y_h < y^{\#}_h$ y_h tends to increase, while for any $y_h > y^{\#}_h$ y_h tends to decrease;
- for any $e_h < e^{\#}_h$ e_h tends to increase, while for any $e_h > e^{\#}_h$ e_h tends to decrease;
- for any $y_b < y^{\#}_b$ y_b tends to increase, while for any $y_b > y^{\#}_b$ y_b tends to decrease;
- for any $e_b < e^{\#}_b$ e_b tends to increase, while for any $e_b > e^{\#}_b$ e_b tends to decrease.

The partial approach methodology illustrated above is eminently suitable for application in international exchange models, because the outgoings and takings of less significant countries – from the economic point of view – may be aggregated and treated as exogenous parameters per calendar year.

9.3. The exchange network as a dynamic space

An economic model possessing the general form **dX/dt** = **A·X** – **K** may be divided into n!/(n-1)!·1! + n!/(n-2)!·2! +... + n!/(n-n)!·n! dynamic sub-systems (dy_i/dt, de_i/dt) defined on the Euclidean space. Each circuit (dy_i/dt, de_i/dt), $i = 1,\ldots, n$, consists of two non-homogeneous linear differential equations, which reproduce income and expenditure (y_i, e_i) of households, businesses, financiers, etc. It follows that each sub-system is characterized by its own (partial) properties. At any given moment, the monetary flows of consumers, producers, bankers, etc., and thus the annual sales of services, products, securities, etc., are determined by the behaviour coefficients **A** of institutional sectors and the lending practices **K** of banking institutions. Each pair of differential equations (dy_i/dt, de_i/dt) defines a dynamic plane (y_i, e_i) where revenue and outlay accelerate or decelerate, unless they coincide with dead point ($y_i^{\#}$, $e_i^{\#}$). The rates of change (dy_i/dt, de_i/dt) of sector receipts and payments (y_i, e_i) are directly proportional to the distance from their respective equilibrium flows $\mathbf{X}_i^{\#} = (y_i^{\#}, e_i^{\#})$. When a dynamic sub-system (dy_i/dt, de_i/dt) is stable, the depicted sector's income and expenditure (y_i, e_i) tend towards a (partial) equilibrium vector ($y_i^{\#}$, $e_i^{\#}$) regardless of the initial combination (y_i^0, e_i^0) of monetary flows. All dynamic planes (y_i, e_i), i.e. the n pairs of differential equations comprising the economic model **dX/dt** = **A·X** – **K**, compose a dynamic space with dimension (R^{2n}, t). When exchange network **dX/dt** = **A·X** – **K** is globally stable, then sector (and thus country and zone) takings and outgoings tend towards vector $\mathbf{X}^*$ regardless of initial vector $\mathbf{X}^0$.

Chapter 10. Mathematical Aspects of Economic Theory

The twelve fundamental assumptions apply to all monetary economies, irrespective of their simplicity or complexity. Euclidean modelling methodology makes possible composition of first-order non-homogeneous linear differential equation systems which possess the general form $\mathbf{dX/dt = A \cdot X - K}$ and duplicate demand and supply of n market goods by n institutional sectors (aggregated under countries or zones). The n pairs of nominal flows stem from opposite and simultaneous accounting entries recorded by banking institutions in nominal units on both sides of their balance sheet. It is highlighted that revenue and outlay do not cancel each other out, despite being opposite and simultaneous. Debits and credits concerning any sector, country, zone, etc., correspond to assets and liabilities, which are always measured in different accounting dimensions or coordinate axes.

Due to inherent linearity and non-homogeneity, every Euclidean flow network has a unique steady state $\mathbf{X}^* = \mathbf{A}^{-1} \cdot \mathbf{K}$ when $|\mathbf{A}| \neq 0$. Equilibrium position $\mathbf{X}^*$ consists of the annual income and expenditure (y_i^*, e_i^*) of n sectors. To each vector $\mathbf{X}^*$ corresponds a vector $\mathbf{R}^*$ consisting of the annual sales of n market goods (i.e. labour, output, liquidity, etc.). Only average quotations are meaningful in macro models because the fine details, which differentiate the various services, products, securities, etc., are lost through aggregation of agents into sectors, countries, zones, etc. Division of a steady-state market flow by the corresponding quotation yields the number of units demanded and supplied per year. Every Euclidean economic model – while duplicating financial and real transactions regarding the exchange of n goods among n sectors, countries or zones – comprises:

- a dynamic system where monetary flows influence each other;
- a vector space consisting of n pairs of unequal monetary flows.

Within each time unit, income and expenditure (y_i, e_i) tend to rise or fall (dy_i/dt, de_i/dt) depending on their distance from the respective equilibriums (y_i^*, e_i^*). Whenever a behaviour coefficient or policy parameter changes, division of receipts and payments among the exchange network's sectors, countries and zones also changes. Internal and external jolts cause endogenous variables (y_1, e_1, … , y_n, e_n) to fluctuate unevenly. If oscillations become smaller, the dynamic interface of revenue and outlay is stable; if oscillations become larger, the system of sector, country and zone interactions is unstable. Stability of $\mathbf{X}^*$ depends on the Routh-Hurwitz determinants of $\mathbf{dX/dt = A \cdot X - K}$ being greater than zero. Behaviour coefficients and policy parameters determine whether Routh-Hurwitz determinants rh_i and equilibrium monetary flows (y_i^*, e_i^*), $i = 1,\ldots, n$, are positive and / or negative. Policymakers always find excuses for adjusting the parameters under their control in order to flaunt their power to move the economic system's equilibrium $\mathbf{X}^*$ from one position to another. As a result, they would like to know how to predict the consequences of their choices. In this connection, it is emphasised that consumers, producers, bankers, etc., insist upon earning and spending positive and adequate amounts relative to the available quantities and average quotations of market goods. Receipts and payments should always be greater than zero so that:

- the twin flows respond in the same direction as the money supply increment;
- interacting sectors, countries and zones survive and live well at the same time.

According to dynamic control theory, policymakers may achieve their goals if, and only if, the exchange network possesses a stable and thus unique steady-state $\mathbf{X}^*$, which consists of elements (y_1^*, e_1^*, … , y_n^*, e_n^*) having the same sign. If, in addition to the aforementioned properties, the number of monetary, fiscal and other policy tools is at least equal to the number of economic policy targets, then the exchange network becomes a perfectly dirigible system. With the assistance of *Mathematica* or other software, it is possible to elucidate the features and implications of Euclidean economic models.

10.1. The Pythagorean doctrine of numbers

Mathematical economic model $\mathbf{dX/dt = A \cdot X - K}$ yields an equilibrium vector $\mathbf{X}^*$ (i.e. the incomes and expenditures of sectors, countries or zones) which originates from the linear transformation of credit policy vector $\mathbf{K}$ (i.e. the apportionment of the liquidity increment among the acceptors of money) by means of structural matrix $\mathbf{A}^{-1}$ (i.e. the inverse of $\mathbf{A}$ portraying the interaction of sector, country or zone takings and outgoings) per time unit. Matrix $\mathbf{A}$ (and thus $\mathbf{A}^{-1}$) reflects the current lifestyle of households, the operational efficiency of entrepreneurs and the financial adeptness of bankers as well as the monetary, fiscal or other policies pursued by the appropriate authorities per annum. Matrix $\mathbf{A}$ (or $\mathbf{A}^{-1}$) – by incorporating the ratios according to which sectors, countries or zones apportion their receipts and payments – mirrors the particular economy's structure during the unit of time. Per calendar year, $t_0 \leq t \leq t_1$, every Euclidean exchange network is associated with unique matrices $\mathbf{A}$ and $\mathbf{A}^{-1}$, and thus unique determinants $|\mathbf{A}|$ and $|\mathbf{A}^{-1}|$, in the same way as every human being is associated with distinctive fingerprints, irises or DNA helices. The last conclusion leads us to the Pythagorean doctrine of numbers (digital theory).

Pythagoras espoused that the nature of the cosmos is mathematical at its deepest level. He believed that everything in the universe is a manifestation of the unit, i.e. the simplest numeral. Pythagoras regarded numbers and forms as collections of units or points (such as pebbles or stars). As a result, every tangible or intangible thing is represented by a combination of one or more integers. Pythagoreans advocated that even abstract ideas, such as harmony, justice and freedom have their respective numerals. In accordance with this doctrine, every economic system comprises an integral whole associated with certain abstract numbers or characteristic magnitudes. For example, in Chapters **5** and **8** we saw that the configuration of Alpha is epitomized by $|\mathbf{A}| = {}^{253109}/_{1368576}$ or its inverse $|\mathbf{A}^{-1}| = {}^{1368576}/_{253109}$. It is recognised that exchange networks may be symbolised by other numbers or magnitudes, in addition to determinants $|\mathbf{A}|$ or $|\mathbf{A}^{-1}|$. In this connection, it is noted that two economies, called Alpha and Beta, having similar exchange structures, are inevitably represented by similar matrices **A** and **B**, and thus by the same determinant $|\mathbf{A}| = |\mathbf{B}|$. A modern application of the Pythagorean doctrine of numbers is given below.

10.2. Measuring the degree of economic convergence

Robert Mundell's Theory of Common Currency Areas has been criticised for failing to provide a comprehensive method for measuring the:

- degree to which two or more countries are converging toward similar structures;
- appropriate time at which they would be ready to adopt a common currency.

During the 2007 - 2015 economic crises, which started with the subprime outrage and peaked with several bank and government bailouts, many commentators argued that Portugal, Italy, Spain and Greece should not have adopted the common European currency because they had not fulfilled the necessary and sufficient conditions of Eurozone membership. Let us now see whether Euclidean methodology makes dealing with such questions possible. Matrices $\mathbf{A}_t$ of general mathematical model $\mathbf{dX}_t/\mathbf{dt} = \mathbf{A}_t \cdot \mathbf{X}_t - \mathbf{K}_t$ consist of sector behaviour coefficients and (as shown in Chapter **6**) fiscal policy parameters. Each matrix $\mathbf{A}_t$, which defines the circulation network per relevant period *t*, is linked to a unique number expressed by determinant $|\mathbf{A}_t|$. Clearly, economic systems can be ranked annually according to the determinants of their structural matrices. If statistical data were collected globally, according to the requirements of Euclidean models, then the hypothesis that «increasing proximity of determinant magnitudes signifies increasing proximity of economic structures» could be tested. In such case, formula (10.1) could be used as an index of structural difference:

$$\big|\, |\mathbf{A}|_t - |\mathbf{\Gamma}|_t \,\big| \,/\, \big|\, |\mathbf{A}|_t + |\mathbf{\Gamma}|_t \,\big| \qquad (10.1)$$

Two identical economies would have zero as index of structural difference. In Chapter **8** we covered two economies called Alpha and Gamma having sector and market interaction determinants $|\mathbf{A}| = {}^{253109}/_{1368576}$ and $|\mathbf{\Gamma}| = {}^{59}/_{864}$, respectively. According to formula (10.1), the index of structural difference would be 46.07% during the particular year. If we had sufficient observations to compute the time-series (10.1), we would be able to argue whether the two exchange networks tended to converge or diverge. For the time being, absence of statistical data prevents us from testing the hypothesis and, thus, adopting the method for assessing:

- the degree to which countries are converging toward analogous structures and
- the appropriate time at which they will be ready to adopt a common currency.

10.3. Plato's theory of forms

Pythagoras's doctrine of numbers is not entirely dissimilar to Plato's theory of forms. Pythagoras believed that tangible things and abstract concepts are made up of numbers. The Pythagorean parallelism between geometry and arithmetic rested upon the rule that the point is to space what the unit is to numbers. This superficial understanding of the world was fatal to further development of geometry and arithmetic by the Pythagoreans. Their particular hypothesis prevented them from exploring the continuity of spaces, of numbers and, ultimately, of scientific and philosophical ideas. As long as irrational numbers, i.e. most square and cubic roots, cannot be approached through arithmetic units or integers, mathematicians should define them using another approach. Thus, Plato rejected the Pythagorean point-unit association and sought to relate geometric continuity to irrational numbers. Plato went on to declare that irrationals are numbers, just as much as integers and their derivatives. He asserted that rational and irrational numbers can be placed in a sequence or line next to each other, if they are seen as generated from geometric triangles.

The Platonic parallelism between (tangible) geometry and (abstract) arithmetic rested upon the rule that every geometric triangle corresponds to a numerical triplet. With the exception of its absolute size, a triangle is fully defined by the proportions of its sides. In this context, Plato proposed that the human mind ultimately perceives things and ideas as forms. He elaborated his breakthrough by suggesting that real and imaginary things may be meaningfully analysed and synthesised by means of triangles, i.e. the simplest shape. By proposing that notions or concepts may be examined via polygons or polyhedrons and, hence, numbers or vectors, Plato forever linked the examination of scientific and philosophical ideas to the visual (plane to see) world of analytic geometry and applied mathematics. The sign "*Let no one ignorant of geometry enter*" was positioned at the door of Plato's Academy in order to emphasise the unbreakable connection between logical thinking and numerical representation.

10.4. The dual world of opposites

The dual nature of economic activity and the twofold dimension of financial phenomena enable reproduction of sector and market as well as country and currency transactions as sets of points on the R^2 Euclidean plane or the (R^2, t) dynamic space. In this context, the pairs of monetary flows, which make up steady-state vector $\mathbf{X}^* = (y_1^*, e_1^*, \ldots, y_n^*, e_n^*)$, are equal to the number of sectors and markets or countries and zones which make up the exchange network. According to the adopted convention, receipts (assets) are measured on the x-axis of abscissas while payments (liabilities) are measured on the y-axis of ordinates. Each pair of sector, country or zone flows (y_i^*, e_i^*), $i = 1, \ldots, n$, defines one of the n vertices of the polygon representing the closed system. The economic polygon's shape – just like any geometric polygon's form – depends on the ratios (angles) among its constituent elements (sides). Specifically, the economic polygon's shape depends on:

- the way institutional sectors, sovereign countries or currency zones apportion their demand for income and supply of expenditure, and
- the way monetary financial institutions apportion the universal liquidity increment among agents, sectors, countries and zones.

The banking balance sheet's incessant modification – in respect of substance and appearance – decides the magnitudes and ratios as well as the directions and signs of household, business, financier, etc., nominal flows. The economic polygon's size and shape depend, respectively, on the yearly issue of liabilities (deposits) and on the latter's distribution as assets (deposits) of sectors, countries and zones by monetary financial institutions. In this framework, central banks – by guiding allotment of annual credit supply among sectors, countries and zones – may ensure that receipts and payments become greater than zero and commensurate with available supplies.

Every geometric polygon with n vertices consists of n!/3!·(n-3)! triangles. Consequently, an economic polygon's area (or the sum of the areas of its constituent triangles) is one of the exchange network's characteristic magnitudes. For example, Alpha's equilibrium position is reproduced by linear system (10.2).

$$\begin{vmatrix} -1 & 0 & -1/7 & 5/6 & 0 & 2023/3168 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ -1/9 & 7/8 & -1 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1/8 & 0 & 1/6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{vmatrix} \cdot \begin{vmatrix} y_h^* \\ e_h^* \\ y_b^* \\ e_b^* \\ y_f^* \\ e_f^* \end{vmatrix} = \begin{vmatrix} 0 \\ 1/2 \\ 0 \\ 1/2 \\ 0 \\ -1 \end{vmatrix} \cdot 10^9 = \mathbf{A}\cdot\mathbf{X}^* = \mathbf{K} \quad (10.2)$$

System (10.2) confirms that structural matrix **A** linearly transforms credit policy vector **K** = $10^9\cdot(0, {}^1/_2, 0, {}^1/_2, 0, -1)$ into vector $\mathbf{X}^* = 10^9\cdot(0.59342, 0.09342; 0.52360, 0.0236; 0.01561, 1.01561)$ which consists of institutional sector incomes and expenditures.

$$\text{Triangle Area} = \begin{vmatrix} y_1 & e_1 & 1 \\ y_2 & e_2 & 1 \\ y_3 & e_3 & 1 \end{vmatrix} = |(½)\cdot(y_1\cdot e_2 + e_1\cdot y_3 + y_2\cdot e_3 - y_1\cdot e_3 - e_1\cdot y_2 - y_3\cdot e_2)| > 0 \quad (10.3)$$

According to analytic geometry, formula (10.3) generates the area of a triangle with vertices (y_1, e_1), (y_2, e_2), (y_3, e_3). In geometry as well as in economics, form plays the most meaningful role because magnitude stems from measuring the configuration or the turnover with a larger or smaller unit. Absolute size is more relevant in practical application than in theoretical analysis. With *Mathematica*'s help, we substitute Alpha's annual flows for (y_1, e_1), (y_2, e_2), (y_3, e_3) in blueprint (10.3) to discover that the area of the triangle depicting the particular network is equal to $1.0473\cdot10^{17}$. Exchange activity is a two dimensional phenomenon linked to both income and expenditure. Therefore, areas calculated by means of formula (6.17) may be more appropriate measures of economic participation by agents, sectors, countries, etc., than either of their twin flows (y_i, e_i) in isolation.

10.5. A third dimension of exchange networks

In disaggregated exchange networks, given the annual behaviour coefficients, policy parameters, average prices and available amounts, Euclidean methodology yields the receipts and payments of consumers, producers, bankers, etc., as well as the clearing quotations and quantity gaps of traded goods. Per time unit, to each equilibrium vector $\mathbf{X}^*$ of sector flows corresponds an equilibrium vector $\mathbf{R}^*$ of market transactions and *vice versa*. At steady states $\mathbf{X}^*$ and $\mathbf{R}^*$:

- sector income and expenditure differ by the amount of bank credit;
- receipts and payments concerning traded goods are inevitably equal.

By dividing the annual sales of services ρ_w^*, products ρ_u^* or securities ρ_s^* by the average salary p_w, price p_u or value p_s, respectively, we obtain the numbers of labour years q_w^*, output pieces q_u^* or liquidity units q_s^* exchanged. In the short run, households, businesses and financiers tend to quote salaries, prices and values without regard to the surplus or deficit of the market good under their control. Pride, sloth and other personality characteristics may play a role in this respect. Due to the problematic nature of quotations, demanded quantities only accidentally coincide with available amounts of services, products and securities per time unit. From one year to the next, quotations are altered by human intervention while quantities vary in accordance with endogenous supply and exogenous factors, such as natural disasters, resource discoveries, warranted growth and diminution rates. Consequently, in order to adequately assess any closed economic system, it is necessary to compare current expenditure flows to those that would develop the productive capacities of sectors, countries and zones without excess or shortfall.

Each market good is distinguished by its special nature; therefore, its quantity should be expressed and measured in terms of a different unit. In disaggregated models, the amount of each market good should be recorded on a distinct coordinate axis and, hence, in another (vector space) dimension. To make matters simpler, more uniform and general, it is better to estimate each good's deficit or surplus as a percentage of the available quantity, i.e. $(q_j - q_j^\#)/q_j^\#$. Given the external parameters, economic model $\mathbf{dX/dt = A \cdot X - K}$ is capable of generating n ordered configurations of three elements y_i, e_i, $(q_j^{\#} - q_j)/q_j^{\#}$, which together epitomize the n sectors, countries or zones for the particular year. Among other ways, economic systems can be portrayed as steady or moving polyhedrons with n vertices [y_i, e_i, $(q_j^{\#} - q_j)/q_j^{\#}$] in the (R^3, t) vector space for each annual set of exogenous variables.

10.6. Wage fund theory

According to the twelve major hypotheses, exchange of services, products, securities etc., is recorded by banking institutions per time unit. Debits and credits correspond to nominal flows which are apportioned among sectors, countries or zones and discounted into financial stocks via the interest rate. Incomes and expenditures as well as assets and liabilities are decided by market preferences of consumers, producers, bankers, etc., as well as by monetary, fiscal and other policies worldwide. Household, business and financier receipts and payments – which originate from selling and buying labour, output and liquidity world-wide – are multiples of the yearly rise of banking system liabilities and the corresponding rise of accepting sector assets. As noted in Section 4.5, the same amount of annual revenue (outlay) may be collected (disbursed) by means of:

- small quantities of expensive services, products and securities, or
- large quantities of inexpensive services, products and securities.

The above deduction reminds us of Wage Fund Theory. According to Jevons's approach and Euclidean methodology, every exchange network yields a specific yearly flow, and thus a specific financial stock, for remuneration of labour. In Chapter **5**, we saw that Alpha's structure allots to the services market annual turnover equal to $\$527.5 \cdot 10^6$ and present value equal to $\$5\,275 \cdot 10^6$ (when the discount rate is $r = {}^{10}/_{100}$), regardless of the reward for labour. Other things being equal, the number of employees (unemployed) is inversely (directly) related to compensation of labour per time unit. The level of salaries defines the dividing line between utilized and wasted work force and, therefore, the need for welfare payments to the latter. On the other hand, the annual discount rate acts like a lens that magnifies (inflates) or shrinks (deflates) the present values of future receipts and payments without affecting real variables other than the gap between demanded and supplied consol units. The last point is further elaborated in Section 12.3.

Chapter 11. Laws of Economy or Laws of Nature?

Mathematical economic models based on the twelve propositions incorporate properties which are not embodied by econometric, DSGE or other reproductions of economic reality. Specifically, Euclidean exchange networks may be built and used in a number of methodologically equivalent ways, i.e. as systems of:

- algebraic equations or matrices,
- geometric polygons or polyhedrons,
- hydraulic flows or circuits,
- dynamic spaces or planes;
- static graphics or animated pictures.

In this chapter, we will discuss important similarities between Euclidean exchange networks, which consist of monetary flows or financial stocks, and dynamic physical systems, which consist of energy flows or material bodies. Among the aims of this monograph is to broaden the horizons of mathematical economic modelling by drawing from analogous scientific approaches.

11.1. Euclidean exchange networks

An elementary exchange network such as (4.1), which involves three sectors and three goods, possesses an equilibrium vector $\mathbf{X}^*$ consisting of three sets of elements (y_i^*, e_i^*), $i = h, b, f$. Per time unit, the income and expenditure pairs – of households, businesses and financiers – are the coordinates of the three vertices, which compose the triangle representing the outcome of transactions on the Euclidean plane R^2. The three vertices of the triangle are determined by the sets of vertical lines $[(y_i^*, 0), (0, e_i^*)]$, which pinpoint the equilibrium positions of the three institutional sectors $i = h, b, f$. For example, steady state $\mathbf{X}^*$ of economy Alpha (where $\alpha_{hw} = {}^8/_9$, $\beta_{hu} = {}^7/_8$, $\alpha_{bu} = {}^6/_7$, $\beta_{bw} = {}^5/_6$, $\beta_{fw} = {}^1/_2$ and $\beta_{fs} = {}^{439}/_{3168}$) consists of monetary flows greater than zero if, and only if, $0.46016 < \kappa_h < 0.72335$. By replacing $\kappa_h = ½$ in equations (5.16), we obtain the sustainable revenues and outlays of consumers, producers and bankers as predetermined multiples (y_i, e_i) of the monetary financial balance sheet's annual rise k_f:

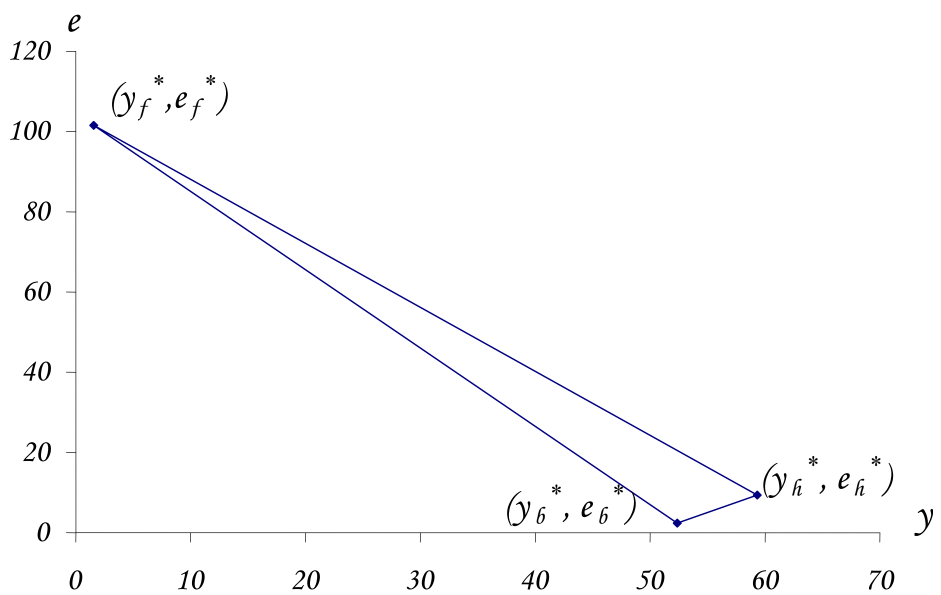
$$\begin{array}{lll} y_h^* = {}^{2102787}/_{3543526}\cdot k_f = y_h \cdot k_f & y_b^* = {}^{265061}/_{506218}\cdot k_f = y_b \cdot k_f & y_f^* = {}^{27661}/_{1771763}\cdot k_f = y_f \cdot k_f \\ e_h^* = {}^{165512}/_{1771763}\cdot k_f = e_h \cdot k_f & e_b^* = {}^{5976}/_{253109}\cdot k_f = e_b \cdot k_f & e_f^* = {}^{1799424}/_{1771763}\cdot k_f = e_f \cdot k_f \end{array} \quad (11.1)$$

The above coordinates (y_h^*, e_h^*), (y_b^*, e_b^*), (y_f^*, e_f^*) form a triangle having:

- perimeter $Y^* = y_h^* + y_b^* + y_f^* = E^* = e_h^* + e_b^* + e_f^*$ equal to $1.13263 \cdot k_f = c \cdot k_f$, where c is the Euclidean multiplier;
- area $(Y^*, 0) \times (0, E^*) = [(y_h^* + y_b^* + y_f^*, 0) \times (0, e_h^* + e_b^* + e_f^*)]$, i.e. economic surface, equal to $0.10473 \cdot k_f^2$.

The triangle's (or polygon's) perimeter and area are, respectively, linear and quadratic functions of the yearly money supply k_f. Figure (11.2) represents Alpha's equilibrium on the Euclidean plane, in the case $k_f = \$100$.

(11.2) Alpha's equilibrium on the Euclidean plane



The difference $y_i^* - e_i^* = k_i$, between a pair of accepting sector *i* flows (or coordinates) is equal to net credit k_i extended to sector *i* (e.g. households *h*, businesses *b, etc.*) by money issuing sector *f* during the relevant period. Due to the twelve fundamental hypotheses, which anchor Euclidean exchange networks, per time unit:

- banking institutions allocate money supply increment k_f between consumers and producers in proportions κ_h and κ_b, respectively;
- sector monetary flows (y_i^*, e_i^*) are equal to the product of bank balance sheet increment k_f and constant proportions (y_i, e_i);
- the banking system's additional liabilities (or deficit k_f) match the accepting sectors' additional assets (or surplus $k_h + k_b$).

Sector equilibrium flows (y_i^*, e_i^*), stemming from general model **dX/dt** = **A·X – K**, comprise a vector space V_{2n}. The vector space consists of the income and expenditure (y_1, e_1; y_2, e_2;...; y_n, e_n) of households, businesses, financiers, etc. Each pair or vertex (y_i^*, e_i^*) subdivides the Euclidean plane (y, e) into quadrants. Within the time unit, i.e. $t_0 \leq t \leq t_0 + 1$, sector receipts and payments (y_i, e_i) tend to increase or decrease depending on whether they are smaller or greater than the corresponding equilibrium flows (y_i^*, e_i^*). Every energised space may be analysed in terms of dynamic planes, the constituent elements (state variables) of which change or move depending on the antithetical forces acting on the entire system. A Euclidean exchange network of the general type **dX/dt = A·X – K** may be subdivided into n dynamic planes (y_i, e_i) where:

- the money issuing sector's revenue and outlay change up to the point where their difference becomes equal to the authorised rise of the said sector's liquid liabilities.
- the money accepting sectors' revenue and outlay change up to the point where the difference becomes equal to the authorised rise of the said sectors' liquid assets.

In an elementary economy such as Alpha, Figure (11.2) enables us to visualise that – within the time unit – as the monetary financial balance sheet is expanded by k_f:

- banking institution outgoings e_f grow more than takings y_f ; consequently, the vertex that symbolises the issuer is pushed upward;
- consumer and producer receipts ($y_h + y_b$) rise more than payments ($e_h + e_b$) ; thus, vertices symbolising acceptors move to the right.

Mathematically speaking, net credit k_f alters the flow coordinates of economic systems in the same way as net force distorts the structural coordinates of physical systems.

11.2. Similarities between money and energy

Any modification of the banking system balance sheet affects receipts and payments of households, businesses and financiers as well as selling and buying of services, products and securities. This chapter explains how units of liquidity credited (debited) to institutional sectors resemble quanta of energy added to (subtracted from) material bodies. *Inter alia*, it will be shown that expansion of bank credit k_f alters an economic system just as the application of heat affects gas molecules in a container. Exchange networks consist of monetary flows and financial stocks, while physical systems consist of energy flows and material bodies. Both are subject to the laws of mathematics because they operate under the same closed framework of action and reaction. For the purpose of facilitating our endeavour to highlight analogies between economic and energy networks, we refer to thermodynamics, the field of physics that studies conversions of energy from one form to another. Conversions of energy are subject to three axioms, called the laws of thermodynamics, outlined in Table (11.3).

(11.3) Laws of thermodynamics

The First Law regulates the quantitative relations between the various forms of energy. According to this law, in a closed system the amount of energy remains stable. All energy conversions produce useful / ordered work and useless / diffused entropy. The Second Law is qualitative because it pertains to the direction of conversions from one form of energy to another. According to this law, without the constant addition of energy, every closed system tends towards a stable equilibrium, i.e. the maximisation of useless / diffused entropy. The Third Law pertains to the redistribution of energy in a closed system. This law asserts that the more energy is consumed, with a consequent increase in entropy, the more difficult is the further production of work.

We know that the following apply to physical systems:

- Energy is simultaneously added and subtracted. The capacity to produce work is directly related to the spatial configuration of a physical system's elements in relation to its spatial centre of balance.
- When energy is added (subtracted), a number of elements retreat from (approach) the physical system's centre of balance. The work produced is the difference between the energy of the configuration before and after the relocation of elements.
- Without external intervention, a system's total energy remains constant. Every quantity of additional (subtracted) energy causes various forms of expansion (contraction), growth (decay), order (disorder), etc., to the system's elements.
- A physical system left on its own tends toward a steady state or a random allocation of its elements.
- Energy tends to flow from the sub-system with surplus to the sub-system with deficit energy, while the total energy remains intact.

In this context, we shall show that:

- An exchange network circulates and balances the annual supply of liquidity similarly to the way a physical system distributes and stabilises an additional quantity of energy.
- An exchange network resembles a thermodynamic machine in the sense that the additional money (energy) is converted into third-party value (accomplished work) and owner-acquired turnover (worthless entropy).
- The antithetical forces operating in exchange networks (physical systems) *de facto* guide the constituent flows (elements) to their steady state (equilibrium position).
- *Ceteris paribus*, an exchange network's steady state changes according to the annual supply of money credited and debited to its various sectors, just as a physical system's steady state changes according to the additional quantity of energy utilised and neutralised by its various components.

As economists, we will investigate how exchange networks circulate and balance money in its polar manifestations and leave it to physicists to explain how their systems distribute and stabilise energy in its opposite forms.

11.3. Utilisation and storage of monetary wealth

In every Euclidean exchange network, vector **K** (reflecting annual credit policy) is linearly transformed by matrix **A** (incorporating yearly behaviour coefficients) in order to yield vector $\mathbf{X}^*$ (consisting of the equilibrium flows of sectors, countries or zones):

$$\mathbf{X}^* = \mathbf{A}^{-1} \cdot \mathbf{K} = (y_1^*, e_1^*, \ldots, y_n^*, e_n^*) = \mathrm{k_f} \cdot (\mathrm{y_1, e_1, \ldots, y_n, e_n}) = \mathrm{k_f} \cdot \mathbf{U} \qquad (11.4)$$

Incremental money (just like additional energy) is created (introduced) and destroyed (absorbed) in two forms (directions) simultaneously. The one component (dimension) cannot exist without the other. As shown in Chapters **4** and **5**, Cramer's rule yields factors $\mathrm{y_i}$ and $\mathrm{e_i}$ that make up circulation vector **U**. Nominal increase $\mathrm{k_f}$ of bank assets and liabilities multiplied by circulation vector **U** yields equilibrium vector $\mathbf{X}^*$, which consists of sector takings and outgoings. In exchange networks, just as in physical systems, energising and accommodating elements appear and interact until a new balance is reached. Let us not forget that to every debit (action) always corresponds an equal but opposite credit (reaction). Steady state receipts and payments (y_i^*, e_i^*), $i = 1, \ldots, n$, are multiples of the yearly liquidity increment k_f. Injections of liquidity (energy) affect all constituent flows (elements) of an economic system (physical structure) in accordance with the exchange network's (ordered configuration's) neutralizing (diffusing) capabilities. It may, therefore, be construed that new money (just like added energy) is dispersed and balanced in its opposite forms, in ways that depend on the exchange network's (physical system's) structure of flows (configuration of elements), in accordance with the First Law of Thermodynamics. Equations (11.5) to (11.7) illustrate how deficits (liabilities) of (funded) issuers (k_f) end up as surpluses (assets) of (funding) acceptors ($k_h + k_b$) and, subsequently, as sector and market receipts and payments per time unit. For example, in economy Alpha:

(i) the exogenous bank balance sheet increment, amounting to $k_f = \$10^9$, is perfectly distributed among sectors as follows:

$$k_f = k_h + k_b \qquad k_f = \kappa_h \cdot k_f + \kappa_b \cdot k_f \qquad k_f = (½) \cdot k_f + (½) \cdot k_f$$
$$k_f = \$10^{10} = \{\$500 + \$500\} \cdot 10^6 \tag{11.5}$$

(ii) the resulting endogenous receipts, payments and sales are perfectly distributed among sectors and markets as follows:

$$c \cdot k_f = Y^* = E^* = R^* \;\Rightarrow\; (y_h + y_b + y_f) \cdot k_f = (e_h + e_b + e_f) \cdot k_f = (\rho_w + \rho_u + \rho_s) \cdot k_f$$
$$(0.510342 + 0.52360 + 0.01561) \cdot k_f = (0.010342 + 0.02360 + 1.01561) \cdot k_f = \tag{11.6}$$
$$= (0.52748 + 0.44880 + 0.15635) \cdot k_f = 1.13263 \cdot \$1 \cdot 10^9 = c \cdot k_f$$

If the banking balance sheet was expanded by $k_f = \$1\,000$ per year, Alpha's gross turnover ($Y^* = E^* = R^*$) of \$1 132.63 would be allocated among Alpha's institutional sectors and market goods as follows:

$$\begin{array}{llll} y_h^* = \$672.12 & y_b^* = \$593.04 & y_f^* = \$17.68 & Y^* \approx \$1\,132.63 \\ e_h^* = \$105.81 & e_b^* = \$26.73 & e_f^* = \$1\,150.31 & E^* \approx \$1\,132.63 \\ \rho_w^* = \$527.48 & \rho_u^* = \$448.80 & \rho_s^* = \$156.35 & R^* \approx \$1\,132.63 \end{array} \tag{11.7}$$

Every economy's sustainable circulation ($Y^* = E^* = R^*$) is equal to its Euclidean multiplier (scale factor) $c = (y_1 + y_2 + \ldots + y_n) = (e_1 + e_2 + \ldots + e_n) = (\rho_1 + \rho_2 + \ldots + \rho_n)$ times the yearly liquidity increment k_f. Multiplier c is i) the scale factor applied to the triangle or polygon depicting the exchange network and ii) reflects the economy's capacity to convert bank liabilities into sector and market flows. The greater the Euclidean multiplier c:

- the bigger the annual liquidity increment's impact on the revenue and outlay of sectors and markets;
- the smaller the money supply change that would be required to eliminate a market surplus or deficit.

Sooner or later, economic systems are confronted by shortages of labour services and other inputs. As soon as resource boundaries are reached, the magnitude of the Euclidean multiplier c is of little benefit to policy makers. Beyond this point, no amount of monetary expansion can augment real wealth.

11.4. Absence of self-propelled economic systems

Looking at equation (11.6), it may appear that \$1 000 of new money creates \$1 132.63 of value added. If economy Alpha were a thermodynamic machine, such a result would imply that an energy input of 1 000 units would increase ordered work by 1 132.63 units. If this were the real outcome, the new money to value added (energy input to ordered work) conversion coefficient of Alpha would be in excess of 100%, a result that is precluded by the Second Law of Thermodynamics. However, \$1 132.63 corresponds to the exchange network's (thermodynamic machine's) nominal turnover and not to its value added. At a disaggregated or individual level, money-accepting agents (i.e. households, businesses, etc.) do not purchase the particular labour, output, etc., they respectively supply. On the contrary, bankers routinely accept (repurchase) the guaranteed bonds they have issued (sold) to third parties. In order to estimate value added by the monetary financial sector, it is necessary to deduct from its gross turnover ρ_s^* the amount of interest $\beta_{fs}\cdot e_f^*$ paid by the central and other banks on the guaranteed securities (deposits) held by banking institutions during the relevant period. Consequently, the monetary financial sector's value added is calculated as follows:

$$\begin{aligned} v_f^* &= \rho_s^* - \beta_{fs}\cdot e_f^* = (\rho_s - \beta_{fs}\cdot e_f)\cdot k_f = \\ &= \left({}^{277013}/_{1771763} - ({}^{439}/_{3168})\cdot {}^{1799424}/_{1771763}\right)\cdot k_f = ({}^{27661}/_{1771763})\cdot k_f \end{aligned} \tag{11.8}$$

Taking the above into consideration, it is concluded that Alpha's value added is:

$$\begin{aligned} V^* &= (v_h^* + v_b^* + v_f^*) = \left(\rho_w^* + \rho_u^* + (\rho_s - \beta_{fs}\cdot e_f)\cdot k_f\right) = \\ &= \left(({}^{934572}/_{1771763}) + ({}^{795183}/_{1771763}) + ({}^{27661}/_{1771763})\right)\cdot k_f = ({}^{1757416}/_{1771763})\cdot k_f \approx 0.99190\cdot k_f \end{aligned} \tag{11.9}$$

From this perspective, it is obvious that every \$1 000 of additional money (energy) generates only \$991.90 of value added (ordered work) in economy (machine) Alpha. It should be remembered that the Second Law of Thermodynamics precludes conversion of energy (money) into work (value) without entropy (loss). According to this Law, a physical system (just like an exchange network) would tend toward null vector $\mathbf{X}^* = (0, 0, \dots, 0, 0)$ unless it constantly received the quantity of energy (money) k_f that would produce the desired homeostasis $\mathbf{X}^* = k_f \cdot (y_1, e_1, \dots, y_n, e_n)$. In economics (like in physics) no nominal stream (material movement) can continue in perpetuity without an external source of money (energy).

11.5. The dead point of an energised configuration

Regardless of dimension, every stable configuration (structure) of elements possesses a centre of mass, energy, etc., where the antithetical forces, pressures, etc., are eliminated. In two-dimensional space, the gravity centre of a set of 2n material bodies or distinct weights is calculated according to the following mathematical formula:

$$\mathbf{M} \cdot \mathbf{L'} = [(m_1 \cdot l_1) + \dots + (m_n \cdot l_n)] - [(m_{n+1} \cdot l_{n+1}) + \dots + (m_{2n} \cdot l_{2n})] = 0 \qquad (11.10)$$

where $\mathbf{M}$ symbolises the vector of 2n material weights comprising the configuration;
$\mathbf{L'}$ symbolises the vector of the distance of elements from the gravity centre;
$i = 1, \dots, 2n$, symbolises the number of material weights comprising the structure;
l_i symbolises the distance of each element from the system's centre of gravity;
m_i symbolises the material weight of each element comprising the configuration.

Steady state (11.10) of a physical system (based on action and reaction) is comparable to steady state $\mathbf{X}^* = \mathbf{A}^{-1} \cdot \mathbf{K}$ of an exchange network (based on debit and credit). In the latter case, liquidity (energy) increment k is apportioned as opposite flows (antithetical forces) among the institutional sectors (constituent elements) in ratios (distances) $y_1, \dots, y_n$ and $e_1, \dots, e_n$ relative to the network's (system's) centre of balance:

$$\begin{aligned} &\mathbf{M} \cdot \mathbf{L'} = [(y_1 \cdot k) + \dots + (y_n \cdot k)] - [(e_1 \cdot k) + \dots + (e_n \cdot k)] \Rightarrow \\ &(y_1 + y_2 + \dots + y_n) \cdot k = (e_1 + e_2 + \dots + e_n) \cdot k \;\; \Rightarrow \;\; Y^* - E^* = 0 \end{aligned} \qquad (11.11)$$

Equation (11.11) demonstrates that the monetary flows comprising an exchange network balance in the same way as the antithetical forces operating in a physical system. At equilibrium $\mathbf{X}^*$, revenues and outlays remain constant because they are simultaneously:

- expanded by the issuing sector's accumulation of monetary liabilities;
- compressed by the accepting sectors' accumulation of monetary assets.

According to the Second Law of Thermodynamics, movement of physical elements toward equilibrium is irreversible. In previous chapters, while discussing economic stability, we may have given the impression that a Euclidean system's Routh-Hurwitz determinants may not be greater than zero at all times. This may have originated from a feeling that maximisation of entropy may not always apply to exchange networks. As far as living organisms such as human beings are concerned, instability and catastrophe go hand in hand. Biological survival – concurrently pursued with other objectives such as economic welfare, political order and social peace – demands certainty. Without predictability, exchange networks composed of agents, sectors, countries, etc., would sooner or later collapse and disappear irretrievably. The intrinsic tendency of antithetical forces operating on every configuration of elements to counterbalance (neutralise) each other may be the universal law regulating all activities known to man. We have repeatedly noted that only equilibrium vectors, which emanate from a stable and dirigible economic system $\mathbf{dX}_t\,/\mathbf{dt} = \mathbf{A}_t \cdot \mathbf{X}_t - \mathbf{K}_t$, can be set as targets because all other revenue, outlay and turnover combinations are untenable. In Chapter **6**, while discussing fiscal policy, we stressed that governments, which focus on achieving their goals, have no alternative but to methodically modify taxation and subsidisation rates as required by the exchange network's stability. Throughout this monograph we have also emphasised that banking institutions have no choice but to allocate and increase credit so as to ensure positive and adequate monetary streams. Thanks to intuitive adjustment of behaviour coefficients and purposeful regulation of policy parameters, a dynamic system's i) Routh-Hurwitz determinants and ii) sector flows tend to become positive as dictated by survival instinct and common sense.

11.6. Differences between steady states

Equilibrium flows of general exchange network $\mathbf{dX/dt = A{\cdot}X} - \mathbf{K}$ comprise an ordered configuration of elements, which may be epitomised either as a polygon on the R^2 plane or as a vector in the R^{2n} space. A stable economic system's receipts and payments (y_1, e_1, y_2, e_2, … , y_n, e_n) keep adjusting until they coincide with the vertices determined by steady state $\mathbf{X}^* = (y_1^*, e_1^*, y_2^*, e_2^*, \dots, y_n^*, e_n^*)$. In Euclidean models, $\mathbf{X}^*$ is always equal to annual liquidity increment k_f multiplied by circulation vector $\mathbf{U}$, which consists of n pairs of constant coefficients (y_i, e_i) as follows:

$$\mathbf{X}^* = (y_1^*,\ldots, y_n, e_1^*,\ldots, e_n^*) = k_f \cdot (y_1, \ldots, y_n, e_1,\ldots, e_n) \cdot = k_f \cdot \mathbf{U} \tag{11.12}$$

The size and shape of polygon $\mathbf{X}^*$ – depicting an exchange network on the R^2 plane – are determined, respectively, by the yearly rise and sector allocation of bank credit as well as by the behaviour coefficients and policy parameters of institutional sectors. With regard to vector $\mathbf{X}^*$ (defined in the R^{2n} space) it is noted that:

$$\begin{aligned} &\text{its length is equal to } \|\mathbf{X}^*\| = (\mathbf{X}^* \cdot \mathbf{X}^*)^{½} = (y_1^2 + e_1^2 + \ldots + y_n^2 + e_n^2)^{½} \cdot k_f, \\ &\text{its norm is equal to } \|\mathbf{U}\| = (y_1^2 + e_1^2 + \ldots + y_n^2 + e_n^2)^{½}, \text{ when } k_f = 1. \end{aligned} \tag{11.13}$$

Let us now compare two equilibrium positions $\mathbf{X}^*$ and $\mathbf{Z}^*$ of a Euclidean economy, which differ only by the annual change of the money supply, i.e. k_f and k_z. According to the above analysis and synthesis, vectors $\mathbf{X}^*$ and $\mathbf{Z}^*$ are, respectively, equal to:

$$\begin{aligned} \mathbf{X}^* &= k_f \cdot (y_1, e_1,\ldots, y_n, e_n) = k_f \cdot \mathbf{U} = (x_1, x_2,\ldots, x_{2n}) \\ \mathbf{Z}^* &= k_z \cdot (y_1, e_1,\ldots, y_n, e_n) = k_z \cdot \mathbf{U} = (z_1, z_2,\ldots, z_{2n}) \end{aligned} \tag{11.14}$$

The distance between the two vectors $\mathbf{X}^*$ and $\mathbf{Z}^*$ is given by the following formula:

$$d(\mathbf{X}^*, \mathbf{Z}^*) = \big((x_1 - z_1)^2 + \ldots. + (x_{2n} - z_{2n})^2\big)^{½} \tag{11.15}$$

By replacing scalar products (11.14) in (11.15), we find that:

$$\begin{aligned} d(\mathbf{X}^*, \mathbf{Z}^*) &= \big(y_1^2 \cdot (k_f - k_z)^2 + e_1^2 \cdot (k_f - k_z)^2 + \ldots + y_n^2 \cdot (k_f - k_z)^2 + e_n^2 \cdot (k_f - k_z)^2\big)^{½} = \\ &= (y_1^2 + e_1^2 + \ldots + y_n^2 + e_n^2)^{½} \cdot |k_f - k_z| = \|\mathbf{X}^*\| - \|\mathbf{Z}^*\| = \|\mathbf{U}\| \cdot |k_f - k_z| \end{aligned} \tag{11.16}$$

Other things being equal, distance between two steady states of an economic system is a direct function of the liquidity increment debited to issuing banks and credited to accepting sectors (of countries or zones), in the same way that distance between two equilibrium positions of a physical system is a direct function of the difference of the amount of energy used as work and accumulated as entropy by its constituent elements. The more exogenous liquidity (energy) approaches zero, the more endogenous monetary (physical) flows (elements) tend toward nothingness (immobility). Economic – physical rates of change diminish with every reduction of the monetary – energy increment. A closed exchange network approaches the empty vector $\mathbf{X}^* = (0,\ldots, 0)$ with each reduction of liquidity in the same way, for example, as a cluster of gas molecules approaches the absolute void with each decrease of temperature, according to the Third Law of Thermodynamics.

Chapter 12. Various Supplementary Observations

According to Euclidean methodology, institutional sectors buy and sell services, products and securities in exchange for monetary credits and debits. The twelve premises enable us to construct and apply exchange networks that fully conform to accounting - statistical in addition to economic - mathematical principles. Sector and market transactions concluded per time unit are replicated by a system of first-order non-homogeneous linear differential equations, i.e. $\mathbf{dX/dt = A{\cdot}X} - \mathbf{K}$. Given behaviour coefficients and policy parameters, Euclidean approach allows assessment of the equilibrium vector's uniqueness, stability and controllability properties. Furthermore, each set of equilibrium flows $\mathbf{X}^*$ yields a set of annual sales $\mathbf{R}^*$. Fine distinctions among different kinds of labour, output, etc., are lost when agents are aggregated under sectors, countries and zones. Properties and outcomes of dynamic model $\mathbf{dX/dt = A{\cdot}X} - \mathbf{K}$ and steady-state solution $\mathbf{X}^* = \mathbf{A}^{-1} \cdot \mathbf{K}$ are unaffected by average prices and available amounts, because the said variables are exogenous scalars.

As a result, dividing the gross turnover of a traded good by the corresponding quotation, we obtain the years of labour, pieces of output or number of consols bought and sold per annum. By comparing the demanded with available quantities, we figure the deficits or surpluses of traded goods. Quantity gaps of services, products and securities are not eliminated automatically. Interplay of economic, political and social forces determines the extent to which market shortages and excesses are bridged by means of price adjustments or policy changes. Banking balance sheet modification **K** – with respect to magnitude and allocation – is linearly transformed by structural matrix **A** into vector **X** consisting of sector, country or zone receipts and payments. By discounting the aforesaid monetary flows, it is possible to estimate assets, liabilities and net financial position of the economy and its constituent parts. With assistance from mathematical software, it is possible to:

- express sector, country or zone monetary flows as functions of exogenous variables;
- obtain the derivatives of endogenous variables with respect to policy parameters;
- design the full utilisation of the exchange network's real and financial resources.

Economic systems rarely evolve by leaps and bounds. Exchange networks, just like living organisms, material clusters, electric circuits, etc., assimilate (radical) change with (great) resistance. In the previous Chapter, we elucidated the closed 'active – reactive' or 'balancing – neutralising' framework within which monetary, fiscal and other policies redistribute sector and market as well as country and currency flows. Euclidean modelling methodology – by combining mathematics and statistics with accounting and computers – has rendered possible duplication of economic systems and solution of relevant problems with simplicity, uniformity and generality. Aiming for the widest possible acceptance, we will show that the approach is capable of dealing with most issues of concern to economists, politicians and the electorate. In this respect, we will discuss theoretical and practical questions concerning the:

1) variety of securities traded by institutional sectors;
2) management of risks threatening banking institutions;
3) multiple roles played by the annual discount rate;
4) existence of and way out of a liquidity trap;
5) simulation of economic history and long-term development.

12.1. Intermediate financial services

Consumers, producers, bankers, etc., distribute their revenues and outlays among real goods and financial securities. Sales and purchases of labour, output, liquidity, etc., by institutional sectors are recorded as debits and credits by the banking system. Doubtful monetary flows generate uncertain present values and *vice versa*. The less safe a category of securities, the less is accepted as means of payment and store of wealth and, consequently, the less is treated as equivalent to money. Guaranteed bonds, which offer a known amount of annual liquidity, may be exchanged with various types of unsecured bonds, which offer a probable sum of capital at any point in time. Lotteries, shares, royalties, options, etc., involve future monetary flows and discounted present values, which are subject to the laws of probability. Many:

- acceptors do not possess the probability distributions which are necessary for computing the expected value of receipts stemming from uncertain assets;
- issuers do not possess the probability distributions which are necessary for computing the expected value of payments arising from uncertain liabilities.

Due to practical, psychological and other reasons, financial contracts subject to chances are daily sold and bought by risk prone and risk averse economic agents. Usually, specialised intermediaries handle each category of uncertain securities. Any number of sectors which trade financial products can be included in general Euclidean model $\mathbf{dX}_t\,/\mathbf{dt} = \mathbf{A}_t \cdot \mathbf{X}_t - \mathbf{K}_t$ provided their takings and outgoings are elevated to certainty equivalent status. Banking institutions are distinguished from financial intermediaries in a number of ways; the main difference being that that the former are allowed to operate as net debtors (issuers of money) while the latter must operate as net creditors (acceptors of money) at all times.

Owing to entirely different purposes, banking institutions and financial intermediaries are subject to dissimilar laws, rules and regulations. According to the twelfth major proposition, every country controls buying and selling of liquid securities between banking institutions on the one hand and households, businesses and the government on the other; this is so because every change regarding magnitude and allocation of the monetary financial balance sheet touches equilibrium, dirigibility and other properties of the exchange network. In contrast, consumers, producers and other accepting sectors (aggregated under countries or zones) are free to trade all sorts of promissory notes / financial contracts among themselves since such transactions do not affect the total supply of liquid money. While building and utilising economic models, banking institutions should always be treated separately from financial intermediaries.

12.2. Dealing with destabilisation risks

In the neighbourhoods of current behaviour coefficients and policy parameters, it is possible to compute the limits of behaviour and policy outside which the exchange network becomes unstable. This monograph has emphasised that any sector, country or zone may destabilise the economic system because numerous flow distributions and fiscal parameters yield negative Routh-Hurwitz determinants. Within this precarious framework, central banks should oblige other monetary institutions to decline purchases of (i.e. allocation of expenditure for) questionable securities and risky investments beyond the harmless minimums. In order to neutralise impending risks, banks should act proactively by apportioning their current and capital transactions within safe bounds. Directors who knowingly violated risk warnings and have burdened monetary institutions with losses should be penalised as they engaged in a ‘game of chance’ to the benefit of some and to the detriment of other economic agents. Bank regulators should ensure that accepting sectors benefit from uninterrupted supply, optimal allocation and appropriate pricing of liquid assets and liabilities per time unit. In addition to containing reckless behaviour, monetary authorities should prevent crises of confidence by obliging banking institutions to maintain sufficient capital to cover likely and unlikely losses. Under all circumstances, *de facto* negative present value of banking system flows ($y_f - e_f$) should be covered by on and off balance sheet capital that includes parliamentary authorisations, government guarantees, deposit insurance, gold reserves, real estate, pledged collateral, etc. By applying a Euclidean model that replicates interaction between money issuing institutions and money accepting sectors (classified under countries and zones) mathematical economists are in position to provide estimates regarding the adequate amount of bank capital per time unit.

12.3. Fivefold role of the discount rate

According to the major premises, banking institutions (entitled to issue perpetual bonds / deposits) are the net debtors while households, businesses and other sectors (obliged to accept perpetual bonds / deposits) are the net creditors of the system. In the action and reaction framework characterising economic exchange, the monetary balance sheet changes according to the annual deficit k_f of banking institutions and the matching surplus $k_f = k_1 + .. + k_{n-1}$ of consumers, producers, governments, etc. For example, in the three-sector model:

$$dM/dt = (e_f - y_f) = (y_h - e_h) + (y_b - e_b) = k_f = k_h + k_b \tag{12.1}$$

Remember that nominal flows of sectors $\mathbf{X}(y_i, e_i)$, $i = 1, \ldots, n$ and markets $\mathbf{R}(\rho_j)$ $j = 1, \ldots, n$, as well as quotation and quantity gaps of traded goods are linear functions of liquidity increment $k_f = dM/dt$, where M is the money supply. At any moment, the circulation network may be in disequilibrium due to erratic human behaviour, inconsistent public policies, unexpected technological discoveries, etc. Consequently, adjustment of annual credit k_f and discount rate r may become necessary in order to bridge demand with supply of liquid securities. Due to mutual dependence and various constraints – arising from the mathematical relationships defining the economic system – any change of discount rate r is related to liquidity increment k_f and *vice versa*. Justifiably, the following rhetorical question arises "*How does the discount rate affect the exchange network*?" According to our methodology, this yearly percentage plays a leading role on the international financial stage (discussed in Section 7.8) and four interconnected parts in the local economic drama:

Firstly, the discount rate corresponds to the *numeraire* proportion that issuers are obliged to pay per annum for every *numeraire* unit borrowed and acceptors are entitled to receive per annum from every *numeraire* unit lent. The value of a consol, which obliges the issuer to pay and entitles the acceptor to receive one unit of currency per year in perpetuity, is equal to the inverse of the annual discount rate, i.e. $p_s = 1/r$. For example, when the discount rate equals:

- 9.09 percent, the value of the consol is $p_s = 1/0.0909 \approx 11.00$
- 4.54 percent, the value of the consol is $p_s = 1/0.0454 \approx 22.03$.

Secondly, the discount rate is the scalar factor used for conversion of annual receipts and payments (monetary flows) into corresponding assets and liabilities (financial stocks) and the contrary. According to the ninth proposition, the discount rate is the *prima fasciae* connection between monetary flows and present values. Fixed coupon consols and variable interest bonds with assured yearly flows, and thus guaranteed present values, are interchangeably used by borrowers and lenders. These archetypal securities are perfectly comparable at the micro-economic valuation level and the macro-economic impact level. As a result, consumers, producers, bankers, etc., use many combinations and variations of constant coupon consols and flexible interest bonds in their daily transactions. Irrespective of their assigned names, most financial assets and liabilities are reducible to linear combinations of the aforesaid security types. Specifically, contracts which provide for:

- pre-determined monetary flows (such as monthly rents, annual leases, hire-purchase agreements, investment depreciations, etc.) approximate the fixed coupon consol;
- fluctuating monetary flows (such as company shares, turnover percentages, patent rights, intellectual royalties, etc.) approximate the variable interest bond.

Thirdly, the number of traded perpetual bonds q_s^* is directly proportional to discount rate r (through $q_s^* = \rho_s^* / p_s$) as opposed to the traded labour years q_w^* and output pieces q_u^*, which are inversely related to average salaries p_w and average prices p_u (due to $q_w^* = \rho_w^*/p_w$ and $q_u^* = \rho_u^*/p_u$), respectively. At liquidity market equilibrium, reflected by (12.2), the supply of outlay for consols e_s^* is equal to the demand for revenue y_s^* from consols:

$$\rho_s^* = e_s^* = (\beta_{hs} \cdot e_h + \beta_{bs} \cdot e_b + \beta_{fs} \cdot e_f) = (\alpha_{hs} \cdot y_h + \alpha_{bs} \cdot y_b + \alpha_{fs} \cdot y_f) = y_s^* \qquad (12.2)$$

The gross turnover of perpetual bonds ρ_s^* divided by the consol value $p_s = 1/r$ (i.e. one currency unit per annum for ever) yields the number of units q_s^* demanded by institutional sectors during a particular year. By substituting Alpha's statistical data (i.e. $\alpha_{hs} = {}^1/_9$, $b_{hs} = {}^1/_8$, $\alpha_{bs} = {}^1/_7$, $b_{bs} = {}^1/_6$, $b_{fs} = {}^{439}/_{3168}$) and sector flows ($y_h^* = 0.59342 \cdot k_f$, $e_h^* = 0.09342 \cdot k_f$, $y_b^* =$

$0.52360 \cdot k_f$, $e_b^* = 0.02360 \cdot k_f$, $y_f^* = 0.01561 \cdot k_f$, $e_f^* = 1.01561 \cdot k_f$) in (12.2), we obtain the annual sales ρ_s^* and thus the demanded quantity q_s^* of consols:

$$q_s^* = \rho_s^* \div p_s = 0.15634 \cdot k_f \div (1/r) = 1.149625 \cdot 10^7$$
$$q_s^* - q_s^\# = 0.15634 \cdot (r \cdot k_f) - 10^7 \qquad (12.3)$$

The number of consols demanded is positively related to the interest rate and to the monetary increment. When $r \approx 7.30\%$ and $k_f = \$10^9$, the difference between units demanded ($q_s^* = 1.149625 \cdot 10^7$) and units supplied ($q_s^\# = 10^7$) is 1 462 500 consols. According to partial equilibrium equation (12.3), excess demand for securities in Alpha may be eliminated by:

i) decreasing the discount rate to 6.40% from 7.30% per year; or
ii) reducing the bank balance sheet increment to $\$0.8698488 \cdot 10^9$; or
iii) any reasonable (r, k_f) combination resulting from (12.3).

Needless to say that solutions ii) and iii) ignore repercussions of k_f on labour and output markets. Even a minute change of the annual money supply affects demand for services and products. If the aforementioned markets should remain untouched, the correct answer would be to let the value of securities rise until the financial paper shortage is bridged.

Fourthly, the discount rate signals the capital gains or losses on the net worth of agents, sectors, etc. While buying and selling services, products and securities, households, businesses, financiers, etc., use the discount rate as an objective factor for linking monetary flows to present values. Variable interest instruments (e.g. bank deposits) are not accompanied by capital gains or losses. The present value of perpetual bonds (issued by banking institutions and accepted by households, businesses, etc.) fluctuates inversely to the discount rate. Abstracting from sudden modification of sector behaviour due to war, disaster, hysteria, etc., we may conclude that national and international banking institutions cause a significant proportion of economic cycles worldwide by varying:

- the expansion rate of their monetary financial balance sheets;
- the allocation of their loans among consumers, producers, etc.;
- the distribution of their current expenses;
- their annual discount rates.

In addition to international portfolio balance (Section 7.8), annual discount rates determine:

- the value of currency units paid or earned in perpetuity;
- the amount of perpetual bonds demanded per annum;
- the multiplier that converts flows into stocks, and thus
- the temporary gains or losses on monetary financial wealth.

12.4. Is there a liquidity trap?

Each year, market behaviour of sectors and credit policy of banks determine receipts from y_s^* and payments on e_s^* securities. The twin monetary flows are functions of different combinations of factors and, hence, do not respond in unison. Externally induced:

- excess demand (shortage) exerts upward pressure on the value of consols so that the central bank may have to engage in liquidity providing (lending) operations;
- excess supply (surplus) exerts downward pressure on the value of consols so that the central bank may have to engage in liquidity absorbing (borrowing) operations.

Most acceptors, including consumers, producers, bureaucrats, etc., overlook the time path of the monetary financial balance sheet, although it is of paramount importance to their economic situation; if they cared they would regularly extrapolate it by multiplying the yearly loan / deposit increment by the inverse of the annual discount rate:

$$M = (k_f / r) \qquad (12.4)$$

As a result of various coincidences, the discount rate may end up approaching zero percent per year. Further inflation of consol / bond / security values may be widely viewed as unacceptable and, hence, undesirable for economic, political and social reasons. In such a case, there is no alternative for the monetary authority but to allow:

- the banking sector to expand its liabilities or annual deficit ($k_f = k_h + k_b$) and
- the accepting sectors to expand their assets or annual surplus ($k_f = k_h + k_b$).

As shown in (4.18), sales of market goods $R^* = (\rho_w^* + \rho_u^* + \rho_s^*) = (\rho_w + \rho_u + \rho_s) \cdot k_f$ rise along with the liquidity increment k_f, leading to absorption of untaken services, products and securities. The liquidity trap, discovered by Keynes more than eighty years ago, is perfectly valid under the twelve propositions of Euclidean methodology. The foregoing remind us of academic discussions from the 1930s to the 1960s, regarding the effectiveness of discount rate decreases compared to money supply increases as means for improving the financial situation of consumers, producers, bankers, etc., during economic depression.

12.5. Economic history and growth

During a designated period of t number of years, the course of an economic system is described by time-series of endogenous variables $\mathbf{X}_t$, $\mathbf{R}_t$, $\mathbf{V}_t$, $\boldsymbol{\pi}_t$, $(q_j^*/q_j^\#)_t$, etc., which stem from t annual sets of exogenous parameters $\mathbf{A}_t$, $\mathbf{K}_t$, $\mathbf{P}_t$ and $\mathbf{Q}_t^\#$. The latter change outside the mathematical model's circulation dynamics in accordance with their own laws of action and reaction. In particular, Euclidean methodology assumes that they:

- evolve separately from the system of monetary flows;
- remain constant within the time unit, i.e. $t_o \leq t \leq t_o+1$;
- change slowly or suddenly in the medium to long run, i.e. $t_o+1 \leq t \leq +\infty$.

Autonomous variation levels may be assessed by means of empirical observations and appropriate arguments covering the entire period under consideration. Many theories of economic history (or growth) focus on alternative hypotheses regarding bygone (or expected) exogenous parameters, which gave rise (will lead) to particular sequences of endogenous vectors. Provided assumptions, estimates or projections have been made about behaviour coefficients, government policies, resource discoveries, technological inventions, etc., during a span of t years, it is possible to reproduce the exchange network's t consecutive steady states using *Mathematica* or other suitable software. Years $t = 1, \ldots, t$ may refer to the depicted economy's past or future. As noted, such replication requires annual data covering the following groups of autonomous variables:

1. income and expenditure coefficients of sectors, countries or zones;
2. monetary, fiscal, exchange and other policy parameters;
3. average quotations of market goods;
4. available years of services, pieces of products and units of securities, etc.

Estimated sets of external variables $\mathbf{A}_t$ and $\mathbf{K}_t$ yield unique solutions $\mathbf{X}^*(y_i^*, e_i^*)_t$ if, and only if, determinants $|\mathbf{A}_t|$ corresponding to years $t = 1, \ldots, t$, are different from zero. From sector flows $\mathbf{X}_t^*$, it is possible to obtain market flows $\mathbf{R}_t^*$. According to the second proposition, average quotations $\mathbf{P}(p_j)_t$, $j = 1, \ldots, n$, are autonomous. Dividing each set of gross turnovers $\mathbf{R}_t^*$ by the corresponding quotations $\mathbf{P}_t$, we figure the quantities $\mathbf{Q}_t^*$ of goods traded during the relevant time units. By comparing traded $\mathbf{Q}_t^*$ with available $\mathbf{Q}_t^{\#}$ amounts, we obtain the sequences of shortages and surpluses and, therefore, inflationary or deflationary pressures concerning markets, countries and zones.

Under well-defined conditions, monetary flows and traded quantities, which stem from the fundamental premises, form vector spaces with time as one of the exogenous factors. Euclidean modelling methodology may generate polyhedrons with n vertices $(y_i^*, e_i^*, \rho_j^*)_t$, $(\rho^*, p_j^*, q_j^*)_t$, $(y_i^*, e_i^*, \pi_j^*)_t$, etc., that present combinations of sector flows, gross turnovers, financial stocks, traded quantities, market gaps, inflation rates, etc., that are relevant to the issues being discussed. An exchange network's historical review or predicted growth may be elucidated by the said series of three-dimensional graphs or animations encapsulating t successive steady states of n sectors, markets, etc., such as:

$$[(y_1^*, e_1^*, (q_1^* - q_1^{\#})/q_1^{\#}), (y_2^*, e_2^*, (q_2^* - q_2^{\#})/q_2^{\#}), \ldots, (y_n^*, e_n^*, (q_n^* - q_n^{\#})/q_n^{\#})]_t \qquad (12.5)$$

Figures 5.22 and 5.23, 6.47 and 6.48, 7.43 and 7.44 as well as 8.62 and 8.63 are examples which illustrate economic activity over time. An economic system's regression or progression during a particular period may be better visualised when current demand is presented as a percentage of capacity supply per annum, as broadly exemplified by Figures 8.62 and 8.63. In every case, the choice of measurement units is very important. Time t should be measured in complete calendar years. Every rate of change should be recorded as a percentage differentiation per annum. Incomes and expenditures as well as assets and liabilities should be measured in units of nominal currency, preferably International Accounting Units (*IAU*s). The mathematical software should enable us to confirm dynamic stability and hence unique existence as well as right turning (same sign) of equilibrium flows, as necessary; it should also be capable of recording numerically and presenting visually the economic system's equilibrium positions over time, including the:

- incomes and expenditures of sectors, countries, etc.;
- annual sales of market goods;
- movement of the corresponding polyhedrons over time;
- percentages of employed and unemployed households;
- shortages or surpluses of products;
- numbers of exchanged and dormant securities, etc.

Provided prerequisites regarding stability and dirigibility are satisfied, the authorities' ability to guide the exchange network to an alternative steady state depends on their power to control a number of applicable instruments that is equal to the number of targeted flows. Under these conditions, scholars may discuss hypotheses and explore options about long-term policy paths. Euclidean modelling methodology, enhanced by computer software, offers the ideal framework for studying scenarios and answering questions relating to economic developments in historical, contemporary or future periods. For example:

- Are there chronological sequences of sector / country / zone behaviour, policy and other variables that reproduce the historical stages of economic growth?
- What happens to the exchange network's successive steady states when the number of institutional sectors and market goods keeps rising?
- In what way is the increasing diversification of income sources and expenditure destinations related to technological progress and market innovation?
- Does apportionment of bank credit among consumers, producers, governments, etc., change slowly (gradually) or rapidly (suddenly) over time?
- What distributions of income and allocations of expenditure would end up benefiting households, businesses, financiers, etc., at each point in time?
- How is economic development influenced by apportionment of bank credit to institutional sectors, sovereign countries and currency areas over time?
- Is there a lasting relationship between a country's share in world monetary expansion and its share in global value added?
- How will climatic conditions modify sector, country and zone behaviour coefficients as well as policy parameters, average prices and real resources?

Euclidean methodology provides researchers with theoretical and practical means with which to construct and apply mathematical models that are capable of duplicating economic reality at any place and time. Academic and professional economists may use such models to solve classic problems, answer current questions and deal with anticipated issues. The sooner specialists are in position to prepare statistics, justify hypotheses and make projections about past and future behaviour coefficients, policy parameters, average quotations and accessible amounts, the sooner will be able to exploit the possibilities offered by Euclidean methodology for the study of economic history and growth.

12.6. Summary and conclusions

This monograph subtitled ***The General Method for Construction and Application of Economic Models*** is based on twelve major propositions assumed to govern closed exchange networks. In such economic systems, called Euclidean, agents classified as consumers, entrepreneurs, bankers and governments, buy and sell labour services, business products, financial securities and public goods for bank money. Nominal flows, real quantities and financial stocks (representing assets and liabilities) may be aggregated under institutional sectors and market platforms as well as sovereign countries and currency zones over successive periods. The calendar year is the naturally recurring, most understood and widely utilized time unit. The fundamental premises, which must be read and interpreted as an integral whole, are summarised below:

Transaction rules

1) Agents measure and compare economic magnitudes using the same accounting unit.
2) Within the time unit, quotations of traded goods are determined exogenously.
3) All transactions among agents are recorded and summarised by the banking system.

Sector behaviour

4) Agents are grouped under sectors and countries while their transactions are also classified by market and currency.
5) The shares of income sources and expenditure destinations stay constant within the time unit but vary from year to year.
6) Agents continuously maximise their assets and minimise their liabilities while modifying their outgoings in accordance with their takings.

Mathematical specifications

7) Agent demand for revenue and supply of outlay are linearly homogeneous functions.
8) Agent income and expenditure are simultaneously determined as a vector.
9) The discount rate is an objective factor linking past, present and future.

Monetary arrangements

10) Banks issue and accept guaranteed securities or perpetual bonds at market value.
11) Banks are entitled to issue securities which the other sectors are obliged to accept.
12) The monetary authority may intervene regarding annual increase, sector allocation and relative cost (remuneration) of bank loans (deposits).

The hypotheses comprise a simple, uniform and general foundation for building and applying economic models which obey the rules of mathematics and accounting. Dealings among n sectors - markets or n countries - zones are reproduced by 2n first-order non-homogeneous linear differential equations. Mathematical models, which duplicate closed exchange networks, possess the form $\mathbf{dX/dt} = \mathbf{A}\cdot\mathbf{X} - \mathbf{K}$, where:

- **dX/dt** is a 2n x 1 column that consists of the changes of sector, country or zone monetary flows during the time unit;
- **A** is a 2n x 2n square matrix that reproduces the economic interaction of sectors, countries or zones;
- **X** is a 2n x 1 column that encompasses the twin flows of sectors, countries or zones;
- **K** is a 2n x 1 column that comprises the yearly increase and sector, country or zone allocation of bank credit per time unit;
- n is the number of interacting sectors - markets or countries - currencies.

Such economic systems operate according to the same principles regardless of degree of detail or level of aggregation. Every exchange network based on the twelve premises has a unique steady state $\mathbf{X}^* = \mathbf{A}^{-1} \cdot \mathbf{K}$, provided $|\mathbf{A}| \neq 0$. Equilibrium vector $\mathbf{X}^*$ of economic system $\mathbf{dX/dt} = \mathbf{A}\cdot\mathbf{X} - \mathbf{K}$ consists of n pairs of income and expenditure (y_i^*, e_i^*). Receipts and payments of sectors, countries and zones differ because of monetary financial operations by banking institutions. On the other hand, pairs of revenue and outlay which concern any traded good are always in perfect balance. The following external factors identify each Euclidean grid of simultaneous buying - selling and borrowing - lending:

- list of interacting sectors and markets as well as countries and currencies;
- sector, country or zone ratios of income sources and expenditure destinations;
- parameters of monetary, fiscal, exchange and other policies;
- available years of labour, pieces of output, units of liquidity, etc.;
- prevailing or average quotations of services, products, securities, etc.

Endogenous takings and outgoings comprise a vector space and thus are fully apportioned among institutional sectors and traded goods as well as sovereign countries and currency zones. Within the time unit, salaries, prices, values, etc., change autonomously because of relentless interference by trade unions, company boards, government bodies and international organisations. The steady-state turnovers of services, products and securities divided by the corresponding salaries, prices or values, yield the years of labour, pieces of output and number of consols traded per annum. Demanded quantities differ from available quantities due to inflexible quotations, real shocks and wrong policies. Consumers, producers and bankers are guided by the shortage or surplus of the economic good under their control in order to post a higher or lower quotation during the subsequent period. Behaviour coefficients and policy parameters decide whether nominal flows of sectors and markets as well as countries and zones converge on, or (worse) diverge from, equilibrium values over time.

Mathematical software is capable of exploring systems of first-order non-homogeneous linear differential equations and providing their steady state, dynamic stability and controllability characteristics. Economic models based on the twelve major propositions yield nominal flows, financial stocks and quantities demanded as functions of behaviour, policy and other exogenous parameters. In line with dynamic control theory, exchange networks consisting of stable (thus unique) and positive flows are perfectly dirigible when the numbers of targeted variables and control instruments coincide. National monetary tools comprise annual expansion, sector allocation and relative cost (remuneration) of loans (deposits) denominated in the home currency. National fiscal tools include withholding tax and subsidisation rates applied to traded goods by each sovereign government. In a world characterised by free trade and investment, local and global economic decisions influence each other because domestic and foreign incomings and outgoings are inexorably linked via international exchange rates and payment systems. The external impact of internal policies is a function of the relative size of the initiating country or currency zone. Freedom of action is constrained by functional dependence among certain instruments as well as by indispensable dynamic stability and uniform direction of endogenous flows.

Granted that quotations are autonomous within the time unit, the main factors affecting capacity utilisation relative to productive potential are the magnitudes and allocations of world liquidity expansion and world aggregate demand. Long and medium term forecasts about equilibrium positions and dynamic properties of economic systems of the general type $\mathbf{dX}_t / \mathbf{dt} = \mathbf{A}_t \cdot \mathbf{X}_t - \mathbf{K_t}$ depend on projections regarding the exogenous parameters over t number of years. Euclidean exchange networks make possible digital animations of the endogenous variables based on scenarios regarding past, present and future estimates of the autonomous factors. Activities ranging from teaching economics to multilateral negotiations may be enhanced by means of animated presentations of the consequences of various kinds of sector behaviours, official policies or other developments. Synergy between mathematical models and modern software enables economists working for international organisations, national governments and central banks to explore and propose globally beneficial policies. Within this framework, sovereign countries and currency zones may be persuaded to coordinate their monetary, fiscal and exchange policies with a view to simultaneously i) optimising their purchasing power, ii) minimising deficits and surpluses as well as iii) ameliorating inflationary and deflationary pressures worldwide.

This monograph has offered examples of systems which encompass several sectors and markets as well as countries and currencies. The ultimate aim has been to assess the method's effectiveness and to promote its acceptance. Euclidean exchange models are capable of incorporating every kind of economic variable; consequently, they are suitable for studying and solving a huge variety of real life problems. Among the theoretical and practical issues examined in this monograph are the following:

- relationships among monetary flows, quantity gaps and market quotations;
- convergence of economic systems striving to adopt a common currency;
- effects of discount and exchange rates on sector, country or zone balance sheets;
- exploration of economic history and growth through automated computations;
- advantages and disadvantages of monetary, fiscal and exchange policies;
- distribution of gains and losses arising from economic globalisation.

Understanding and implementing the twelve major propositions presupposes in-depth knowledge of economics and mathematics as well as extensive experience with accounting statistics. For the time being, diffusion and application of the new modelling methodology are hampered by shortage of worldwide data and absence of track record. Hopefully, the aforementioned and various other obstacles will be removed in the near future so as to enable mathematical economists to:

- duplicate and compare exchange networks around the world;
- forecast developments regarding sectors, markets, countries and currencies;
- facilitate worldwide dialogue about monetary, fiscal and other kinds of policies.

Scientific discoveries provide tools to those aspiring to improve human welfare. Since the 1930s, thousands of econometric, DSGE and other models have been built and tested by amateurs and professionals. Despite their acknowledged usefulness, all these simulation approaches have been questioned on several and various grounds. Exchange networks based on the twelve major propositions have made possible systematic forecasting and policy formulation. International financial organisations (such as the IMF, World Bank, ECB, etc.) are urged to develop a mathematical model of the global economy and to collect the necessary statistical data in an effort to advise i) banks on the magnitudes and allocations of their annual balance sheet increments as well as ii) governments on withholding taxes, subsidisation rates and other public policies that would optimise supply of goods and minimise change of quotations worldwide.

Alphabetic Index

One page summary

The General Method for Construction and Application of Economic Models is the revised and updated version of ***Euclidean Economics*** published in Greek by the University of Cyprus in November 2006. The monograph establishes that the science of economics can be studied and utilised on the basis of a minimum of fundamental hypotheses and the universal laws of mathematics. In order to reveal the strength of the Euclidean approach, references and bibliography are intentionally excluded. In this conjunction, in-depth knowledge of economics and extensive experience with mathematics are essential. Starting from twelve major propositions, it is illustrated that closed economies are replicated by systems of 2n first-order non-homogeneous linear differential equations, which simulate household, business, financier and government receipts and payments.

Such exchange networks, called Euclidean, are defined in the (R^{2n}, t) vector space, where n is the number of institutional sectors and corresponding markets or sovereign countries and corresponding currencies, while t is the elapsed time; they reproduce takings and outgoings as functions of behaviour coefficients and policy parameters per relevant period. Elements of irrationality or idiosyncrasy that often characterise agents are reflected by the coefficients of behaviour and the inflexibility of quotations. Division of annual turnovers of services, products and securities by average salaries, prices and values, yields the demanded years of labour, pieces of output and units of securities. Quantities demanded only fortuitously coincide with quantities available. The potential inflation or deflation rate is proportional to the weighted deficit or surplus of market goods.

The equilibrium vector's uniqueness, stability and controllability characteristics are also functions of sector, country or zone behaviour coefficients as well as monetary, fiscal and other policy parameters. Modern software enables examination of the economic system's steady state, dynamic stability and controllability properties under various scenarios. Digital animation of the endogenous variables is possible on the basis of historical, projected or hypothetical data about the exogenous variables. The monograph is intended for academic and professional economists as well as for undergraduate and postgraduate physicists, engineers and other scientists engaged in simulation theory and practice. It is also relevant to financial and government organisations interested in improving their forecasting capabilities through all-inclusive economic models.